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A
SCHOOL ARITHMETIC

BY

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AND

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AUTHORS OF "A SCHOOL GEOMETRY," "A TEXT-BOOK OF EUCLID'S ELEMENTS," ETC.

WITH ANSWERS

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PREFACE.

In the present work our object has been to provide the essentials of a School Course of Arithmetic, suited to modern requirements, and in accordance with the recommendations of the Mathematical Association.

The book is not intended for pupils who have had no previous teaching in the subject. First lessons in Arithmetic can never be efficient and adequate unless the teaching is expressed verbally, direct from teacher to pupil, and largely illustrated by means of oral exercises. Partly for this reason, and also in order to keep our work within a reasonable compass, we have not gone as fully into the details of the "First Four Rules" as is necessary for learners quite new to the subject. Our first two chapters furnish a course of revision lessons suitable for all who have a first elementary knowledge as far as Reduction. Here we have reviewed fundamental principles, and in their application we have endeavoured to enforce none but the best methods. From this point we believe our treatment will be found sufficiently full for pupils of all classes.

With regard to the succession of the different parts of the subject, there is no recognised order which would satisfy all teachers, or which would be equally serviceable for all learners. Consequently we have made no attempt to lay down a course which may be followed without deviation from beginning to end; but we believe that the order we have adopted is that which will give teachers the least trouble in finding what they want.

Our main plan has been to group together for convenience such portions of the subject as are closely allied, and to follow a natural exposition and development without resorting to needless subdivision. For example, all the essentials of Fractions are introduced and disposed of in Chapters v. and vi., and Decimals are discussed fully in Chapters viii.–xi. But it is not intended that the study of Fractions should be completed before Decimals are begun; it will probably be found convenient if the simpler parts of these two sections are read concurrently. Still less do we suggest that all that deals with Contracted Methods and Approximation in Chapters x. and xi. should be mastered before entering on some later chapters: such, for example, as those on Proportion, Percentage, and Interest. The full Table of Contents will enable teachers to map out for themselves the course best suited to their own classes.

The following special features may be mentioned:

(i) In dealing with Compound Quantities, the British and Foreign Weights and Measures are introduced and illustrated side by side. Tables which are becoming obsolete or rarely used have been excluded, while Linear and Square Measure have been simplified by the omission of all cases of reduction from poles to yards, or yards to poles.

(ii) In view of the fact that the use of symbols is no longer prohibited in examinations in Arithmetic, algebraical methods are used whenever they seem conducive to clearness and simplicity. As no previous algebraical knowledge is assumed, the necessary processes are explained simply, as occasion requires.

(iii) In Fractions very frequent use is made of diagrams for the purpose of illustration.

(iv) Decimals and the Metric System are discussed together, metric weights and measures being constantly used in illustration of decimal notation.

(v) Great care has been bestowed on the sections dealing with Decimal Approximation. We believe that in this part of the work we have gone further than has hitherto been attempted in an English school book.

(vi) The Method of Aliquot Parts (or 'Practice') is not made the subject of a separate chapter, but arises naturally as a convenient form of multiplication in connection with Fractions and Decimals. See pages 126–128, 213–215.

(vii) The principles of Ratio, Proportion, and Percentage are introduced in a simple manner in connection with Fractions (see pages 121–126), and are more fully developed in later sections.

(viii) Though the Unitary Method is dealt with in an early chapter, it gives way later to more direct and scientific methods in all questions depending on Proportion.

(ix) A chapter is given explaining all that is necessary for a clear understanding of Simple Graphs. In subsequent chapters graphical methods are frequently used for the purpose of illustration—especially in connection with Proportion and Variation. See pages 307–315, 377–380, 448–453.

(x) The use of Four-Figure Logarithms is fully explained. Tables of Logarithms and Antilogarithms are supplied, together with a large variety of examples for practice in logarithmic work.

(xi) We have given no formal treatment of Recurring Decimals, or of Cube Root. Such cases of cube root as occur in the examples can be treated by factors or by the use of logarithms.

(xii) A short section on the properties of Continued Fractions has been given in Chapter xx., in order to meet the requirements of some of the harder Civil Service Examinations.

We offer no apology for the extent to which we have made use of easy algebraical methods. The time is probably not far distant when Arithmetic and the easier parts of Algebra will be taught together, and correlated in examination papers.

It is a matter of common experience that a mechanical use of graphs can be attained without any clear notions of the principles upon which graphical solutions depend; pupils are apt to be content with the success of such methods without really knowing why they succeed. We hope that our introduction to graphs in Chapter xv. will be found to put the subject on a sound and satisfactory basis. Until a pupil can grasp the idea of two connected variables there is no tangible principle upon which a study of graphs can be founded, but the Algebra necessary for enforcing this principle is so simple that it ought not to deter any intelligent reader.

Considerable care has been bestowed on the selection and classification of the examples. In each exercise it will be found that the initial examples are usually quite straightforward and easy, so that pupils may get the requisite practice in new principles and processes without being deterred by purely arithmetical detail. It is hoped that the numerous oral examples in the early chapters (and especially in Fractions and Decimals) may do something to prevent a mechanical application of rules before the notation and first principles are thoroughly grasped.

Those of the examples which are not original have been taken mainly from examination papers of recent date. We are indebted to several examining bodies for permission to use examples from their papers. In particular, our thanks are due to the Oxford Delegacy and the Cambridge Syndicate in regard to questions set in the University Local Examinations; and to the Controller of H.M. Stationery Office for the use of examination questions set by the Civil Service Commissioners. The Examinations under the Civil Service Commission have for some time past been noted for examples of fresh and instructive types, and we have thought it worth while to indicate such examples in our pages by a distinctive mark.

Nearly all the answers have been checked by working the examples from the proof sheets. We gratefully acknowledge much valuable help in this work of verification, and we venture to hope that any remaining errors will prove to be few and unimportant.

H. S. HALL.
F. H. STEVENS.

May 1908.

CONTENTS.

PART I.

Chapter I.

PAGE

Revision of the Four Rules for Simple Quantities - - 1

Addition (pp. 1, 2). Subtraction, Complementary Addition (pp. 2-4). Multiplication, Verification, Casting out the Nines (pp. 5-8). Division, Italian Method, Partition, Quotition (pp. 9–11).

Miscellaneous Examples I. Simple Quantities - - - 13

Chapter II.

Compound Quantities - - - - - - - - - 16

Tables, British and Foreign. Money (pp. 16–30). Tables of Length (pp. 30–34). Tables of Weight and Capacity (pp. 35–37). Tables of Square Measure (pp. 38–42). Land Measure (pp. 42–44). Tables of Cubic Measure (pp. 44–47).

Chapter III.

Use of Signs and Symbols - - - - - - - - 48

Introductory Definitions and Notation (pp. 48-51). Negative Quantities (pp. 52–54). Use of Brackets, Multiplication Formulæ (pp. 54-58). Illustrations leading up to Equations (p. 59). Equations, Simple Applications (pp. 60-63).

Chapter IV.

Factors. Prime Numbers. Highest Common Factor. Lowest Common Multiple - - - - - - - - - 64

Factors, Prime Numbers (p. 64). Tests of Divisibility (pp. 65-68). Resolution into Prime Factors (pp. 68–70). Square and Cube Root by Factors (pp. 71–73). Highest Common Factor (pp. 73–77). Lowest Common Multiple (pp. 77–80).

Chapter V.

PAGE
FRACTIONS - - - - - - - - - - - 81

Definitions and Oral Examples (pp. 81-84). Reduction to Lowest Terms (pp. 86-88). Proper and Improper Fractions, Mixed Numbers (pp. 88, 89). Comparison of Fractions (pp. 90, 91). Addition and Subtraction of Fractions (pp. 91-98). Multiplication and Division by an Integer, Graphical Illustrations (pp. 98-102). Multiplication by a Fraction, Compound Fractions, Graphical Illustrations (pp. 102-109). Division by a Fraction (pp. 109-111). Combined Processes, Complex Fractions (pp. 112-115). Fractional Equations (pp. 115-117).

Chapter VI.

FRACTIONS CONTINUED. COMPOUND QUANTITIES - - - - 118

Fractions of Concrete Quantities (pp. 118-121). Ratio, Proportion, Ratios as Percentages (pp. 121-126). Aliquot Parts, Practice (pp. 126-128).

Miscellaneous Examples II. - - - - - - - 129

Chapter VII.

THE UNITARY METHOD - - - - - - - - - 135

Chapter VIII.

DECIMALS - - - - - - - - - - - 143

Numeration and Notation, Oral Examples (pp. 142-146). Place-Value illustrated by the Metric Table of Length (pp. 146-148). Multiplication and Division by Powers of Ten, Standard Form (pp. 148-152). Illustrations from Metric Square Measure, Cubic Measure, Capacity, Weight (pp. 152-155). Addition and Subtraction (pp. 156-158). Multiplication (pp. 158-163). Division (pp. 164-170). Decimals and Fractions (pp. 170-173).

Chapter IX.

DECIMAL REDUCTION OF COMPOUND QUANTITIES - - - - 174

Illustrative Examples (pp. 174-178). Decimalization of Money at sight (pp. 178-182).

Chapter X.

PAGE

Decimal Approximations - - - - - - - - 183

Significant Figures, Absolute Error, Relative Error, Percentage Error (pp. 183-188). Contracted Addition and Subtraction (pp. 188-190). Contracted Multiplication (pp. 190-193). Contracted Division (pp. 194-198). Further Examples on Contracted Work (pp. 198-201). Approximate Data, Limits of Errors (pp. 201-208).

Chapter XI.

SOME APPLICATIONS OF DECIMAL PROCESSES - - - - - 209

Decimals of Compound Quantities, Ratios, Percentages (pp. 209-213). Practice Decimalized (pp. 213-216). Further Applications, Formulæ (pp. 216-221).

Chapter XII.

EASY PROBLEMS - - - - - - - - - - - 222

Examples on Work and Time (pp. 222-224). Examples on Time and Distance (pp. 224-227). Averages (pp. 228, 229). Miscellaneous Examples (pp. 230-236).

Miscellaneous Examples III. - - - - - - - 237

PART II.

Chapter XIII.

SQUARE ROOT - - - - - - - - - - - 241

Square Root of Integers (pp. 241-244). Square Root of Decimals, Approximate Square Roots (pp. 245-249). Surds (pp. 249, 250). Miscellaneous Examples on Square Root (pp. 251-253).

Chapter XIV.

PAGE

AREAS AND VOLUMES - - - - - - - - - - 254

Rectangular Areas (pp. 254–259). Carpeting Floors, Papering Walls (pp. 260–263). Miscellaneous Examples on Areas (pp. 264–266). Cubes and Cuboids (pp. 267–270). Specific Gravity, Weight, Miscellaneous Examples on Volumes (pp. 271–274).

Chapter XV.

GRAPHS - - - - - - - - - - - - 275

Axes, Coordinates, Plotting Points (pp. 275–279). Definition of a Graph, Graph of a Straight Line (pp. 279–283). Graphs of Statistics, Practical Applications (pp. 284–291).

Chapter XVI.

RATIO AND PROPORTION - - - - - - - - 292

Definitions, Easy Examples (pp. 292–295). Simple Proportion, Variation (pp. 295–306). Tests of Proportionality, Graphical Illustrations (pp. 307–315). Compound Proportion (pp. 315–318). Proportional Division (pp. 319–323). Miscellaneous Applications of Proportion and Variation (pp. 324–328).

Miscellaneous Examples IV. EXERCISES FOR REVISION - 329

Chapter XVII.

PERCENTAGES - - - - - - - - - - - 339

Definitions, Examples on Percentages (pp. 339–344). Profit and Loss (pp. 345–351). Miscellaneous Examples on Percentages (pp. 351–354).

Chapter XVIII.

INTEREST AND DISCOUNT - - - - - - - - 355

Simple Interest (pp. 355–360). Inverse Cases of Simple Interest (pp. 361–365). Discount, Bills of Exchange (pp. 366–370). Compound Interest (pp. 370–374). Inverse Cases, Miscellaneous Examples on Interest and Discount (pp. 374–377). Graphical Illustrations (pp. 377–380).

Chapter XIX.

PAGE

STOCKS AND SHARES - - - - - - - - - - 381

Definitions, Examples on Stocks (pp. 381–388). Brokerage, Examples involving Brokerage (pp. 389–393). Shares (pp. 393–395). Miscellaneous Examples on Stocks and Shares (pp. 396–398).

Miscellaneous Examples V. EXERCISES FOR REVISION - 399

Chapter XX.

CONTINUED FRACTIONS. FURTHER NOTES ON APPROXIMATE METHODS - - - - - - - - - - 409

Simplification of Continued Fractions (pp. 409, 410). Properties of Continued Fractions, Convergents (pp. 411–415). Approximation by Aliquot Parts, Reduction Formulæ, Evaluation of Series (pp. 415–419). Further Notes on Decimal Approximation (pp. 419–425).

Chapter XXI.

LOGARITHMS - - - - - - - - - - - 426

Definitions, Laws of Indices, General Properties of Logarithms (pp. 426–432). Common Logarithms (pp. 432–435). Use of Four-Figure Tables (pp. 436–440). Applications of Logarithms (pp. 440–443). Tables of Logarithms and Antilogarithms (pp. 444–447).

Miscellaneous Examples VI. - - - - - - - 448

Answers.

PART I.

CHAPTER I.

INTRODUCTORY.

Revision of the Four Rules for Simple Quantities.

1. It is assumed that pupils who use this book will have learnt the rules for Addition, Subtraction, Multiplication, and Division for both Simple and Compound Quantities. Consequently no formal explanation of these rules will be given. This introductory chapter will be devoted to enforcing some points of method and arrangement in connection with abstract numbers and simple quantities. It will also be assumed that the pupil is acquainted with the meanings of the signs +, −, ×, ÷ ; the symbols =, ∴ ; and the simplest use of brackets ( ).

2. **Addition.** The only way to secure rapid work in addition is to dispense with all that is not actually necessary in the mental calculation.

Suppose it is required to add the series of numbers in the margin. It is undesirable to say mentally 1 and 8 are **9**; 9 and 7 are **16**; 16 and 7 are **23**, and so on.

All that is really necessary is as follows:

1st column. 9, 16, 23, 28, 31. Set down **1** and carry the 3 tens to the next column.

2nd column. 12, 14, 20, 29, 34, 42. Set down **2** and carry 4.

3rd column. 10, 18, 22, **29**.

$$\begin{array}{r} 783 \\ 55 \\ 497 \\ 867 \\ 628 \\ 91 \\ \hline 2921 \end{array}$$

This process can be still further shortened when the eye is trained to combine two or more numbers in a single step at any part of a column. This is especially useful when a pair or more of such numbers combine to make up 10.

For example, to add the numbers given in the left-hand margin. In the right-hand margin the way in which the numbers are grouped together is shewn by brackets.

MENTAL WORK.

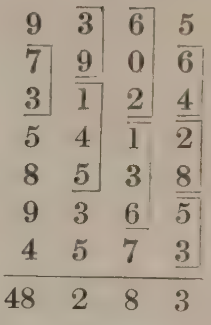
9365
7906
3124
5412
8538
9365
4573
48283

1st col. 8, 18, 28, 33. Set down **3** and carry 3.

2nd col. 10, 20, 28. Set down **8** and carry 2.

3rd col. 7, 10, 20, 32. Set down **2** and carry 3.

4th col. 7, 16, 24, 29, 39, **48**.



To check the accuracy of addition the columns should be added from top to bottom as well as from bottom to top.

3. It is very useful to be able to add a series of numbers horizontally from left to right, or from right to left, without placing them in column. The process is the same as before, but it is less easy to group two or more numbers together. Beginners are recommended not to attempt such grouping, or to confine the grouping to pairs which make up 10.

EXAMPLE. *Add together the following numbers:*

3124, 6835, 4787, 9138.

Working from right to left, the mental process is

15, 20, **24**; 5, 13, 16, **18**; 2, 9, 17, **18**; 10, 20, **23**;

and the answer is 23884, the figures in deeper type being those which are successively written down, beginning with that in the units' place.

4. Subtraction. Suppose it is required to subtract 279 from 751. Probably most beginners go through the following mental process:

9 from 11, **2**; 8 from 15, **7**; 3 from 7, **4**.

751
279
472

But very few can give the reason for this process, or explain clearly why the 1 in the first step is treated as 11, or the 7 in the second as 8.

The method depends upon the following principle:

The difference between any two numbers is unaltered if each of them is increased by the same amount.

Hence, when we find that 9 cannot be taken from 1, we increase the 1 by 10 *units* (making 11). At the same time we add a compensating *ten* to the lower number *in the tens' column* (making the 7 *tens* into 8 *tens*).

Again, since 8 *tens* cannot be taken from 5 *tens* we increase the 5 *tens* by 10 *tens* (making 15 tens). At the same time we add a compensating *hundred* to the lower number *in the hundreds'* column (making the 2 *hundreds* into 3 *hundreds*).

5. Another method of subtraction known as **Complementary Addition** has great advantages and should be acquired at the outset. In the foregoing example we found the difference between 751 and 279. Now this difference may be regarded as the number which must be *added to* 279 in order *to make up* 751.

Hence instead of saying '9 from 11,' we mentally ask what must be *added* to 9 to make 11; then what must be added to 8 (carrying 1) to make 15, and so on.

The mental process will then be

$$\begin{array}{r} 751 \\ 279 \\ \hline 472 \end{array}$$

9 and **2**, 11; 8 and **7**, 15; 3 and **4**, 7;

the digits in deeper type being those which are successively written down in the result.

This method is sometimes referred to as the **Austrian method,** and in some subsequent rules is indispensable for rapid work. We give two more examples to illustrate its use.

EXAMPLE 1. *Subtract* 3265 *from* 6182.

MENTAL WORK.

5 and **7**, 12. Set down 7, and carry 1 to the tens' column of the lower number.

7 and **1**, 8. Set down 1; (nothing to carry).

2 and **9**, 11. Set down 9, and carry 1.

4 and **2**, 6. Set down 2.

$$\begin{array}{r} 6182 \\ 3265 \\ \hline 2917 \end{array}$$

EXAMPLE 2. *Subtract the sum of* 2346, 4658, 3245, *and* 1026 *from* 13682.

Write down the last number and draw a line under it; then place the other numbers under the line. Then as in previous examples, we have

11, 19, 25, and **7**, 32. Set down 7 and carry 3.

5, 9, 14, 18, and **0**, 18. Set down 0 and carry 1.

3, 9, 12, and **4**, 16. Set down 4 and carry 1.

2, 5, 9, 11, and **2**, 13. Set down 2.

$$\begin{array}{r} 13682 \\ \hline 2346 \\ 4658 \\ 3245 \\ 1026 \\ \hline 2407 \end{array}$$

To check the result, now add the numbers in the five lower rows and the sum should be the number in the top row.

6. The complementary method is also useful in subtraction of money. [See Chap. II., Art. 23.] It is, in fact, nothing more than the usual 'shop method' of giving change. Suppose a purchaser buys an article worth $8\frac{1}{2}d.$ and tenders a half-crown as payment. The shopkeeper would probably give

a halfpenny, a threepenny piece, a shilling, a sixpence,

saying as he put down the several coins :

'**9** pence,' '**1** shilling,' '**2** shillings,' '**half a crown.**'

He thus *makes up* to even money at each stage.

The following Examples will furnish practice in the Austrian method of subtraction.

EXAMPLES I. a.

1. Subtract (i) 3289 from 7291 ; (ii) 43702 from 90000.

2. The sum of two numbers is 2735 and the less is 857 ; what is the greater ?

3. From 529 take the sum of 27, 39, 47.

4. In a school of 250 boys, four Houses contain respectively 60, 45, 42, 37 boys ; how many day-boys are there ?

5. Subtract the sum of 2034, 4316, 3184 from 21351.

6. In a battle 10,000 men were engaged ; of these 227 were killed, 563 were wounded, and 87 were taken prisoners. How many returned to camp unwounded ?

7. Find the value of 75683 – 2317 – 4184 – 1609.

8. Find the difference between 98469 and the sum of 12065, 235, 4837, 62038.

9. A country has an area of 32,300,000 acres ; it consists of 11,425,000 acres of arable land, 15,237,000 acres of meadow, 2,975,250 acres of waste land, and the rest is water. How many acres of water are there ?

Write down the number in the missing line in each of the following examples in addition :

10.
2971
4832
60791
*****
83050

11.
45907
38652
70945
*****
170000

12.
583
31276
8975
*****
68732

7. Multiplication. Suppose it is required to multiply 3297 by 628. This means that we have to find the value of 3297 taken 628 times. Now the multiplier $628 = 600 + 20 + 8$, so that we have to take the multiplicand 600 times, 20 times, and 8 times, and then add the results. In other words we have to multiply 3297 by 6 hundreds, 2 tens, 8 units, and then add the partial products.

In theory the order in which these separate multiplications is performed is immaterial, but there are great advantages in keeping the order above indicated, that is, *beginning with the figure of highest place-value in the multiplier*; in the present case hundreds before tens, tens before units.

The work will stand as follows:

$$\begin{array}{r@{\;=\;}l}
3297 & \\
629 & \\
\hline
1978200 & 3297 \times 600 \\
65940 & 3297 \times 20 \\
29673 & 3297 \times 9 \\
\hline
2073813 & 3297 \times 629
\end{array}$$

Or more briefly thus:

$$\begin{array}{r}
3297 \\
629 \\
\hline
19782 \\
6594 \\
29673 \\
\hline
2073813
\end{array}$$

Place the numbers so that units, tens, hundreds of the multiplier are respectively under units, tens, hundreds of the multiplicand. The several partial products and their sum are shewn in the work on the left-hand side.

The ciphers may be omitted as in the arrangement on the right-hand side if it is remembered that *in each line of work the first figure is set down immediately below the figure of the multiplier which is being used to obtain the partial product.* The other figures then fall into their proper places naturally.

NOTE. Though the order of the successive lines of work may be varied without altering the final result, it should be noticed that the order here recommended has the advantage of giving the partial products *in the order of their importance.* Thus in the above example the first line gives a rough approximation to the result required; the first and second taken together give another approximation closer than the first; while the first, second, and third combine to give the correct product. In forming such approximations care must be taken not to omit ciphers at the end.

$$\begin{array}{r}
12567 \\
4325 \\
\hline
50268000 \\
3770100 \\
\hline
54038100
\end{array}$$

Thus a first approximation to the product of 12567 and 4325 is 50268000 - - - - - - - - 50268000

And a second approximation is 54038100 - - - 54038100

8. Some **special artifices** for shortening work may here b noticed.

To multiply by 5. Since $10=5\times 2$, the required result will b obtained by multiplying by 10 and dividing the result by 2. Tha is, we have only to place *one* cipher to the right of the multiplican and divide by 2. The process is entirely mental.

To multiply by 25. Since $100=25\times 4$, we have only to place *two* ciphers to the right of the multiplicand and divide by 4.

To multiply by 125. Since $1000=125\times 8$, we have only to place *three* ciphers to the right of the multiplicand and divide by 8.

NOTE. The converse processes for division by 5, 25, 125 may also be used, but when the division is not exact, beginners will find the remainders puzzling.

9. The product of three or more numbers is called their **continued product**; and each number is called a **factor** of the product. The factors may be taken in any order.

EXAMPLE. *Find the value of the following continued products:*

(i) $125\times 650\times 8\times 2$. (ii) $4\times 2\times 76\times 50\times 25\times 15$.

(i) By changing the order of the factors,

$$\text{the product}=650\times 2\times 125\times 8=1300\times 1000=1300000.$$

(ii)
$$\begin{aligned}\text{The product}&=76\times 50\times 2\times 25\times 4\times 15\\&=76\times 100\times 100\times 15=760000\times 15\\&=11400000.\end{aligned}$$

The last result is obtained by halving the multiplicand and multiplying by 30.

10. The following examples shew how the work of multiplication may be abbreviated in special cases.

EXAMPLE. *Multiply* 7342 (i) *by* 99; (ii) *by* 998.

(i)
$$\begin{array}{r}734200\\7342\\\hline 726858\end{array}$$

Here $99=100-1$. Hence we have only to subtract the multiplicand from the multiplicand $\times$ 100.

(ii)
$$\begin{array}{r}7342000\\14684\\\hline 7327316\end{array}$$

Since $998=1000-2$, we place *three* ciphers after the multiplicand and subtract twice the multiplicand from the number so obtained.

A similar method may be used for multipliers such as 59, 78, 87, which differ very slightly from *some multiple of* 10.

11. It will sometimes be found that the digits of the multiplier suggest an order and arrangement which will shorten the work.

EXAMPLE. *Multiply* 3297 *by* 639.

$$\begin{array}{r} 3297 \\ 639 \\ \hline 29673 \\ 207711 \\ \hline 2106783 \end{array}$$

Here $639 = 9 + 630 = 9 + 9 \times 70$. Hence in this case it is convenient to form the first partial product by beginning with the unit figure 9. The next partial product is 70 times the first. That is, we have only to multiply the first product by 7 tens, and the first resulting figure will be set down in the tens' place.

12. **Verification.** The best way of testing the accuracy of a result in multiplication is the method known as "casting out the nines." We shall give an example of the use of this rule, deferring the explanation of the principle on which it depends to a later chapter.

To 'cast out nines' from a number such as 4$\cancel{9}$73$\cancel{9}$68, we add up the digits dropping every 9, and the sum of two or more digits which make up 9. Also whenever the sum is more than 9, we subtract 9, and go on with the remainder.

Thus first crossing out the two digits 9 in the number, we have counting from the right, 14 (subtract 9), **5**, 8, 15 (subtract 9), **6**, 10 (subtract 9), and **1** is left.

EXAMPLE. *Multiply* 40397 *by* 5684 *and test the accuracy of the result by casting out nines.*

$$\begin{array}{r} 40397 \\ 5684 \\ \hline 201985 \\ 242382 \\ 323176 \\ 161588 \\ \hline 229616548 \end{array}$$

Cast out nines from multiplicand and multiplier.

$\left.\begin{array}{l} 7,\ 10,\ 1,\ \mathbf{5} \\ 12,\ 3,\ \cancel{9},\ \mathbf{5} \end{array}\right\}$ Multiply the two remainders and cast out nines: 25, **7**.

Now cast out nines from the product.

12, 3, 8, 14, 5, 6, 12, 3, 5, **7**.

If the two remainders obtained by this process are different, the work is certainly wrong; if they agree, the result is *probably* correct; but the rule does not give an absolute test of accuracy. For example, an interchange of two or more digits in the complete product would vitiate the answer, without altering the remainder on casting out nines.

13. The following example illustrates a method of combining multiplication and subtraction *in a single operation.* The method is of great importance and will be frequently used.

EXAMPLE. *Multiply* 48327 *by* 3 *and subtract the result from* 162615.

WORK IN FULL.
48327 | 162615
144981
17634

ABBREVIATED WORK.
48327 | 162615
17634

The process is as follows :
3 sevens, 21, and **4**, 25. Set down **4**, and carry 2;
3 twos, 6, 8, and **3**, 11. Set down **3**, and carry 1;
3 threes, 9, 10, and **6**, 16. Set down **6**, and carry 1;
3 eights, 24, 25, and **7**, 32. Set down **7**, and carry 3;
3 fours, 12, 15, and **1**, 16. Set down **1**.

Addition may, of course, be combined with multiplication in the same way, but the combination is less useful than that illustrated above.

EXAMPLES I. b.

Find the value of the following products, in each case beginning with the left-hand digit of the multiplier. Test the results by casting out nines.

1. 247 × 35.
2. 1032 × 324.
3. 2083 × 619.
4. 6234 × 836.
5. 91253 × 478.
6. 8132 × 2561.
7. 50327 × 7030.
8. 43097 × 8203.
9. 79306 × 8009.

Write down a first approximation to the following products. (See Art. 7, NOTE.)

10. 6345 × 74.
11. 2893 × 821.
12. 4385 × 6301.

Multiply, as shortly as you can,

13. 4275 by 5, 25, 125.
14. 96084 by 5, 25, 125.
15. 3652 by 98, 999.
16. 84703 by 99, 59, 798.

17. Shew that to multiply by 625 we may affix 4 ciphers to the multiplicand and then divide the result by 16.

[*In Examples* 18–29 *each is to be worked in a single step.*]

18. Multiply 256 by 3, and subtract the result from 980.
19. Multiply 478 by 5, ,, ,, ,, 2583.
20. Multiply 4036 by 4, ,, ,, ,, 20000.
21. Multiply 6871 by 6, ,, ,, ,, 43127.
22. Multiply 9386 by 8, ,, ,, ,, 102364.
23. Multiply 5376 by 9, and add the result to 357.

Write down the result of

24. $58356 - 9240 \times 6$.

25. $864083 - 107920 \times 8$.

26. $97832 - 13938 \times 7$.

27. $100000 - 14832 \times 6$.

28. $4832 \times 8 + 1236$.

29. $57345 \times 6 + 24302$.

30. Multiply (using only two partial products)

(i) 69432 by 567 ; (ii) 803050 by 9036.

14. Division. In short division beginners do not usually find much difficulty except with regard to remainders when the divisor is the product of two or more factors. The examples which follow are given in order to explain how the remainders are dealt with, and to give the pupil models for reference.

EXAMPLE 1. *Divide* 67441 *by* 56, *or* 7×8.

$$\begin{array}{r|l} 7 & 67441 \\ \hline 8 & \;\;9634 \text{ groups of } 7\text{, and } 3 \textit{ units} \text{ over.} \\ \hline & \;\;\;1204 \text{ groups of } 56\text{, and } 2 \textit{ sevens} \text{ over.} \end{array}$$

$$\begin{array}{r|l} 7 & 67441 \\ \hline 8 & \;\;9634 \ldots 3 \\ \hline & \;\;1204 \ldots 2 \end{array}\Big\} 17$$

Complete remainder $= 2$ sevens $+ 3$ units $= 7 \times 2 + 3 = 17$.

In practice the only work necessary is that shewn on the right.

EXAMPLE 2. *Divide* 92467 *by* 168, *or* $4 \times 6 \times 7$.

$$\begin{array}{r|l} 4 & 92467 \\ \hline 6 & 23116 \text{ groups of } 4\text{, and } 3 \textit{ units} \text{ over.} \\ \hline 7 & \;\;3852 \text{ groups of } 24\text{, and } 4 \textit{ fours} \text{ over.} \\ \hline & \;\;\;\;550 \text{ groups of } 168\text{, and } 2 \textit{ twenty-fours} \text{ over.} \end{array}$$

$$\left.\begin{array}{r|l} 4 & 92467 \\ \hline 6 & 23116 \ldots 3 \\ \hline 7 & \;\;3852 \ldots 4 \\ \hline & \;\;\;\;550 \ldots 2 \end{array}\right\} 67$$

Complete remainder $= 2$ twenty-fours $+ 4$ fours $+ 3$ units

$$= 24 \times 2 + 4 \times 4 + 3$$

$$= 67.$$

It will be seen that the following rule is applicable in each case :

Multiply each partial remainder by all the divisors preceding its own, and add the results.

15. In Long Division the pupil is recommended to learn and practice the **Italian Method**, in which at each stage of the work corresponding to a digit in the quotient, the multiplication and subtraction are combined in a single process as explained in Art. 13. The following example exhibits the work thus abbreviated side by side with the ordinary full working.

EXAMPLE. *Divide* 784946 *by* 358.

WORK IN FULL.

$$\begin{array}{r}
2192 \\
358\,)\,784946 \\
\underline{716} \\
689 \\
\underline{358} \\
3314 \\
\underline{3222} \\
926 \\
\underline{716} \\
210
\end{array}$$

ITALIAN METHOD.

$$\begin{array}{r}
2192\phantom{\text{ remainder.}} \\
358\,)\,784946\phantom{\text{ remainder.}} \\
\underline{689}\phantom{\text{ remainder.}} \\
\underline{3314}\phantom{\text{ remainder.}} \\
\underline{926}\phantom{\text{ remainder.}} \\
210\text{ remainder.}
\end{array}$$

Quotient = 2192,
remainder = 210.

To perform the 1st stage of division we shall require 3 digits of the dividend; thus the first digit of the quotient will represent thousands, and it is convenient to place it in the place of thousands, that is *over the last digit of the dividend used in the 1st stage.* The other digits of the quotient will then follow in their proper places. The mental work is as follows:

2 eights, 16, and **8**, 24; carry 2. 2 fives, 10, 12, and **6**, 18; carry 1. 2 threes, 6, 7. Bring down 9 from the dividend.

1 eight, 8, and **1**, 9; carry 0. 1 five, 5, and **3**, 8; carry 0.

1 three, 3, and **3**, 6. Bring down 4 from the dividend, and continue the process.

16. When Division is *exact*, the Dividend, Divisor, and Quotient are connected by the relation

$$\textit{Dividend} \div \textit{Divisor} = \textit{Quotient},$$

or

$$\textit{Dividend} = \textit{Divisor} \times \textit{Quotient}.$$

When Division is *not exact*, the relation is

$$\textit{Dividend} = (\textit{Divisor} \times \textit{Quotient}) + \textit{Remainder}.$$

Or using symbols, if a number N is divided by a divisor D, with quotient Q, and remainder R

$$N = D \times Q + R$$

from which result any one of the four quantities can be determined when the numerical values of the other three are known. Such a general relation is called a **formula**.

EXAMPLE. *The number* 217 *when divided by a certain number has a quotient* 16 *and remainder* 9; *what is the divisor?*

Since there is a remainder 9 on division, it follows that 217 is greater by 9 than 16 times the divisor. That is, 208 is 16 times the divisor, hence the divisor is 13.

17. Two distinct questions are answered by division of one number by another. For instance

(i) *If £493 is divided into 29 equal parts what is* **the value of each part?**

Here $£493 \div 29 = £17.$

The divisor is *abstract*, and the quotient *concrete* (being of the same denomination as the dividend). This is called **partition.**

(ii) *If £493 is divided into shares of £29 each,* **how many** *such shares are there?*

Here $£493 \div £29 = 17.$

The divisor is *concrete* and the quotient *abstract*. This is called **quotition.**

In brief, **partition** answers the question "What is the value of each *part*?"

Quotition answers the question "*How many times?*"

EXAMPLES I. c.

In Examples 1–12 find the quotient and remainder, using short division.

1. $3267 \div 42.$ **2.** $4248 \div 56.$ **3.** $1373 \div 72.$

4. $8950 \div 63.$ **5.** $7356 \div 49.$ **6.** $56703 \div 54.$

7. $4785 \div 108.$ **8.** $8412 \div 144.$ **9.** $3207 \div 128.$

10. $15831 \div 105.$ **11.** $6320 \div (7 \times 8 \times 11).$ **12.** $9873 \div (5 \times 9 \times 11).$

Perform the following divisions by the Italian method.

13. $6099 \div 19.$ **14.** $2495 \div 23.$ **15.** $8593 \div 41.$

16. $86765 \div 37.$ **17.** $22919 \div 53.$ **18.** $93802 \div 91.$

19. $103672 \div 119.$ **20.** $141431 \div 233.$ **21.** $177399 \div 207.$

22. $136160 \div 471.$ **23.** $710580 \div 593.$ **24.** $876543 \div 123.$

18. In the foregoing pages the following fundamental principles of Arithmetic have been assumed without formal proof or illustration:

(i) *Additions and Subtractions may be made in any order, so long as the subtractions are possible.*

Thus $15+8-6=8+15-6=15-6+8=8-6+15.$

Or, more generally, if we use letters to denote numbers,

$$a+b-c=b+a-c=a-c+b=b-c+a.$$

(ii) *The factors of a product may be taken in any order.*

Thus $5\times7=7\times5$; $8\times9=9\times8$.

Similarly $5\times7\times3=7\times5\times3=7\times3\times5$.

(iii) *When a number consisting of several parts is to be multiplied by another number, the result may be obtained by multiplying each of the parts of the multiplicand separately, and then taking the sum of the partial products.*

Thus $(4+3)\times5=4\times5+3\times5$.

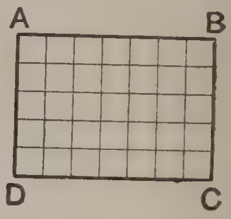
Similarly $(4+3+7)\times5=4\times5+3\times5+7\times5$;

and $(4-3+7)\times5=4\times5-3\times5+7\times5$.

Of these (i) may be accepted without proof.

The reason for (ii) and (iii) may be thus explained.

(ii) *To prove that* $5\times7=7\times5$.



In the adjoining diagram let each square represent a unit. Then the rectangle ABCD contains 35 units.

There are 7 columns each containing 5 squares, and 5 rows each containing 7 squares.

If we add up by columns the number of squares in ABCD is 5 multiplied by 7.

If we add up by rows the number of squares in ABCD is 7 multiplied by 5.

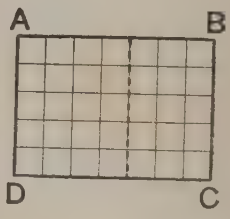
Thus $5\times7=7\times5$.

And since the reasoning is quite general, we may say

$$a\times b=b\times a,$$

where a and b denote any two numbers.

(iii) *To prove that* $(4+3)\times5=4\times5+3\times5$.



In the adjoining diagram the rectangle ABCD has 4+3 squares in each row, and there are 5 rows.

∴ the total number of squares $=(4+3)\times5$.

Again the dotted line divides the rectangle into two smaller ones containing 4 squares and 3 squares in a row respectively; and in each rectangle there are 5 rows.

∴ the total number of squares $=4\times5+3\times5$.

Thus $(4+3)\times5=4\times5+3\times5$.

Or, generally, $(a+b)\times c=a\times c+b\times c$.

MISCELLANEOUS EXAMPLES I.

SIMPLE QUANTITIES.

1. Add together ninety-nine millions; nine thousand and ten; seventy thousand and eighty; eight hundred thousand and nine; six hundred and eighty thousand and twenty-one. State in words by how much the sum falls short of two hundred millions.

2. What number taken from seventy-two thousand, six hundred and seventeen will leave forty thousand, six hundred and three?

3. The populations of the different parishes and districts of a large town are 16640, 321, 3750, 3906, 5144, 2684, 13360, 391, and 5797 respectively: by how many does the whole town's population fall short of a million?

4. A man leaves £17875 amongst his six sons. If each of the five younger sons receives £2500, what is the portion of the eldest?

5. A man leaves £30,000; to his wife he gives £10,500, to his eldest son £4000, and to his other sons and daughters £8750. There are also two debts of £3036 and £287 to be paid. If the rest of his property is left to charitable objects, find the amount so left.

6. Find the sum, difference, product, and quotient of two dozen dozen and half a dozen dozen.

7. How many times must 257 be added to 3785 so that the sum may be 7126?

8. How many times must 332 be subtracted from 18944 so that the difference may be 5000?

9. One number is greater than another by 155, and their sum is 547: find them.

10. Divide 500 into two parts whose difference shall be 26.

11. A firm of three partners earns £4500 a year, and their expenses amount to £2064. If the profits are equally divided, what is the share of each?

12. A greengrocer has 500 oranges. He has orders for 200, 175, 90, and 5 dozen. How many more must he buy to complete his orders?

13. In a page of a newspaper there are 7 columns, and in each column 172 lines, and in each line an average of 50 letters. How many letters go to a page?

14. Find the sum of 7 consecutive odd numbers of which 27 is the fourth.

15. Write down 12 consecutive numbers beginning with 35, and find the sum of the *even* numbers in the series.

16. Two horses and a cart cost £52, and each of the horses cost 6 times as much as the cart; what did each horse cost?

17. Three hats and two caps cost 22*s.* and each hat cost 3 times as much as a cap; find the price of a hat and cap respectively.

18. A father and son together earn 36 shillings a week. If the father earns 14 shillings more than the son, find the wages of each.

19. Multiply 203547 by 567 in two lines. [See Art. 11.]

20. Multiply 357895 by 9998 as shortly as possible.

21. Divide 643281 by 512 and give the remainder, using factors and short division.

22. Find the number next above 987654321 which is exactly divisible by 164609.

23. What is the nearest whole number to one million which is exactly divisible by 234?

24. How much must be added to five hundred thousand in order that the sum may be divisible by 117?

25. In a division sum the quotient is 420, the divisor is 7564, and the remainder 5199; find the dividend. [See Art. 16.]

26. Find the quotient when the dividend is 1021357, the divisor 2035, and the remainder 1822.

27. What number is the same multiple of 7 that 43120 is of 55?

28. Multiply 31729 (i) by 41696; (ii) by 52575, each in three lines.

29. After filling 71 casks from a tank, 15 gallons remained; if each cask contained 18 gallons, what did the tank hold at first?

30. *Write down* the value of (i) $4 \times 737 \times 25$;
(ii) $8 \times 625 \times 50$.

31. *Write down* the quotient and remainder when 3785036 is divided by the product of 25 and 40.

32. What is the least number which must be added to 2486132 in order that the sum may be exactly divisible by 4125?

33. What is the least number which must be subtracted from 28241 in order that the remainder may be exactly divisible by 3472?

34. *Write down* the product of 555666778 and 999.

35. *Write down* a number such that if multiplied by 65 and then divided by 13 the result is 4832160.

36. How old is a man who 7 years ago was twice as old as his sister now aged 15?

37. Five years hence a man will be five times as old as his son now 4 years old. How old was the father when the son was born?

38. The sum of the ages of a father and son is 50 years. Five years ago the father's age was three times the son's. What are their present ages?

39. Along a hedge, 220 yards in length, 23 trees are planted at equal distances, the first tree being at the beginning of the hedge. What is the distance between the trees in yards?

40. Find, as shortly as possible, the continued product of 16, 64, 125, and 625.

41. If 109 is multiplied by a certain number it is increased by 2071. Find the multiplier.

42. A boy had to divide 76428 by 123. He copied a figure wrong in the divisor, and obtained as his quotient 611 with remainder 53. What mistake did he make?

43. Complete the division below by writing in the missing numbers in the first and second lines:

$$\begin{array}{r|l} 5 & \underline{} \\ 8 & \underline{} \text{ remainder 4.} \\ & 1718 \text{ remainder 2.} \end{array}$$

44. A certain number is divided by 165 by short division. The quotient is 262, the first remainder is 2, the second is 4, and the third is 8. Find the number.

45. *A* has £85 and *B* has £39; how many pounds must *A* give to *B* in order that they may each have the same sum of money?

46. A man bought 270 eggs at the rate of 15 for a shilling and the same number at 18 for a shilling. He found that 81 were damaged and he sold the rest at the rate of 27 for 2 shillings; how much did he gain?

47. In the number 2345678 insert a cipher, somewhere between the 2 and the 8; first, so as to make the greatest possible difference in the number; next, so as to make the smallest possible difference, and find the difference of these two differences.

48. The aggregate score of three cricketers *A*, *B*, and *C* was 149. If *B* and *C* together made 76 runs, and *A* and *C* made 103, what was the score of each?

49. *A*, *B*, and *C* together made 168 runs in a cricket match. *C* made twice as many runs as *A*, and *B*'s score was half of *A*'s; what was the score of each? If in the same match there were 15 "extras," and the total score was 255, what was the average score of the remaining batsmen?

50. With 4 wickets to fall, an innings is "declared" for a total of 233. If there were 12 extras and the not-out batsmen had made 56 and 45 respectively, find the average score made by the other batsmen.

51. A certain number when divided by 8 has a remainder 5, and the quotient divided by 9 has a remainder 7; what will be the respective remainders when the order of division is reversed?

52. In a long division sum the dividend is 529565, and the successive remainders from first to last are 246, 222, 542. Find the divisor and quotient.

CHAPTER II.

COMPOUND QUANTITIES.

19. THE pupil is supposed to be already acquainted with the general principles of Reduction and of the four Fundamental Rules as applied to Compound Quantities. The object of this chapter is to provide examples for revision, and in particular to illustrate the advantages of *Decimal* Tables (that is to say, Tables based upon the number 10) over the older British Standards.

Money.

BRITISH.

4 farthings = 1 penny (*d.*)
12 pence = 1 shilling (*s.*)
20 shillings = 1 pound (£)
or sovereign.

UNITED STATES.

100 cents (¢) = 1 dollar ($)

FRANCE.

100 centimes (c.) = 1 franc (fr.)

NOTES.

(i) One **dollar** is worth a little under 4*s.* 2*d.*, so that 1 cent is nearly equivalent to 1 halfpenny.

(ii) One **franc** is worth a little over 9½*d.*, so that 25 francs are nearly equivalent to £1.

Ten centimes are worth about 1*d.*

An American *cent* (½*d.*) should be carefully distinguished from a French *centime* ($\frac{1}{10}$*d.*).

20. Reduction. A quantity expressed in a single denomination is said to be **simple.**

For instance, £7, 11 feet, 20 acres are simple quantities.

A quantity expressed in two or more denominations is said to be **compound.**

For instance, £7. 5*s.* 6*d.*, 2 yds. 1 ft. 3 in. are compound quantities.

I. *Reduction of a compound sum to its lowest denomination.*

EXAMPLE. *Reduce* (i) £45. 13*s*. $7\frac{1}{2}$*d. to farthings.*
(ii) 45 francs 13 centimes *to centimes.*
(iii) 73 dollars 4 cents *to cents.*

BRITISH COINAGE.

(i)
$$\begin{array}{rl} \pounds45.\ 13s.\ 7\tfrac{1}{2}d. & \\ \underline{20} & \\ 913 & \text{shillings.} \\ \underline{12} & \\ 10963 & \text{pence.} \\ \underline{4} & \\ \underline{\underline{43854}} & \text{farthings.} \end{array}$$

DECIMAL SYSTEM.

(ii)
$$\begin{array}{rl} 45\ \text{fr.}\ 13\ \text{c.} & \\ \underline{100} & \\ \underline{\underline{4513}} & \end{array}$$

(iii)
$$\begin{array}{rl} 73\ \$\ 4\ ¢. & \\ \underline{100} & \\ \underline{\underline{7304}} & \text{cents.} \end{array}$$

Observe carefully that while in (i) the process requires the numerical work of multiplication and addition, in (ii) and (iii) *the required results may be at once written down without any numerical work whatever.*

Thus

56 dollars 70 cents (generally written \$56,70) = 5670 cents.

183 francs 9 centimes (generally written fr. 183,09) = 18309 centimes.

II. *Reduction of a simple sum to a compound sum in higher denominations.*

EXAMPLE. *Reduce* (i) 27306 farthings *to* £. *s. d.*
(ii) 27306 centimes *to francs and centimes.*

BRITISH COINAGE.

(i)
$$\begin{array}{r|l} 4 & 27306\ \text{farthings} \\ \hline 12 & 6826\ \text{pence} + \tfrac{1}{2}d. \\ \hline 20 & 568\ \text{shillings} + 10d. \\ \hline & 28\ \text{pounds} + 8s. \\ \hline \end{array}$$

∴ 27306 farthings = $\underline{\pounds28.\ 8s.\ 10\tfrac{1}{2}d.}$

DECIMAL SYSTEM.

(ii)
$$\begin{array}{r|l} 100 & 273,06\ \text{centimes.} \\ \hline & \underline{\underline{273\ \text{fr.}\ \ 6\ \text{c.}}} \end{array}$$

Here again the required result *may be at once written down*, by merely marking off two figures from the right.

Similarly

3280 cents = \$32,80, that is, 32 dols. 80 cents.

14703 centimes = fr. 147,03, that is, 147 francs 3 centimes.

EXAMPLES II. a.

(*Examples 1–12 should be taken orally.*)

1. How many *shillings* are there in £5; £3. 12*s.*; £4. 15*s.*; £7. 11*s.*; £9. 17*s.*; 15 guineas?

2. How many *pence* are there in 1*s.* 7*d.*; 2*s.* 6*d.*; 3*s.* 11*d.*; 7*s.* 2*d.*; 9*s.* 7*d.*; £10; £1. 5*s.*?

3. How many *centimes* are there in 2 fr. 54 c.; 13 fr. 27 c.; 8 fr. 50 c.; 80 fr. 50 c.; 2 fr. 4 c.; 18 fr. 9 c.?

4. How many *cents* are there in 23 dols. 14 cents; 23 $. 4 ¢.; 70 $. 70 ¢.; 7 $. 7 ¢.; 70 $. 7 ¢.?

5. Reduce to *pounds* and *shillings*: 140 shillings; 190*s.*; 270*s.*; 374*s.*; 551*s.*; 637*s.*

6. Reduce to *shillings* and *pence*: 60 pence; 74*d.*; 113*d.*; 132*d.*; 151*d.*; 219*d.*

7. Reduce to *francs* and *centimes*: 375 centimes; 1482 c.; 1010 c.; 370 c.; 307 c.; 3007 c.; 30070 c.

8. Reduce to *dollars* and *cents*: 946 cents; 2735 ¢.; 4800 ¢.; 4080 ¢.; 40800 ¢.; 4008 ¢.; 40008 ¢.

9. How many *sixpences* are there in 11*s.* 6*d.*; 15*s.* 6*d.*; 19*s.* 6*d.*; £5; £3. 10*s.*; £4. 9*s.* 6*d.*?

10. How many *threepences* are there in 7*s.*; 13*s.*; 8*s.* 6*d.*; 15*s.* 3*d.*; £1. 10*s.*; one guinea?

11. How many *florins* are there in 18*s.*; 34*s.*; £1. 4*s.*; £6. 8*s.*; £3. 14*s.*; £4. 12*s.*?

12. How many *half-crowns* are there in 5*s.*; 15*s.*; £1; 7*s.* 6*d.*; 12*s.* 6*d.*; 15*s.*; 25*s.*; £2. 2*s.* 6*d.*?

Reduce to pence

13. £5. 3*s.* 10*d.*
14. £10. 1*s.* 8*d.*
15. £73. 2*s.* 11*d.*
16. 12 half-crowns.
17. 29 guineas.
18. 31 florins.

Reduce to half-pence

19. £13. 7*s.* 11*d.*
20. £8. 2*s.* 7½*d.*
21. £19. 4*s.* 3½*d.*

Reduce to farthings

22. £6. 13*s.* 11½*d.*
23. £4. 11*s.* 3¾*d.*
24. £9. 2*s.* 7¼*d.*

Express as £. *s.* *d.*

25. 2190 pence.
26. 3286 pence.
27. 4636 pence.
28. 3480 half-pence.
29. 4577 half-pence.
30. 8283 half-pence.
31. 6561 farthings.
32. 9193 farthings.
33. 21927 farthings.

Find the number of sixpences in

34. £33. 13*s*. 6*d*. **35.** £87. 0*s*. 6*d*. **36.** £91. 12*s*.

Find the number of half-crowns in

37. £723. 10*s*. **38.** £561. 12s. 6*d*. **39.** £1920. 17*s*. 6*d*.

Reduce to £. *s*. *d*.

40. 297 sixpences. **41.** 801 threepences. **42.** 1144 fourpences.

43. 1359 florins. **44.** 3449 half-crowns. **45.** 16243 half-crowns.

46. How many guineas are worth £399?

47. How many pounds are equivalent to 740 guineas?

48. How many half-crowns are equivalent to 3470 florins?

49. Reckoning 25 francs as equivalent to £1, estimate in pounds sterling an income of 100,000 francs.

50. How many francs and centimes should I get in exchange for £37. 10*s*.?

21. **The Four Compound Rules.** In applying the rules of Addition, Subtraction, Multiplication, and Division to compound quantities, the advantage of Decimal Tables lies in the fact that all processes of reduction required in carrying the work from one denomination to the next are performed automatically.

22. Addition.

EXAMPLE. *Compare the following additions in British and Decimal Tables.*

BRITISH.

	£.	*s*.	*d*.
(i)	43	9	4½
	7	17	8½
	103	14	3
	19	8	11½
	281	13	2
	456	3	5½

DECIMAL.

	fr. c.
(ii)	87,61
	407, 3
	8,30
	129,81
	14,73
	647,48

In (ii), on adding the centimes, we get **2**48 c., that is, **2** francs 48 centimes; thus the number of *francs* to be carried from the second column of figures is the same as the number of *hundreds* which we should carry from that column if we were adding *simple quantities*. The mental process is therefore merely that of adding abstract numbers.

EXAMPLES II. b.

ADDITION.

Add the following sums of money (writing down the answers only):

1.

£.	s.	d.
4	18	3
	13	7
15	8	11
	19	4

2.

£.	s.	d.
13	7	4
7	0	8
	17	5
18	2	2

3.

fr.	c.
61,	18
3,	08
17,	30
23,	64

4.

£.	s.	d.
3	9	2½
5	10	7
13	11	4½
4	0	9
15	9	3½

5.

£.	s.	d.
17	19	9
19	8	3½
	15	6
25	9	3
36	7	2½

6.

fr.	c.
47,	81
102,	09
37,	45
800,	00
42,	07

7.

£.	s.	d.
25	6	4¾
18	9	2
20	10	2½
30	19	6½
15	8	4½
32	17	9¾

8.

£.	s.	d.
213	13	6½
147	9	3
13	11	10¾
290	8	7¼
316	7	9½
18	9	11

9.

$.	¢.
318,	07
46,	28
201,	10
13,	93
75,	22
81,	00

Find the total of the following payments:

10. £12. 9*s.* 5*d.*, £14. 3*s.* 7*d.*, £8. 0*s.* 11*d.*, £7. 5*s.* 4*d.*, £3. 6*s.* 4*d.*, and £6. 2*s.* 5*d.*

11. fr. 62,04, fr. 49,00, fr. 192,70, 23 c., fr. 6,03.

12. $42,32, $79,09, 18 ¢., $40,90, $100,01.

13. Add the following sums of money in columns, also in rows. Find and check the grand total.

(i)

£.	s.	d.
9632	11	9
803	5	7
8070	13	10
985	6	0
6732	17	8
573	0	11
7306	15	4
3572	8	2
1384	4	9
2222	2	2
1009	19	11
3808	7	6

(ii)

£.	s.	d.
235	12	9
1783	9	4
673	0	5
409	15	0
9356	7	7
853	19	3
4576	3	6
5847	16	8
328	5	11
793	13	2
5489	11	10
6357	18	4

(iii)

£.	s.	d.
3149	17	2
672	14	9
548	4	3
19	18	7
893	6	8
2749	9	1
134	17	4
6924	2	9
4867	12	6
439	19	10
9578	9	2
21	7	11

23. Subtraction.

EXAMPLE 1. *Compare the following processes of subtraction in British and Decimal Tables.*

BRITISH.

	£.	*s.*	*d.*
(i)	5 .	6 .	$4\frac{1}{2}$
	3 .	8 .	$9\frac{3}{4}$
	1 .	17 .	$6\frac{3}{4}$

DECIMAL.

	\$. ¢.
(ii)	108,11
	89,17
	\$18,94

In each case the method of *Complementary Addition* should be used. [Art. 5.]

(i) 1*st step.* Since 3*f.* cannot be taken from 2*f.* we add 4*f.* to the farthings in the top line, and 1*d.* to the pence in the lower line.

2*nd step.* Since 10*d.* cannot be taken from 4*d.*, we add 12*d.* to the pence in the top line, and 1*s.* to the shillings in the lower line.

3*rd step.* We add 20*s.* to the shillings in the top line, and £1 to the pounds in the lower line.

The mental work is briefly as follows:

3*f.* and **3***f.*, 6*f.* = $1\frac{1}{2}$*d.* Set down 3*f.* ($\frac{3}{4}$*d.*) and carry 1*d.*

10*d.* and **6***d.*, 16*d.* = 1*s.* 4*d.* Set down 6*d.*, and carry 1*s.*

9*s.* and **17***s.*, 26*s.* = £1. 6*s.* Set down 17*s.*, and carry £1.

£4 and £**1**, £5. Set down £1.

(ii) Here the process is simply that of finding how many cents must be added to 8917 cents to make 10811 cents.

EXAMPLE 2. *Subtract the sum of* £3. 6*s.* 2*d.*, £4. 18*s.* $8\frac{1}{2}$*d.*, £5. 13*s.* $6\frac{1}{2}$*d.*, *and* £2. 7*s.* $8\frac{1}{2}$*d. from* £25. 17*s.* $4\frac{1}{2}$*d.*

Write down £25. 17*s.* $4\frac{1}{2}$*d.* and draw a line under it; then place the other sums of money under the line. Then we have

2*f.*, 4*f.*, 6*f.*, = $1\frac{1}{2}$*d.*, and **0***f.* = $1\frac{1}{2}$*d.*; carry 1*d.*

9*d.*, 15*d.*, 23*d.*, 25*d.*, = 2*s.* 1*d.*, and **3***d.* = 2*s.* 4*d.*; carry 2*s.*

9*s.*, 12*s.*, 20*s.*, 26*s.*, 36*s.*, 46*s.*, = £2. 6*s.*, and **11***s.* = £2. 17*s.*; carry £2.

£4, £9, £13, £16 and £**9** = £25.

£.	*s.*	*d.*
25 .	17 .	$4\frac{1}{2}$
3 .	6 .	2
4 .	18 .	$8\frac{1}{2}$
5 .	13 .	$6\frac{1}{2}$
2 .	7 .	$8\frac{1}{2}$
9 .	11 .	3

In the work of each column the figures in deeper type are those to be set down in the answer.

The result may be checked as in Ex. 2 of Art. 5.

EXAMPLES II. c.

SUBTRACTION.

Find in each case the difference between the following sums of money:

1.
£. s. *d.*
28 . 14 . 3
18 . 17 . 7

2.
£. s. *d.*
403 . 11 . 4
317 . 12 . 5½

3.
£. s. *d.*
80 . 0 . 1¼
74 . 19 . 0½

4.
fr. c.
486,27
197,89

5.
fr. c.
750,00
89,25

6.
$. ¢.
2308,07
639,48

7. If I pay a bill of £3. 13*s.* 5*d.* with a five-pound note, how much change should I get?

8. What must be added to £93. 18*s.* 4½*d.* to make up £100?

9. Find the excess of fr. 375,50 over fr. 368,72.

10. By how much does $699,99 fall short of $1000,00?

In the following subtraction sums supply the missing figures in the places marked by asterisks.

11.
£. s. *d.*
7*5 . * . *
496 . 5 . 4
288 . 17 . 10

12.
£. s. *d.*
3** . * . *
295 . 12 . 9
76 . 19 . 7

13.
£. s. *d.*
565 . 15 . 11
*** . * . *
240 . 19 . 2

In the following examples the sums of money below the dotted line are to be added together and their sum is to be subtracted from the top line. (Only the answer is to be written down.)

14.
£. s. *d.*
105 . 13 . 2
17 . 2 . 9
43 . 5 . 4
16 . 9 . 8

15.
£. s. *d.*
129 . 14 . 5
28 . 19 . 11
75 . 9 . 2
15 . 5 . 4

16.
£. s. *d.*
678 . 15 . 6½
354 . 13 . 3¼
213 . 10 . 4½
20 . 6 . 4¾

17. Subtract the sum of £29. 12*s.* 7*d.*, £19. 0*s.* 10*d.*, £37. 17*s.* 11*d.* from £97. 8*s.* 3*d.*

18. Having a balance of £269. 15*s.* 6*d.* at my bank, I draw cheques for £75. 7*s.* 5*d.*, £37. 15*s.* 1*d.*, £42. 4*s.* 2*d.* and £17. 12*s.* 6*d.* How much have I left?

24. Multiplication. CASE I. *When the multiplier can be resolved into convenient factors.*

EXAMPLE. *Multiply* (i) £16. 18*s*. 6½*d*. *by* 35; (ii) 16 dollars 18 cents *by* 35.

BRITISH.

	£.	s.	d.
(i)	16 .	18 .	6½
			7
	118 .	9 .	9½
			5
	592 .	8 .	11½

DECIMAL.

	$. ¢.
(ii)	16,18
	7
	113,26
	5
	$566,30

In (i) the multiplication is retarded by the necessity of mentally reducing pence to shillings, and shillings to pounds. In (ii) the mental process throughout is the same as if the multiplicand were the *simple quantity* 1618 cents.

CASE II. *When the multiplier is large or not readily expressed in factors.*

EXAMPLE. *Multiply* (i) £27. 6*s*. 5¾*d*. *by* 817; (ii) 27 francs 6 centimes *by* 817.

(i) 4 | 2451 farthings ....(i) 612...**3**	Multiply 817 by the number of farthings. The result is reduced to pence, giving 612*d*. and 3*f*. over.
4085 12 | 4697 pence ....(ii) 391...**5**	Multiply 817 by the number of pence and add in the 612*d*. The result is reduced to shillings, giving 391*s*. and 5*d*. over.
4902 20 | 5293 shillings ...(iii) 264...**13**	Multiply 817 by the number of shillings and add in the 391*s*. The result is reduced to pounds, giving £264 and 13*s*. over.
1634 5719 22323 pounds .....(iv)	Multiply 817 by the number of pounds and add in the £264, giving a total of £22323.
£22323. 13*s*. 5¾*d*. *Ans.*	

	fr. c.
(ii)	27,06
	8 17
	21648,
	270,6
	189,42
	fr. 22108,02
	=22108 fr. 2 c. *Ans.*

Remembering that 27 fr. 6 c. =27**06** centimes, we arrange the work as shewn on the left.

The multiplicand is now treated as the simple number 2706, and the work proceeds as in ordinary multiplication.

NOTE. Work in Compound Multiplication may in many cases be shortened by the method of *Aliquot Parts* explained in Chapter VI.

EXAMPLES II. d.

MULTIPLICATION.

Multiply

1. £15. 7*s*. 11*d*. (i) by 3; (ii) by 7; (iii) by 11.

2. £26. 9*s*. 4½*d*. (i) by 4; (ii) by 7; (iii) by 9.

3. 208 fr. 15 c. (i) by 5; (ii) by 9; (iii) by 12.

4. 417 dols. 8 cents (i) by 6; (ii) by 8; (iii) by 10.

Multiply

5. £28. 16*s*. 3*d*. by 16.

6. £26. 0*s*. 4½*d*. by 20.

7. £23. 2*s*. 7*d*. by 49.

8. £41. 5*s*. 2½*d*. by 72.

9. fr. 738,05 by 88.

10. $507,09 by 96.

11. Multiply (i) £23. 2*s*. 7*d*. (ii) 23 fr. 27 c. by 10; by 100; by 1000.

12. Multiply £41. 5*s*. 2½*d*. (i) by 8. *Hence* obtain the values of (ii) £41. 5*s*. 2½*d*. × 9; (iii) £41. 5*s*. 2½*d*. × 17.

Multiply

13. £37. 15*s*. 7*d*. by 29.

14. £23. 2*s*. 7*d*. by 79.

15. £27. 6*s*. 4*d*. by 122.

16. £46. 5*s*. 2*d*. by 177.

17. £82. 10*s*. 5*d*. by 141.

18. £37. 0*s*. 2½*d*. by 286.

19. fr. 40,21 by 177.

20. fr. 76,03 by 307.

21. Twenty-nine members of a club agree to subscribe £3. 2*s*. 6*d*. each towards the extinction of a debt of £100. How much more will be wanted?

22. A tradesman bought 96 yards of cloth at 2*s*. 9½*d*. a yard and sold the whole at 3*s*. 3*d*. a yard. What was the total profit?

23. Six men work 290 days in the year for a daily wage of $2,35 each: what is the total sum earned?

24. A company has 6500 Ordinary shares and 5250 Preference shares. What sum will be required to pay a dividend of 5*s*. 3*d*. on the former and 4*s*. 6*d*. on the latter?

25. Division. The advantage of decimal tables is further seen on comparing the following examples in Division.

Example 1. (Partition.) *Divide* (i) £783. 9*s.* 5*d.*, (ii) 783 fr. 90 c. *into* 47 *equal parts, and find the remainder.*

British.

```
             £.    s.   d.
(i)   47 ) 783 . 9 . 5 ( £16.
           47
           ---
           313
           282
           ---
            31
            20
           ---
      47 ) 629 ( 13s.
           47
           ---
           159
           141
           ---
            18
            12
           ---
      47 ) 221 ( 4d.
           188
           ---
            33
             4
           ---
      47 ) 132 ( 2f.
            94
           ---
    remr. = 38f. = 9½d.
  ∴ each part = £16. 13s. 4½d.,
        with 9½d. over.
```

Decimal.

```
             fr.  c.
(ii)  47 ) 783,90 ( 16 fr. 67 c.
           47
           ---
           313
           282
           ---
            319
            282
            ---
             370
             329
             ---
    remr. = 41 centimes.
  ∴ each part = 16 fr. 67 c.,
    with 41 centimes over.
```

Throughout the division the dividend is considered as the simple quantity 78390 centimes.

Note. In each case the written work might be abridged by the Italian method.

Example 2. (Quotition.) (i) *How many times is* £2. 14*s.* 7*d. contained in* £564. 19*s.* 1*d.*? (ii) *How many times is* 31 fr. 28 c. *contained in* 6475 fr. 1 c.?

In each case reduce both sums of money to their highest common denomination, and divide the second sum by the first.

```
      £.   s.   d.          £.    s.   d.
(i)   2 . 14 . 7           564 . 19 . 1          655 ) 135589 ( 207
     20                     20                         1310
     ---                  ------                       ----
     54s.                 11299s.                        4589
     12                      12                          4585
     ----                 -------                        ----
     655d.                135589d.                   remr. = 4d.
```

Ans. 207 times, remainder 4*d.*

(ii) 31 fr. 28 c. = 3128 centimes;
6475 fr. 1 c. = 647501 centimes.

Ans. 207 times, remainder 5 c.

```
3128 ) 647501 ( 207
       6256
       -----
        21901
        21896
        -----
remr. = 5 centimes.
```

Observe that in (ii) the necessary reduction is done *at sight.*

EXAMPLES II. e.

DIVISION (*Partition*).

Divide

1. £37. 15*s.* 5*d.* by 7.
2. £51. 16*s.* 6*d.* by 9.
3. £54. 15*s.* 0*d.* by 12.
4. £260. 3*s.* 9*d.* by 10.
5. 109 fr. 92 c. by 6.
6. 509 fr. 8 c. by 11.

7. If £99. 15*s.* is equally divided among 10 men, and the same amount equally divided among 12 women, by how much does the share of each man exceed that of each woman?

8. How much must be added to *one-ninth* of £250. 10*s.* to make *one-eighth* of the same sum?

Divide

9. £409. 14*s.* 8*d.* by 56.
10. £2970. 15*s.* by 72.
11. fr. 2213,75 by 55.
12. $8643,84 by 96.

13. Write down at sight (in francs and centimes)
(i) *one-hundredth,* (ii) *one-tenth* of fr. 7308.

14. If £2312. 18*s.* 4*d.* is divided (i) into 100, (ii) into 1000 equal parts (omitting fractions of a penny), find the value of each part, and the remainder, if any.

Divide as far as pence (or centimes, or cents), noting remainders:

15. £326. 4*s.* 7*d.* by 100.
16. 608 fr. 23 c. by 100.
17. £439. 9*s.* 1*d.* by 19.
18. £695. 6*s.* 11*d.* by 43.
19. £6643. 5*s.* 3*d.* by 161.
20. £2497 by 397.
21. fr. 4936,70 by 267.
22. $8271,00 by 407.

23. A company having made a net profit of £26,872. 15*s.* carries forward £2116. 10*s.* to the next year's account, and distributes the remainder equally among 300 shareholders. How much does each receive?

(*Quotition.*)

How many times is the first of the following sums contained in the second?

[Be careful in each case to reduce both sums to their *highest* common denomination; for instance in Ex. 24 to *half-sovereigns*, in Ex. 25 to *half-crowns*, and so on.]

24. £5. 10*s*. in £93. 10*s*.?

25. £3. 7*s*. 6*d*. in £37. 12*s*. 6*d*.?

26. £4. 5*s*. in £148. 15*s*.?

27. 3*s*. 6*d*. in £4. 0*s*. 6*d*.?

28. £7. 14*s*. in £346. 10*s*.?

29. £2. 10*s*. 7*d*. in £35. 8*s*. 2*d*.?

30. fr. 47,03 in fr. 3386,16?

31. fr. 23,80 in fr. 2641,80?

32. How many payments each of £33. 18*s*. 1*d*. can be made out of £779. 16*s*. 9*d*., and what will be the remainder?

33. How many times can £7. 11*s*. 2*d*. be subtracted from £100, and what will be the remainder?

34. If coal is £1. 3*s*. 8*d*. a ton, what is the greatest number of tons I can buy for £86, and how much money should I have over?

35. By distributing £2416. 13*s*. 4*d*., a company can pay a dividend of 4*s*. 10*d*. per share. How many shares are there?

EXAMPLES II. f.

COMPOUND RULES. MONEY.

(*Examples* 1–15 *should be taken orally.*)

1. How many six-pences are there in 15*s*. 6*d*., in 29*s*. 6*d*., in £3. 11*s*., in £2. 6*s*. 6*d*., in £5. 11*s*. 6*d*.?

2. Add together

(i) 4*d*., 5*d*., 7*d*., 9*d*., 11*d*.

(ii) 6*d*., 9½*d*., 10*d*., 10½*d*.

(iii) 3½*d*., 7½*d*., 6*d*., 8½*d*., 13½*d*.

(iv) 1*s*. 2*d*., 2*s*. 6*d*., 3*s*., 3*s*. 4*d*.

3. What change should be given when

(i) 3¾*d*. is taken out of 1 shilling?
(ii) 1*s*. 1½*d*. .... 1 florin?
(iii) 1*s*. 8½*d*. .... half-a-crown?
(iv) 4*s*. 10½*d*. .... half-a-sovereign?
(v) 12*s*. 7*d*. .... 1 sovereign?
(vi) £3. 3*s*. 3*d*. .... a five-pound note?

4. Give the cost of a dozen things at

(i) 5*d*. each; (ii) 7*d*. each; (iii) 3½*d*. each;
(iv) 9½*d*. each; (v) 4¼*d*. each; (vi) 6¾*d*. each;
(vii) 11½*d*. each; (viii) 1*s*. 4*d*. each; (ix) 1*s*. 7*d*. each;
(x) 1*s*. 2½*d*. each; (xi) 2*s*. 8*d*. each; (xii) 3*s*. 6½*d*. each.

5. Give the cost of 20 things at
(i) 4*s.* each; (ii) 5*s.* 6*d.* each; (iii) 2*s.* 3*d.* each;
(iv) 7*s.* 6*d.* each; (v) £1. 4*s.* each; (vi) £2. 5*s.* 6*d.* each.

6. State the value of 8 half-crowns, 3 half-crowns, 5 half-crowns, 7 half-crowns, 11 half-crowns, 13 half-crowns, 25 half-crowns.

7. Add together
(i) 2*s.* 6*d.*, 7*s.* 6*d.*, 3*s.* 8*d.*, and 6*s.* 4*d.*
(ii) 1*s.* 2*d.*, 3*s.* 10*d.*, 4*s.* 3*d.*, 5*s.* 9*d.*, and 10*s.*
(iii) £1. 3*s.*, £2. 7*s.*, £2. 12*s.*, £3. 8*s.*, £4. 15*s.*, and £5. 5*s.*

8. At a stationer's shop I spend 2*s.* 3*d.* and 6*s.* 8*d.*; what change should I get out of half-a-sovereign?

9. For things costing 3*s.* 6*d.*, 4*s.* 8*d.*, and 11*s.* 4*d.* I tender one sovereign. What change should I get?

10. Third-class fare is a penny a mile. If for a third single ticket from Paddington to Exeter (194 miles) I tender a sovereign, what is my change?

11. Reckoning first-class fare at twopence a mile, what should be the change out of a sovereign after paying for a first single ticket from Oxford to Bristol (76 miles)?

12. What is the cost of 6 yards of silk at 4*s.* 8*d.* a yard?

13. What is the cost of 9 lbs. of tea at 2*s.* 4*d.* a pound?

14. What is the cost of 10 tons of coal at 24*s.* 6*d.* a ton?

15. Five guineas are to be equally distributed among 21 persons: how much should each get?

16. I have £3. 2*s.* 6*d.* in my purse; after paying two bills of £1. 7*s.* 5*d.* and 19*s.* 2*d.*, what have I left?

17. Subtract the sum of £2. 6*s.* 7*d.*, £3. 2*s.* 2*d.* from £6. 13*s.* 10*d.*

18. By how much is the sum of £43. 2*s.* 7¼*d.* and £76. 17*s.* 2½*d.* less than £120?

19. I have £95 in the bank; what is my balance after writing cheques for £24. 6*s.* 11*d.*, £13. 17*s.* 4*d.*, £32. 5*s.* 7*d.*, and £5. 2*s.* 8*d.*?

20. What is the cost of 18 yards of linen at 2*s.* 8*d.* per yard?

21. How much does a man earn in 55 days of 8 hours each at 5*d.* per hour?

22. What amount of money can be equally distributed amongst 14 persons if each is to receive £15. 7*s.* 11*d.*?

23. Divide £3744. 13*s.* by 42, using short division.

24. A sum of £31. 11*s.* 6*d.* is equally divided between 36 men: how much does each receive?

25. If I buy three jugs at 1*s.* 3*d.*, 1*s.* 6*d.*, and 1*s.* 9*d.* respectively, and 4 dishes at 1*s.* 5*d.* each, how much do I spend altogether?

26. How many books at 3*s.* 6*d.* each can be bought for £35. 17*s.* 6*d.*?

27. A Christmas dole of £7. 8*s.* 6*d.* was divided amongst 297 poor children. How much did each receive?

28. How many half-crowns are there in £561. 12*s.* 6*d.*?

29. How many payments of £5. 11*s.* 3*d.* each can be made with a sum of £483. 19*s.* 4*d.*, and how much will be left?

30. With £60 to begin with, for how many days can a man spend 5*s.* 6*d.* a day, and how much will then be left?

31. In how many weeks can a man save 15 guineas to buy a bicycle if he saves 5*s.* 10*d.* a week?

32. How many persons can each receive £1. 3*s.* out of a sum of £170. 10*s.* 2*d.*? And if the balance is distributed equally among them, how much more will each receive?

33. At a school treat a sum of £11. 14*s.* 9*d.* was used in giving a threepenny piece to each child. How many children were there?

34. Write down the sum of money in the missing line of the following examples in addition.

	£.	*s.*	*d.*		£.	*s.*	*d.*		£.	*s.*	*d.*
(i)	15	7	8	(ii)	6	8	2	(iii)	127	19	5
	13	19	2		45	7	5		46	15	11
	15	6	10		12	13	6		364	8	7
	—	—	—		—	—	—		—	—	—
	67	19	4		100	0	0		595	10	6

35. To how many persons may the sum of £19. 19*s.* 11*d.* be paid out of £10,000? How much may each of 25 other persons receive from the residue?

36. A sum of £43. 4*s.* 5*d.* is made up of an equal number of pounds, shillings, and pence. How many are there of each?

37. In four successive years a business cleared £401. 11*s.* 10*d.*, £491. 8*s.* 8*d.*, £528. 6*s.* 11*d.*, and £719. 4*s.* 3*d.* respectively. What were its average annual profits?

38. A man's yearly income is £741. 12*s.* 6*d.* and his daily expenditure a guinea and a half. How much does he save in an ordinary year?

39. Find the total cost of three dozen pocket-handkerchiefs at 1*s.* 10*d.* each, two-and-a-half dozen collars at $9\frac{1}{2}$*d.*, and one dozen neckties at 1*s.* $11\frac{3}{4}$*d.* each.

40. Divide £67 between *A* and *B* so that *A* may have £13 more than *B*.

41. How must £265 be divided between A, B, and C so that A may have £40 more than B, and B £15 more than C?

42. A certain sum of money was divided among A, B, and C; A and B received £18. 19*s.*; A and C, £17. 13*s.* 6*d.*; B and C, £15. 12s. 6*d.* How much did each receive?

43. If the income tax is 11*d.* in every £, what amount of tax would a man have to pay whose income was £850?

44. At 11*d.* in the £ a man has to pay income tax amounting to £45. 16*s.* 8*d.*; what is his income?

45. A tradesman buying articles at the rate of £3. 7*s.* 6*d.* per score, sells at £2. 2*s.* 6*d.* per dozen. What profit does he make on one such article? What profit on 1000 articles?

46. Cloth was bought at 2*s.* 9½*d.* a yard and sold at 3*s.* 3*d.* If the total profit came to £2. 4*s.*, how many yards were bought and sold?

47. A wine merchant mixes 29 gallons of wine which cost him 15*s.* 8*d.* a gallon with 17 gallons at 23*s.* 4*d.* a gallon. What is the value of one gallon of the mixture?

48. A grocer mixes 11 lbs. of tea at 2*s.* 6*d.* with 15 lbs. at 1*s.* 10*d.* and 14 lbs. at 2*s.* 1*d.* per pound. If he sells the whole at 2*s.* 4*d.* per pound, what does he gain per pound, and what is his total gain?

49. To bring goods from Zanzibar to Unyamwezi costs £200 per ton, and, in consequence, calico which costs 1¼*d.* a yard in Zanzibar costs 5*d.* a yard in Unyamwezi. How many yards are there to the ton?

50. In the first four months the takings of a business are respectively £335. 2*s.* 1½*d.*, £371. 15*s.* 11*d.*, £401. 11*s.* 5½*d.*, and £446. 11*s.* 6*d.* What must be the average takings for the remaining months in order that the total receipts for the year may be £5000?

26. Tables of Length.

BRITISH.

12 Inches (in.) = 1 Foot (ft.).
3 Feet = 1 Yard (yd.).
1760 Yards = 1 Mile (mi.).

The following measures are sometimes used:

1 furlong = 220 yards;
so that 8 furlongs = 1 mile.

1 pole = 5½ yards;
so that 40 poles = 1 furlong.

1 chain = 22 yards = 100 links.

Short lengths will be usually expressed in *feet* and *inches*; as 7 ft. 2 in. (or, less commonly, in *yards*, *feet*, and *inches*, as 2 yds. 1 ft. 2 in.).

Long distances will be generally expressed in *miles* and *yards*, as 7 mi. 480 yds.

METRIC.

10 *milli*-metres (mm.) = 1 *centi*-metre (cm.).
10 centimetres = 1 *deci*-metre (dm.).
10 decimetres = 1 **metre** (m.).
10 metres = 1 *deca*-metre (Dm.).
10 decametres = 1 *hecto*-metre (Hm.).
10 hectometres = 1 *kilo*-metre (Km.).

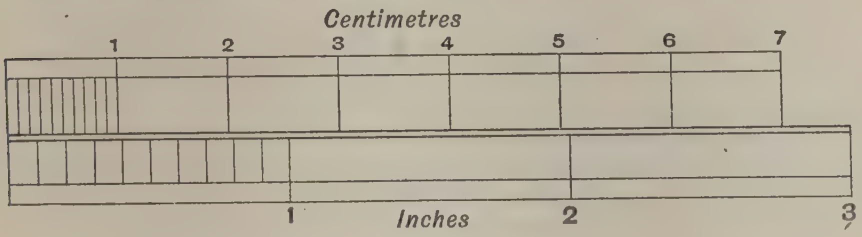
Here the metre is considered as the *Standard unit of length*, and accordingly gives its name to the whole system.

Short lengths will be expressed in *metres*, *decimetres*, and *centimetres*; as 5 m. 3 dm. 7 cm.: *shorter* lengths in *centimetres* and *millimetres*; as 8 cm. 4 mm.

Long distances will usually be expressed in *kilometres* and *metres*; as 9 Km. 356 m. (decametres and hectometres being little used).

NOTE. **1 metre = $39\frac{3}{8}$ inches** (nearly); so that the metric unit of length is roughly $3\frac{1}{2}$ inches longer than the *yard*.

A scale comparing inches and centimetres is given below.



In this scale the first inch is subdivided into 10 equal parts.

The first centimetre is also subdivided into tenths, that is to say, into millimetres.

The diagram shews that 1 centimetre is slightly less than *four* tenths of an inch.

27. The Metric Table may be arranged thus:

1 decametre = 10 metres.
1 hectometre = 100 metres.
1 kilometre = 1000 metres.

1 decimetre = $\frac{1}{10}$ metre.
1 centimetre = $\frac{1}{100}$ metre.
1 millimetre = $\frac{1}{1000}$ metre.

Here, the *metre* being taken as the unit, the higher denominations are shewn by *Greek* prefixes: *Deca-* (10), *Hecto-* (100), *Kilo-* (1000); while the corresponding subdivisions are shewn by *Latin* prefixes; *deci-* ($\frac{1}{10}$), *centi-* ($\frac{1}{100}$), *milli-* ($\frac{1}{1000}$).

For the sake of greater clearness it is usual to write the prefixes for the higher denominations with a capital letter.

Observe that the Metric Table of Length is constructed on the same plan as the *Numeration Table*; that is to say, each denomination is *ten* times the denomination next below it, just as in any abstract number each digit has a "place-value" ten times that of the next digit on its right.

For example,

Just as 3765 = 3 *thousands* + 7 *hundreds* + 6 *tens* + 5 *units*,
so 3765 metres = 3 Km. + 7 Hm. + 6 Dm. + 5 m.
Similarly 4080 metres = 4 Km. + 0 Hm. + 8 Dm. + 0 m.
Conversely, 5 Km. 9 Hm. 1 Dm. 2 m. = 5912 metres.
Similarly 7 Km. 3 m. = 7003 metres.

In like manner,

8 m. 5 dm. 6 cm. = 856 centimetres = 8560 millimetres.

Conversely,

7090 millimetres = 709 centimetres = 7 m. 0 dm. 9 cm.

Hence in the Metric Table all processes of *Reduction* follow the laws of *Numeration*, and can be performed at sight. Consequently *Addition*, *Subtraction*, *Multiplication*, and *Division* in *Compound* metric quantities differ in no way from the corresponding work in *Simple* quantities. Herein lies the overwhelming advantage of a decimal over a non-decimal system of Tables.

EXAMPLE 1. *Compare the following additions in British and Metric Tables of Length.*

BRITISH.

	mi.		yds.
(i)	3	.	450
	17	.	718
	9	.	822
	21	.	540
			2530 (1 mi.
			1760
	51 mi.		770 yds.

METRIC.

	Km.		metres.
(ii)	3	.	450
	17	.	718
	9	.	822
	21	.	540
	52	.	530

In (i) we obtain on addition 2530 yards. This we divide by 1760, setting down the remainder 770 yds. and carrying the quotient 1 mile to the next column.

In (ii) we obtain 2530 metres, namely 2 **Km.** 530 **metres**; so that the number of metres set down and the number of kilometres to carry are just the same as if the quantities to be added were *simple*.

EXAMPLE 2. *Multiply* (i) 7 yds. 2 ft. 5 in. *by* 11, (ii) 7 m. 2 dm. 5 cm. *by* 11.

BRITISH.

$$\text{(i)}\quad \begin{array}{rrr} \text{yds.} & \text{ft.} & \text{in.} \\ 7\ . & 2\ . & 5 \\ & & 11 \\ \hline 85\ . & 2\ . & 7 \\ \hline \end{array}$$

METRIC.

$$\text{(ii)}\quad \begin{array}{rrr} \text{m.} & \text{dm.} & \text{cm.} \\ 7\ . & 2\ . & 5 \\ & & 11 \\ \hline 79\ . & 7\ . & 5 \\ \hline \end{array}$$

In (i), to get the carrying figure at each stage, inches must be reduced to feet, and feet to yards.

In (ii) the carrying figures are the same as if the multiplicand were the simple quantity 725 cm.;

for 55 cm. = 5 dm. 5 cm.; set down **5**, and carry 5.

And 27 dm. = 2 m. 7 dm.; set down **7**, and carry 2.

EXAMPLES II. g.

COMPOUND RULES. LENGTH.

Reduce to inches

1. 8 ft. 5 in.
2. 23 ft. 7 in.
3. 19 yds. 2 ft. 11 in.

Write down the number of centimetres in

4. 7 metres.
5. 8 decimetres.
6. 7 m. 8 dm. 5 cm.
7. 6 m. 9 dm. 3 cm.
8. 6 m. 0 dm. 3 cm.
9. 6 m. 9 dm. 0 cm.
10. 5 m. 3 dm.
11. 5 m. 3 cm.
12. 5 dm. 3 cm.

Reduce to yards

13. $8\frac{1}{2}$ miles.
14. 11 mi. 250 yds.
15. 4 fur. 111 yds.

Write down the number of metres in

16. 7 kilometres.
17. 8 hectometres.
18. 3 decametres.
19. 7 Km. 8 Hm. 3 Dm.
20. 7 Km. 8 Hm. 3 m.
21. 7 Km. 8 Dm. 3 m.
22. 7 Km. 830 m.
23. 14 Km. 14 m.
24. 141 Km. 4 m.

25. Reduce (i) 171 in., (ii) 493 in. to yards, feet, and inches.

Write down in compound quantities

26. 723 centimetres.
27. 813 centimetres.
28. 905 centimetres.
29. 7020 millimetres.
30. 702 millimetres.
31. 5173 metres.
32. 5002 metres.
33. 5200 metres.
34. 5020 metres.

35. How many inches are there in a quarter of a mile? How many feet in five miles and a quarter?

36. How many chains are there in a mile? How many inches in chain? The length of 1 link is between 7 inches and 8 inche which of these is nearer to the actual value? (Shew work justify your answer.)

Add together the following quantities:

37.

yds.	ft.	in.
4	2	8
7	1	3
8	0	7
11	1	9
3	0	4

38.

m.	dm.	cm.
4	2	8
7	1	3
8	0	7
11	1	9
3	0	4

39.

dm.	cm.	mm.
4	2	8
7	1	3
8	0	7
11	1	9
3	0	4

40.

miles.	yds.
5	100
2	375
3	950
12	876
1	219

41.

Km.	metres.
5	100
2	375
3	950
12	876
1	219

42.

fur.	yds.
4	117
6	75
5	80
7	122
3	35

43. Subtract (i) 4 yds. 2 ft. 9 in. from 16 yds. 2 ft. 0 in.
(ii) 4 m. 2 dm. 9 cm. from 16 m. 2 dm. 0 cm.

44. By how much does 7 mi. 980 yds. fall short of 10 miles? And how much does 10 kilometres exceed 7 Km. 980 m.?

45. Multiply (i) 5 yds. 1 ft. 7 in. (ii) 5 m. 1 dm. 7 cm. by 9.

46. Divide (i) 35 yds. 1 ft. 7 in. (ii) 35 m. 1 dm. 7 cm. by 11; an find the remainder in each case.

47. A circular track is 2 fur. 88 yds. round. How far will a cycli have ridden when he has gone round 40 times?

48. If telegraph posts stand at intervals of 25 metres, how man kilometres apart are the 1st and 241st posts?

49. A train is travelling at the rate of 36 miles an hour: how man yards will it run in half a minute?

50. How many pieces of calico each 14 ft. 6 in. long can be cut from roll of 50 yards, and how much will be left?

51. A carriage wheel is 8 ft. 3 in. in circumference: how man revolutions does it make in travelling 7 mi. 1331 yds.?

52. A surveyor measuring the four sides of a field finds them to 5 ch. 50 lks., 5 ch. 10 lks., 4 ch. 75 lks., and 4 ch. 65 lk What is the distance all round the field (i) in chains, (ii) yards?

53. The average speed of a certain mail-boat is 18 knots an hour, knot being 2026 yds. 2 ft. Two passengers estimate her dail run at 497 miles and 498 miles respectively: which estimate nearer the truth?

54. A gang of men began laying a section of railway, 67 mi. 500 yds. in length, on the morning of January 1st, and just finished by the end of the month. What was the average advance per day?

Had the length of the section been 67 Km. 500 m., and the daily advance 2 Km. 177 m., how much would have remained to be laid at the end of the month?

55. A cage is lowered into a mine by a steel rope run off a drum 44 ft. 6 in. in circumference. If the drum makes 2 revolutions in every 3 seconds, and the cage descends in 45 seconds, what is the depth of the shaft?

Tables of Weight and Capacity.

28. Weight.

British. (*Avoirdupois.*)

16 ounces (oz.) = 1 pound (lb.).
28 pounds = 1 quarter (qr.).
4 quarters = 1 hundredweight (cwt.).
20 hundredweight = 1 ton.

The following weights are sometimes used:

1 oz. = 16 drams (dr.).
1 lb. = 7000 grains (gr.).
1 stone = 14 lbs.

Metric.

10 *milli*-grams (mg.) = 1 *centi*-gram (cg.).
10 centigrams = 1 *deci*-gram (dg.).
10 decigrams = 1 **gram** (g.).
10 grams = 1 *deca*-gram (Dg.).
10 decagrams = 1 *hecto*-gram (Hg.).
10 hectograms = 1 *kilo*-gram (Kg.).

The following weights are sometimes used:

100 Kg. = 1 quintal.
1000 Kg. = 1 tonne.

Notes.

(i) The **gram** is the Metric unit of weight; and the prefixes *Deca-*, *Hecto-*, *Kilo-*, *deci-*, *centi-*, *milli-*, are used with the same meaning as in the Metric Table of Length.

(ii) **1 gram** = **$15\frac{1}{2}$ grains** (nearly).
∴ **1 kilogram** = **$2\frac{1}{5}$ pounds** (nearly).

Hence the British *ton* and the Metric *tonne* are nearly equal, for

1 ton (British) = 2240 lbs.
1 tonne (Metric) = 2200 lbs. (nearly).

Further information as to the connection between the Metric and the old British systems will be found in Chapter VIII.

(iii) For the precious metals a special ounce, called the *ounce Troy*, is at present used.

1 oz. Troy = 480 grains.

29. Capacity.

BRITISH.

2 pints (pts.) = 1 quart (qt.).
4 quarts = 1 gallon (gal.).

These measures are used bo for liquid and dry goods. T following are used for dry goo only :

1 peck = 2 gallons.
1 bushel = 8 gallons.

METRIC.

10 *centi*-litres (cl.) = 1 *deci*-litre (dl.).
10 decilitres = 1 **litre** (l.).
10 litres = 1 *deca*-litre (Dl.).
10 decalitres = 1 *hecto*-litre (Hl.).
10 hectolitres = 1 *kilo*-litre (Kl.).

These measures are us for liquid and dry goo alike.

NOTES.

In the British system :

A **gallon** is capable of containing **10 lbs.** weight of pure water.

An ordinary tumbler contains about half-a-pint, so that 16 tumbl hold about 1 gallon.

In the Metric system :

The **litre** is the metric unit of capacity.

1 litre = 1¾ pints (nearly).

EXAMPLES II. h.

COMPOUND RULES. WEIGHT. CAPACITY.

Reduce to pounds

1. 9 cwt. 17 lbs. **2.** 4¼ tons. **3.** 2 tons 3 cwt. 2 q

Write down the number of grams in

4. 5 kilograms. **5.** 9 hectograms. **6.** 3 Kg. 9 Hg.
7. 6 Hg. 41 g. **8.** 6 Hg. 4 Dg. 1 g. **9.** 3¼ kilograms.

Express

10. 400 oz. in pounds.
11. 6000 lbs. in tons, hundred-weight, and pounds.
12. 355 pints in gallons, quarts, and pints.

Write down in compound quantities

13. 407 grams.
14. 4007 grams.
15. 4700 grams.
16. 4070 grams.
17. 357 litres.
18. 2903 litres.

19. Work out the equivalents of (i) 1 ton (British), (ii) 1 tonne (metric) in pounds; remembering that 1 tonne = 1000 kilograms, and 1 kilogram = $2\frac{1}{5}$ lbs.

Add together the following quantities:

20.

cwt.	qrs.	lbs.
18	3	13
7	0	17
10	2	9
5	1	12

21.

Kg.	Hg.	Dg.
8	0	2
3	7	9
4	8	6
7	3	8

22.

l.	dl.	cl.
8	0	2
3	7	9
4	8	6
7	3	8

23. What weight must be added to 11 cwt. 47 lbs. to make up 1 ton?

And by how much does 1 kilogram exceed 9 Hg. 8 Dg. 7 g.?

In the following examples the quantities below the dotted line are to be added together and their sum is to be subtracted from the quantity in the top line. (Only the answer is to be written down.)

24.

tons	cwt.	qrs.
215	13	3
14	5	2
23	12	1
47	15	0
90	17	2

25.

l.	dl.	cl.
68	3	8
15	0	6
20	8	9
7	4	5
14	0	7

26.

gm.	dg.	cg.
47	2	8
9	4	6
18	2	9
6	5	3
12	9	1

27. One man weighs 12 stone 8 lbs., another 166 lbs. By how much does the first man's weight exceed the other's? And what is their average weight?

28. A small copper wire, evenly wound on a reel, goes just 48 times round. If the circumference of the reel is 4 ft. 3 in., and the wire weighs 3 ounces to the yard, what are the length and weight of the wire on the reel?

29. A steel rope capable of working a load of 2 tons 10 cwt. weighs $2\frac{1}{2}$ lbs. to the yard. What would be the total weight of 280 yards of the rope together with the load above mentioned?

30. How many trucks each capable of carrying 6 cwt. 50 lbs. may be loaded up from a stack of bricks weighing 10 tons?

31. A collier cuts out an average of 3 tons 5 cwt. of coal per How many trollies each carrying 13 cwt. will he fill in a working m of 27 days?

32. From a tank containing 2850 gallons water is drawn off pipe which discharges a pint each second. The discharge tap is tur on at 9 o'clock, when will the tank be empty?

33. An engine of 1 horse-power will raise a weight of 55 lbs. thro 10 feet in 1 second. How many tons would it raise the same heigh 1 hour? [Give a rough answer to the nearest ton only.]

30. Tables of Square Measure.

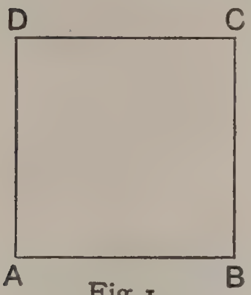


Fig. 1.

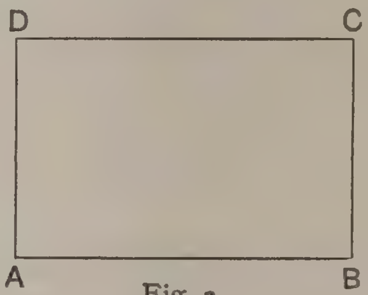


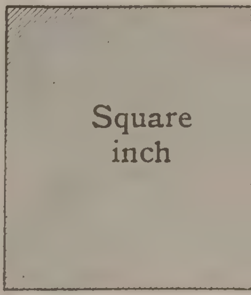
Fig. 2.

The familiar figures represented above are known as a **squa** and a **rectangle.**

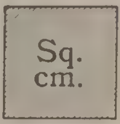
In the square (Fig. 1) the length AB is equal to the breadth A

In the rectangle (Fig. 2) the length AB and the breadth AD unequal.

A **square inch** is the *amount of surface* enclosed within a square of which each side is one inch in length.



Similarly a **square centimetre** is the space enclosed within a square on a side of one centimetre.

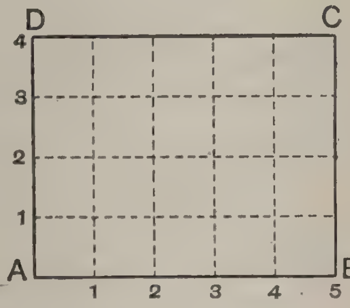


The terms *square yard*, *square foot*, *square metre* are to be understo in the same sense.

31. An inch of *length* is called a **linear** inch to distinguish it from an inch of *surface* or *square inch.*

The **area** of any figure is the *amount of surface* enclosed within its bounding lines. This is measured by the number of square inches or square centimetres (or other units of square measure) it contains.

32. *To find the* **area of a rectangle,** *having given its* **length** *and* **breadth.**



Here ABCD represents a rectangle whose length AB is 5 inches, and breadth AD is 4 inches.

Divide AB and AD into equal parts representing inches; and through the points of division draw parallels as shewn by the dotted lines in the diagram.

The rectangle ABCD is then divided into equal compartments representing *square inches.*

Now there are 4 rows, each containing 5 squares;

∴ the rectangle contains (5 × 4) square inches.

Similarly if the length $= l$ *linear* units, and the breadth b *linear* units,

then the *area* $= (l \times b)$ units of *square* measure.

This we may abridge by saying

area *of rectangle* = **length** × **breadth.**

Examples.

(i) If the page of a book is 7 inches long by 5 inches wide,

its *area* is 7 × 5, or 35, *square* inches.

(ii) If the top of a table is 6 feet long by 4 feet wide,

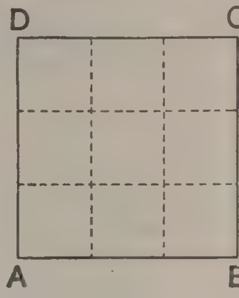
its *area* is 6 × 4, or 24, *square* feet.

(iii) If a court yard is 30 metres long by 20 metres wide,

its *area* is 30 × 20, or 600, *square* metres.

Note. It is correct to write "*area* = (30 × 20) *square* metres"; but it is incorrect to write "*area* = 30 *metres* × 20 *metres.*" For a multiplier cannot be a *concrete* quantity, nor can any process of multiplication convert units of one kind (such as metres) into units of another kind (such as *square* metres).

33. We can now shew how a Table of *Square* Measure i derived from the corresponding Table of *Linear* Measure.



The figure **ABCD** in the margin represents a square of which each side is 1 *yard.* That is, **ABCD** represents a *square yard.*

Divide **AB** and **AD** into equal parts representing feet, and complete the figure as explained in Art. 31.

Then the square **ABCD** is divided into equal compartments representing *square feet.*

Now there are 3 rows each containing 3 squares ;

$\therefore$ the whole square contains 3×3, or 9, *square feet.**

Hence from

1 yd. = 3 ft., we derive 1 *square* yd. = 3^2, or 9, *square* ft.

Similarly from

1 ft. = 12 in., we derive 1 *square* ft. = 12^2, or 144, *square* in.

1 m. = 10 dm., ,, 1 *square* m. = 10^2, or 100, *square* dm.

Thus we have the following **Tables of Square Measure.**

BRITISH.	METRIC.
12^2, or 144, sq. in. = 1 sq. ft.	10^2, or 100, sq. mm. = 1 sq. cm.
3^2, or 9, sq. ft. = 1 sq. yd.	10^2, or 100, sq. cm. = 1 sq. dm.
$(1760)^2$ sq. yds. = 1 sq. mi.	10^2, or 100, sq. dm. = 1 sq. m.
	and so on.

NOTES.

(i) A beginner often confuses the expressions *three feet square* and *three square feet.*

Three feet square denotes the square on a side of 3 feet, that is, the whole square represented by **ABCD** in the last diagram, viz. 9 sq. feet.

Three square feet denotes an area represented by *one row* of squares in the same diagram.

(ii) Figures of different *shapes* may have the same area. We imply nothing as to the shape of a figure when we say that its *area* is 3 square feet.

---

*The product of a number multiplied by itself is called the *square* of that number.
Thus 3×3 is read "3 *squared*," and may be written 3^2.
4×4 ,, "4 squared," ,, ,, ,, 4^2.

EXAMPLES II. k.

COMPOUND RULES. SQUARE MEASURE.

1. Point out rectangular objects and surfaces in the room, and measure or estimate their length and breadth.

2. What is meant by the *area* of a figure, and in what units is it expressed?

Draw figures on squared paper to shew that a figure whose area is 1 square inch is not itself necessarily square.

3. Draw figures on squared paper to shew why
(i) 1 square foot $=12^2$, or 144, square inches.
(ii) 1 square metre $=10^2$, or 100, square decimetres.

4. State the area of the following in square feet:
(i) A door, 8 feet high and 4 feet wide.
(ii) The top of a table, 6 feet long by 5 feet broad.
(iii) A window, 7 feet long by 3 feet wide.
(iv) A wall, 20 feet long by 12 feet high.

5. A sheet of paper is 12 inches long by 8 inches wide; what is its area in square inches?

6. The length of a rectangular play-ground is 60 metres, and its breadth is 45 metres; how many square metres are there in its surface?

7. What is the area of a square play-ground of which each side measures 90 yards?

8. A slate is a foot long and 8 inches wide; how many square inches are there on one side of it?

9. A plank is 4 feet long and 10 inches wide; how many square inches are there in its upper surface?

10. A drinking-trough is 4 yards long and 4 feet wide. Give the area of the water surface in square feet.

11. A strip of silk is 5 metres long by 8 decimetres wide; how many square decimetres are there in it?

Reduce to square inches

12. 8 sq. ft. **13.** $10\frac{1}{4}$ sq. ft.
14. 7 sq. yds. **15.** 15 sq. ft. 40 sq. in.
16. 2 sq. yds. 7 sq. ft. **17.** 3 sq. yds. 7 sq. ft. 104 sq. in.

Write down in square centimetres the equivalents of

18. 14 sq. decimetres. **19.** 14 sq. metres. **20.** 14 sq. m. 14 sq. dm.
21. 5 sq. m. 16 sq. dm. 17 sq. cm. **22.** 5 sq. m. 6 sq. dm. 7 sq. cm.

Reduce

23. 1100 square feet to sq. yds. and sq. ft.

24. 1248 square inches to sq. ft. and sq. in.

25. 726084 square inches to sq. yds., sq. ft., sq. in.

Write down in compound quantities

26. 5648 sq. cm. 27. 564800 sq. cm. 28. 560408 sq. cm.

Add together

29.

sq. yds.	sq. ft.	sq. in.
10	7	120
18	5	115
21	8	72
32	4	130

30.

sq. m.	sq. dm.	sq. cm.
3	14	30
11	8	9
4	80	41
7	97	20

31. How many square metres are there in (i) a square decametre (ii) a square hectometre? (iii) a square kilometre?

32. What is the total area of 12 plots of ground each containing (i) 80 sq. yds. 7 sq. ft., (ii) 80 sq. m. 7 sq. dm.?

33. How many times is (i) 5 sq. yds. 8 sq. ft. contained in 70 sq. yds 6 sq. ft.?

(ii) 7 sq. m. 18 sq. dm. contained in 86 sq. m. 16 sq. dm.?

34. A garden contains 16 flower beds, a lawn, and gravel paths. The whole area of the garden is 1300 sq. yds; the area of each bed is 50 sq. yds. 5 sq. ft., and the paths take up 241 sq. yds. 1 sq. ft. What is the area of the lawn?

Find the areas of the following rectangular spaces:

35. Length 30 ft., breadth 25 ft. [Ans. in sq. yds., sq. ft.]

36. Length 22 in., breadth 18 in. [Ans. in sq. ft., sq. in.]

37. Length 22 dm., breadth 18 dm. [Ans. in sq. m., sq. dm.]

38. Length 50 m., breadth 28 dm. [Ans. in sq. m.]

39. If the length of a card-board is 19 inches and its breadth 15 inches, by how much does its area fall short of 2 square feet?

40. The floor of a room has an area of 600 sq. ft., and its length is 30 ft.; what is its breadth?

34. **Land Measure.** In the British standard land is measured in acres, roods, and square poles.

40 sq. poles = 1 rood,
4 roods = 1 acre.

We have seen that a *linear* pole $= 5\frac{1}{2}$ yards; hence 1 *square* pole is the *area of a square on a side of* $5\frac{1}{2}$ yards.

Thus 1 sq. pole contains $(5\frac{1}{2})^2$, namely $30\frac{1}{4}$, square yards.

From these data the student should prove that

(i) **1 acre = 4840 sq. yds.**; (ii) **1 square mile = 640 acres.**

Note. There is no *linear* unit corresponding either to a *rood* or an *acre* of surface.

It should however be observed that an acre is *nearly equal to the area of a square on a side of* 70 *yards*:

for 1 acre = **4840** square yards;
the square on 70 yards = 4900 square yards.

35. Again we have seen on page 30 that 1 chain = 22 yards.

∴ 1 *square* chain $= (22)^2$, or 484, *square* yards.

Now 1 acre = 4840 square yards.

∴ **1 acre = 10 square chains.**

Similarly, since 1 chain = 100 links.

∴ 1 *square* chain $= (100)^2$, or 10,000, *square* links.

Hence 1 acre = 100,000 square links.

36. The metric unit of area for land measure is the **square decametre**, called an **Are.**

∴ 1 Are (A.) $= 10^2$, or 100, square metres.

1 hectare (Ha.) = 100 ares = 10,000 square metres.

Note. 1 sq. metre = 1550 sq. inches (nearly).
1 hectare = $2\frac{1}{2}$ acres (nearly).

Further information as to metric land measure with examples for practice will be given in Chapter VIII.

EXAMPLES II. k. (*Continued.*)

41. Give calculations to shew that

(i) 1 acre = 4840 sq. yds.; (ii) 1 sq. mile = 640 acres.

42. About how many square poles should you think there are in the area of your school-room floor? Give a reason for your estimate.

43. Try to estimate in acres the area of any play-ground, field, garden, or quadrangle that may be well known to you (remembering that 1 acre is nearly equal to a square on a side of 70 yards).

44. Find the area in acres of the following rectangular fields:

(i) Length 110 yards, breadth 44 yards.
(ii) Length 121 yards, breadth 120 yards.
(iii) Length 220 yards, breadth 99 yards.
(iv) Length 12 chains, breadth 5 chains.

45. Reduce (i) 5 acres 2 roods to sq. yds.
(ii) 3 Ha. 74 A. to sq. metres.

Add together the following quantities:

46.

ac.	r.	p.
	2	36
73	1	25
224	3	32
	3	27

47.

Square links.
14,350
32,600
40,250
12,800

48.

Ha.	A.
2	13
4	65
22	3
	19

49. Find the total area of 100 allotments

(i) of 3 ac. 2 r. 10 p. each; (ii) of 3 Ha. 14 A. each.

50. How many allotments each of 6 ac. 0 r. 25 p. may be made from an estate of 215 ac. 1 r. 35 p.?

37. Tables of Cubic Measure.

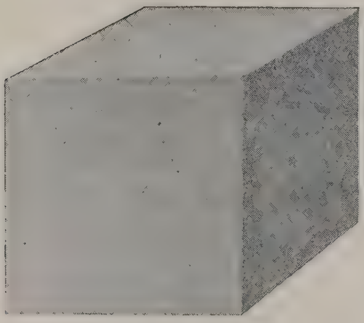


FIG. 1.

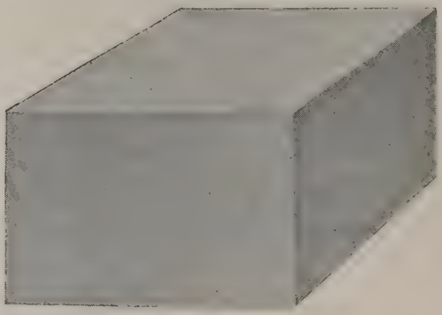


FIG. 2.

The solid figures represented above are known as **rectangular** blocks. If, as in Fig. 1, the length, breadth, and height are all equal, the figure is called a **cube.** If, as in Fig. 2, the length, breadth, and height are not all equal, the figure is called a **cuboid.**

A **cubic inch** is the amount of space enclosed within the faces of a cube of which each edge is 1 inch.

The terms cubic yard, cubic foot, cubic metre, cubic centimetre are to be understood in the same sense.

38. The **volume** of any solid figure is the amount of space enclosed within its bounding faces. This is measured by the number of cubic inches, or cubic centimetres (or other units of cubic measure) it contains.

39. *To find the* **volume** *of a rectangular block, having given its* **length, breadth,** *and* **height.**

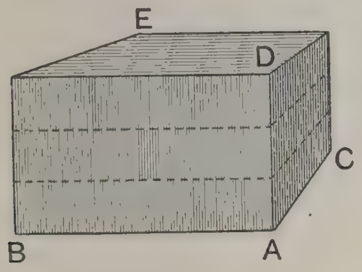


FIG. 1.

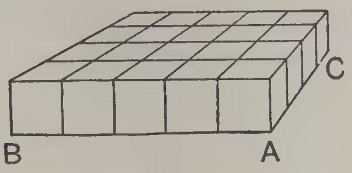


FIG. 2.

Consider the rectangular block whose length AB is 5 inches, breadth AC 4 inches, height AD 3 inches.

Fig. 1 shews that the solid may be divided into 3 equal *slices*, each 1 inch thick.

And each slice may be subdivided (as in Fig. 2) into cubical blocks whose edges are 1 inch, that is, into cubic inches.

Now the number of cubic inches in each slice is 5×4; so that the number of cubic inches in the whole solid is $5 \times 4 \times 3$, or 60.

Similarly if the length $= l$ linear units, the breadth $= b$ linear units, and the height $= h$ linear units,

then the *volume* $= (l \times b \times h)$ units of *cubic* measure.

This we may abridge by saying

volume *of rectangular solid* = **length** × **breadth** × **height.**

EXAMPLES.

(i) If a rectangular block of wood is 8 inches long, by 4 inches wide, and 3 inches thick, the amount of wood in it is given by

volume $= 8 \times 4 \times 3$, or 96, *cubic* inches.

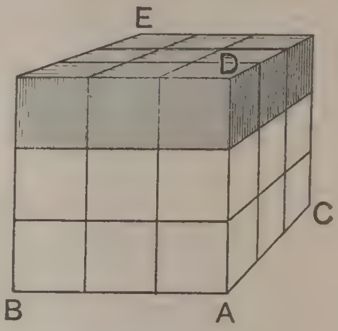
(ii) If the internal dimensions of a tank are as follows: length 6 feet, breadth 5 feet, depth 4 feet, the amount of water it can contain is given by

volume $= 6 \times 5 \times 4$, or 120, *cubic* feet.

(iii) If a room is 7 metres long, by 5 metres wide, and 4 metres high, the air-space within it is given by

volume $= 7 \times 5 \times 4$, or 140, *cubic* metres.

40. We can now shew how a Table of *Cubic* Measure is derived from the corresponding Table of *Linear* Measure.



The fig. ABCDE represents a cube of which each side is 1 *yard.* That is, ABCDE represents 1 *cubic yard.*

The diagram shews that the whole cube may be divided into 3 horizontal slices, each 1 foot thick; and each slice may be subdivided into cubical blocks whose edges are 1 foot, that is, into *cubic feet.*

∴ the whole cube contains $3 \times 3 \times 3$, or 27, *cubic feet.**

Hence from

1 yd. = 3 ft., we derive 1 *cubic* yd. = 3^3, or 27, *cubic* ft.

Similarly from

1 ft. = 12 in., we derive 1 *cubic* ft. = 12^3, or 1728, *cubic* in.

1 m. = 10 dm., ,, 1 *cubic* m. = 10^3, or 1000, *cubic* m.

and so on.

NOTES.

The following equivalents connecting **cubic** measures with measures of *capacity* and *weight* should be noted.

In the British system:

(i) **1 cubic inch** of **distilled water** (under average conditions of temperature and pressure) weighs about **252¼ grains.**

Now **1 gallon** contains **10 lbs.** of pure **water**; hence it may be shewn that

(ii) **1 gallon** is nearly equivalent to **277½ cubic inches.**

(iii) **1 cubic foot** of **water** weighs **996½ ounces**, nearly, (or **1000 ounces**, roughly).

In the Metric System:

(i) **1 cubic centimetre** of water weighs **1 gram.**

(ii) **1 litre** is equivalent to **1 cubic decimetre**, or 1000 cubic centimetres.

(iii) **1 litre** of water weighs **1 kilogram.**

---

* The continued product of three factors, each equal to a given number, is called the **cube** of that number. Thus

$3 \times 3 \times 3$ is read "3 *cubed*," and may be written 3^3.

$10 \times 10 \times 10$,, "10 cubed," ,, ,, ,, 10^3.

EXAMPLES II. 1.

Compound Rules. Cubic Measure.

1. What is meant by a *cubic foot*? Define the *volume* of a solid figure, and state in what units it may be expressed.

2. Draw a figure to represent a cube on a side of 1 metre; and explain how it may be subdivided into 1000 cubic centimetres.

3. Find the volume of the following:

(i) A block of stone measuring 12 inches in length, 10 inches in width, and 5 inches in thickness.

(ii) The content of a box measuring internally 4 feet long, 3 feet broad, and 2 feet deep.

(iii) The air-space in a room of which the length is 20 feet, the breadth 15 feet, and the height 12 feet.

(iv) A cubical mass of masonry of which each edge measures 5 feet.

4. How many cubic inches are there in

(i) 1 cubic yard; (ii) 10 cu. ft. 620 cu. in.?

5. Express six million cubic centimetres in terms of cubic metres.

6. In cutting a certain trench, 700 cubic yards of earth were excavated. What weight of earth was removed, reckoning 21 cubic feet to the ton?

7. (i) Given that 1 litre is equivalent to 1 cubic decimetre, how many litres are contained in 1 cubic metre?

(ii) Given that 1 cubic centimetre of water weighs 1 gram, what is the weight of 1 cubic metre of water?

8. One gallon of water weighs 10 lbs. In 1 ton of water how many gallons are there?

9. A coal truck carries 8 tons, and 42 cubic feet of coal go to the ton. How many trucks could be filled after cutting out 112 cubic yards of coal?

10. How many cubic inches of deal are there per foot in planking 10 inches wide and 3 inches thick?

11. A piece of squared timber is 7 yards long, and 1 foot both in width and thickness. How many cubic feet does it contain? And what is its weight (in cwt.) at the rate of 64 lbs. per cubic foot?

12. A tank is 5 feet long, 4 feet wide, and 3 feet deep; and I estimate that it can contain $1\frac{1}{2}$ tons of water. Supposing that 1 cubic foot of water weighs 1000 ounces, by how many pounds do I overestimate the weight of water?

CHAPTER III.

USE OF SIGNS AND SYMBOLS. NEGATIVE QUANTITIES. EQUATIONS.

41. IT is often convenient in Arithmetic to represent numbers by *letters*, for by this means we may express Arithmetical laws in such a way as to shew that they are *general*, that is to say, true for all numbers whatever; while if *figures* only were used, our conclusions might be true only for the special numbers so expressed.

For example, $5 \times 7 = 7 \times 5$ merely states that 5 multiplied by 7 gives the same result as 7 multiplied by 5. But if we allow a and b to stand for any numbers whatever, then $a \times b = b \times a$ states the same principle quite generally, that is, not for 5 and 7 only, but for any and every pair of numbers.

42. Any collection of numbers (or of letters standing for numbers) connected by the signs $+$, $-$, $\times$, $\div$, is called an **expression.**

Thus $14 + 2 - 3$, $a + b - c$, $45 + 4 \times 3 - 6 \div 2$, are expressions.

Parts of an expression separated by the signs $+$ or $-$ are called **terms.**

43. When two or more numbers are multiplied together to form a product, each is called a **factor** of the product.

Thus 2, 5, 7 are factors of $2 \times 5 \times 7$, or 70; and a, b, c are factors of the product $a \times b \times c$.

44. When *letters* stand for numbers the sign of multiplication between factors is often omitted.

For instance, $a \times b$, or $b \times a$, may be written ab.
$a \times b \times c$, or $c \times b \times a$, ,, ,, abc.
Similarly $7 \times a$, or $a \times 7$, ,, ,, $7a$.

NOTE. The omission of the sign × cannot be allowed if all the factors are expressed by figures. For instance, 75 already means *seventy-five*, and must not therefore stand for 7×5. In such cases the sign × is often replaced by a dot. Thus $2 \times 5 \times 7$ may be written $2 . 5 . 7$.

EXAMPLES. If $a=3$, $b=5$, and $c=8$,

then $ab=3\times5=15$; and $abc=3\times5\times8=120$;

$9b=9\times5=45$; $11c=11\times8=88$;

$7bc=7\times5\times8=280$; $12abc=12\times3\times5\times8=1440$;

$7a+4b=7\times3+4\times5=21+20=41$;

$4c+9b-13a=32+45-39=38$.

45. The product obtained by multiplying together several factors all equal to the same number is called a **power** of that number.

Thus 4×4 is called the **second power** of 4,

$6\times6\times6$ .... **third power** of 6,

$a\times a\times a\times a$ .... **fourth power** of a,

and so on.

For the sake of brevity the following notation is used:

$$4\times4=4^2;\ 6\times6\times6=6^3;\ a\times a\times a\times a=a^4;$$

and the small figure which indicates the number of equal factors is called the **index** of the power.

Thus in 2^3, 4^5, x^6 the *indices* are 3, 5, and 6 respectively.

46. The *second* and *third* powers of a number are known as its **square** and **cube** respectively.

Thus the square of 8, or $8^2=8\times8=64$;

the cube of 7, or $7^3=7\times7\times7=343$.

EXAMPLES. If $a=5$, $b=9$, and $c=4$,

then $7a^3=7\times a\times a\times a=7\times5\times5\times5=875$;

$8bc^2=8\times b\times c\times c=8\times9\times4\times4=1152$;

$$3a^2+2b^2+c^3=3\times5\times5+2\times9\times9+4\times4\times4$$
$$=75+162+64=301.$$

47. In such expressions as $5x$, $7a^3$, $8bc^2$, the *numerical* multipliers 5, 7, 8 are called the **coefficients** of x, a^3, and bc^2 respectively.

48. Division is often expressed by writing the divisor *under* or *after* the dividend with a line between them.

Thus $20\div4$ may be written $\frac{20}{4}$, or $20/4$;

$a\div b$ .... $\frac{a}{b}$, or a/b.

EXAMPLES III. a.

(*Most of the following examples may be taken orally.*)

1. Explain the difference in meaning between $3a$ and a^3, when a stands for any number.

2. If x and y represent two numbers, how would you express *eleven times* x, y *multiplied by thirteen*, *the product* of x and y, $x \times x$, and $y \times y \times y \times y$?

3. If a, b, and c stand for numbers, explain in words the meaning of

$$4a+3b, \quad 12ab, \quad 7c^3, \quad 9b^2c, \quad \frac{bc}{a}.$$

Find the value of

4. $4 \cdot 5 \cdot 6$. **5.** 5^3. **6.** $4^2 \times 5$. **7.** $5^2 \times 2^3$.

8. $1+3+3^2+3^3$. **9.** $2 \cdot 3^2+3 \cdot 2^2+2^3 \cdot 3^2$.

If $a=5$, $b=6$, $c=1$, $x=2$, $y=3$, find the value of

10. $10x$. **11.** y^2. **12.** xa. **13.** $7b$.

14. $4xy$. **15.** c^3. **16.** bx^2. **17.** $3ax^2$.

18. acy. **19.** $4a^2y$. **20.** $9c^2a$. **21.** $11ax^3$.

22. If x denotes a certain number, how would you express (i) the number next *above* x, (ii) the next above that, (iii) the number that is greater than x by 5?

How would you express (i) the number next *below* x, (ii) the next below that, (iii) the number less than x by 5?

23. I possess £40. How many pounds shall I have left (i) if I spend £15, (ii) if I spend £a, (iii) if I first spend £15 and then spend £a?

24. How many shillings have I left if out of £1 I spend (i) 5 shillings, (ii) 11 shillings, (iii) y shillings?

25. A boy is 13 years old. How old will he be (i) in 5 years, (ii) in 9 years, (iii) in k years? How old was he (i) 5 years ago, (ii) 9 years ago, (iii) k years ago?

26. What is the cost of 15 books (i) at 2*s*. each, (ii) at 5 shillings each, (iii) at x shillings each?

27. I want to give 5 apples to each of m children; how many apples are required? How many apples would be required if I wanted to give m apples to each of 5 children?

28. How would you express (i) the number of shillings in £a, (ii) the number of pence in p shillings, (iii) the number of cwt. in z tons?

29. How many pounds are there (i) in 100 shillings, (ii) in x shillings? How many shillings are there (i) in 72 pence, (ii) in y pence?

If $m=4$, $n=1$, $p=5$, $q=8$, find the value of

30. $3m^2n$. **31.** n^4. **32.** $7p^2m$. **33.** $3mnpq$.

34. $\frac{q}{2}$. **35.** $\frac{q}{m}$. **36.** $\frac{6m}{q}$. **37.** $\frac{2q}{mn}$.

38. A man is p years old: how old will he be in q years' time? How old was he q years ago?

39. Out of 3 shillings I spend z pence; how many pence have I left? How many pence should I have left if instead of 3 shillings I had x shillings to start with?

40. A man walks 4 miles an hour: how far will he walk at the same rate in 3 hours? How far in y hours?

41. A book-seller buys 13 copies of a book for 39 shillings: how much does he pay for each? What would he pay for each copy (i) if he bought x for 39 shillings, (ii) if he bought x copies for y shillings?

42. How much do I pay per mile, if I travel (i) 9 miles for 36 pence, (ii) 11 miles for 55 pence, (iii) m miles for n pence?

43. How many square inches are there in a sheet of paper (i) 12 inches long by 8 inches wide, (ii) 12 inches long by y inches wide, (iii) x inches long by y inches wide?

44. There are a boys and b girls. To each boy I give 5 shillings, to each girl I give 4 shillings. How many shillings do I give in all?

45. If a man earns 30 shillings a week, and a boy 10 shillings, at this rate what are the weekly earnings (i) of 3 men and 5 boys, (ii) of x men and y boys?

46. If tea costs 2*s.* 6*d.* a pound and coffee 1*s.* 6*d.* a pound, how many *pence* must be paid for x lbs. of tea and y lbs. of coffee?

47. Suppose a day's work consists of five lessons, and a boy's mark for each of them is 0; what is his score for the day? What is the value (i) of 0×3, (ii) of 0×7, (iii) of 9×0, (iv) of $0 \times$ a million?

If $x=3$, $y=5$, $z=0$; $p=1$, $q=7$, $r=9$, find the value of

48. $11z$. **49.** qz. **50.** $4xy+5z$. **51.** $7zy+qx$.

52. $4x+2p+r-z$. **53.** $13p-2y+3r-10x$.

54. $5xq-y+22-7r$. **55.** $93-x^2-3y^2-8p^3$.

56. $100p+10q+r$. **57.** $100r+10q+p$.

58. $3y+y^3-\frac{100}{y}$. **59.** $\frac{4r}{x}+5zq^2p-\frac{q^2}{q}$.

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60. What powers of 2 are 8 and 32? What powers of 3 are 9 and 81? Express 25 and 625 as powers of 5.

61. "*Every power of* 1 *is* 1." What does this mean? Shew why it is true.

62. Prove that $4^3 \times 4^2 = 4^5$. And in the same way shew that $a^3 \times a^2 = a^5$, where a stands for any whole number.

63. Simplify

(i) $2^3 \times 2 \times 2$. (ii) $3^2 \times 3 \times 3^2$. (iii) $a^3 \times a^2 \times a$.

49. Quantities which are preceded by the sign + are said to be **positive**; those to which the sign − is prefixed are said to be **negative**. When no sign is prefixed the + sign is to be understood. These signs are frequently used to denote a *quality* possessed by the quantities to which they are attached, as explained in the following illustrations:

(i) Suppose a trader gains £100 and then loses £70, the result of his trading is a *gain* of £30, that is +£100 − £70 = +£30; and the +£30 denotes that he is £30 better off than when he began.

If, however, he had first gained £70 and then lost £100, the result of his trading would be a *loss* of £30, that is, +£70 − £100 = −£30, and the −£30 denotes that he is £30 worse off than when he began, or that he now has a *debt* of £30.

Thus we see that the −£30 denotes a quantity *equal in magnitude, but opposite in character* to the +£30.

Again if the trader had first gained £70 and then lost £70, the loss would exactly balance the gain, that is +£70 − £70 = £0. Thus he would be in the same position as when he began.

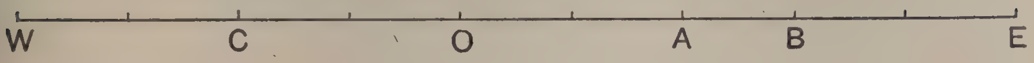
(ii) Again, suppose a man to walk 5 miles due East and then to walk back 3 miles due West, his position relative to the starting point would be +5 miles − 3 miles, or +2 miles, the +2 miles denoting the distance he was due East of his starting point.

If he had walked 3 miles due East and then back 5 miles due West, his position relative to the starting point would be +3 miles − 5 miles, or −2 miles, the −2 miles denoting the distance he was due West of his starting point.

Thus we see that −2 miles denotes a distance *equal in magnitude, but opposite in direction* to that denoted by 2 miles.

In general, if we represent positive quantities by a series of steps taken in one direction, we may represent negative quantities by a series of steps taken in the opposite direction.

This may be illustrated graphically as follows:



Let O be the starting point and let the line WOE be marked in centimetres, each centimetre representing 1 mile. Also let the direction OE (from left to right) be considered as due East, and OW (from right to left) as due West.

Then 5 miles Eastwards followed by 3 miles Westwards will be represented by motion from O to E followed by motion in the opposite direction from E to A.

Thus $\mathrm{OA} = \mathrm{OE} - \mathrm{EA} = +5 \text{ cm.} - 3 \text{ cm.} = +2 \text{ cm.}$

which represents $+2$ miles.

Again, 3 miles Eastwards followed by 5 miles Westwards will be represented by motion from O to B followed by motion in the opposite direction from B to C.

Thus $\mathrm{OC} = \mathrm{OB} - \mathrm{BC} = +3 \text{ cm.} - 5 \text{ cm.} = -2 \text{ cm.}$

which represents -2 miles.

(iii) On a Centigrade thermometer 15° C. means 15° *above* the freezing point, and $-15°$ C. denotes 15° *below* the freezing point.

From the above examples it will be understood that $+5$, for example, will denote a quantity *greater* than 0 by 5 units, whereas -5 will denote a quantity that is *less* than 0 by 5 units, the two quantities being of the same *absolute value* but *of opposite character*.

50. The value of an expression is the same in whatever order the terms are taken.

Thus, for example, $-£4 + £9 + £5 - £3 - £2$ may be regarded as a combination of two gains and three losses, and the final result is the same in whatever order the losses and gains occur.

More generally, if we use letters to represent numbers, expressions, such as $a - b - c$, $-b + a - c$, $-b - c + a$, might be used for a *gain*, represented by a, together with losses represented by b and c.

Or the same expression might be used to denote a steps in one direction together with b steps and c steps in the opposite direction. In each case the result is the same in whatever order the terms are considered.

The following Examples will serve to illustrate the principles here explained.

EXAMPLES III. b.

1. A trader gains £20, loses £42, and then gains £10; a second trader loses £35, gains £50, and then loses £12. In each case write down the result of the three transactions.

2. A merchant has a gain represented by £a, losses represented by £b and £c, and then a gain represented by £d. Express in symbols the final result of his trading.

3. A man walks 7 miles due South, then 8 miles due North, and finally 4 miles due South. What is his final position relatively to the starting point? Illustrate by a diagram on the scale of Art. 49, giving his distance in centimetres measured from O, the starting point.

4. Two cricket counties play 16 matches; one wins 10 and loses 6, and the other wins 7 and loses 9. Express the two results, allowing a gain of one point for a win and a loss of one point for a defeat.

5. In the night a Centigrade thermometer falls to $-8°$, and in the day-time it rises to $12°$. How many degrees are there between the readings?

6. A Centigrade thermometer rises to $9°$ in the day-time and falls $15°$ during the night; what is the night reading?

7. Two men each fire 20 shots at a mark and agree to register 4 points for every hit and to deduct 3 points for every miss. One hits the mark 12 times, the other 8 times. Express their separate scores by means of the proper signs.

8. A man who can row 5 miles an hour in still water rows for one hour *against* a stream flowing at the rate of 2 miles an hour. He then turns, and, using the same force, rows *with* the stream for one hour. Illustrate by a diagram, and express his final distance from his starting point with the proper sign.

9. A has £a, and B has £b. A owes £x to B, and B owes £y to A. How many pounds will each man have when their debts are paid?

The Use of Brackets.

51. Brackets ( ) are used to indicate that the terms enclosed within them are to be considered as one quantity.

The expression $8+(13+5)$ means that 13 and 5 are to be added and their sum added to 8. It is clear that 13 and 5 may be added to 8 separately or together without altering the result.

Thus $$8+(13+5)=8+13+5=26.$$

Similarly, if the letters a, b, c are used to represent any numbers, $a+(b+c)$ means that the sum of b and c is to be added to a; and since b and c may be added separately or together, it follows that

$$a+(b+c)=a+b+c.$$

Again, $8+(13-5)$ means that to 8 we are to add the excess of 13 over 5; now if we add 13 to 8 we have added too much by 5, and must therefore take 5 from the result.

Thus $8+(13-5)=8+13-5=16$.

Similarly $a+(b-c)$ means that to a we are to add b, diminished by c. If therefore we add b we must afterwards subtract c.

Thus $a+(b-c)=a+b-c$.

These results shew that *when an expression within brackets is preceded by the sign +, the brackets may be removed without making any change in the expression.*

Conversely, *any part of an expression may be enclosed within brackets and the sign + prefixed, the sign of every term within the brackets remaining unaltered.*

Thus $9-7+8-5=9-7+(8-5)$

and $a-b+c-d=a-b+(c-d)$.

52. The expression $9-(3+2)$ means that from 9 we are to subtract the sum of 3 and 2. To take the sum of 3 and 2 from 9 gives the same result as to subtract them *separately* from 9.

Thus $9-(3+2)=9-3-2$.

Similarly $a-(b+c)=a-b-c$.

Again, $9-(3-2)$ means that from 9 we are to subtract the excess of 3 over 2. If we subtract 3 we shall have taken away too much by 2, and must therefore add 2 to obtain the correct result.

Thus $9-(3-2)=9-3+2$.

Similarly $a-(b-c)=a-b+c$.

These results shew that *when an expression within brackets is preceded by the sign −, the brackets may be removed if the sign of every term within the brackets is changed.*

Conversely, *any part of an expression may be enclosed within brackets and the sign − prefixed, provided the sign of every term within the brackets is changed.*

Thus $9-7-3+8-5=9+8-7-3-5=9+8-(7+3+5)$,

and the expression $a-b+c+d-e$ may be written in any of the following ways:

$$a-(+b-c-d+e),$$
$$a-b-(-c-d+e),$$
$$a-b+c-(-d+e).$$

53. Sometimes a line called a **vinculum** is drawn over numbers or symbols which are to be taken together.

Thus $a-(b+c)$ and $a-\overline{b+c}$ are used with the same meaning.

54. Suppose A owes £5 to B. Then A's debt is represented by $-$£5. Next suppose that B forgives this debt, that is to say, *takes it away* or *subtracts it*. This act is represented by $-$(debt of £5) or $-(-$£5), and is evidently equivalent to *giving* £5 to A, for now A is £5 better off than before.

Thus $$-(-5)=+5.$$

Similarly we conclude that $-(-a)$ is equivalent to $+a$.

55. When all the terms of an expression *taken as a whole* are to be multiplied or divided by a given number, the expression is enclosed in brackets.

Thus the expression $(9+5)\times 6$, which may also be written $6(9+5)$, means that *the sum of* 9 *and* 5, namely 14, is to be multiplied by 6, giving the result 84.

Now we have seen in Art. 18 (iii) that the same value is obtained by multiplying the two terms 9 and 5 separately by 6, and adding the results.

Thus $$6(9+5)=54+30=84.$$

Or, generally, $$m(a+b)=am+bm.$$

In the same way $$6(9-5)=6\times 4 \quad =24,$$

and $$6(9-5)=54-30=24.$$

Or, generally, $$m(a-b)=am-bm.$$

Conversely, just as 11 *tens* + 8 *tens* = 19 *tens*,

so $$11x+8x=19x.$$

Similarly $$15y-3y=12y,$$

and $$15y-3y+2y=14y.$$

56. When the terms of an expression *differ only in their coefficients* they can always be combined as in the last article. The process is known as **collecting coefficients.**

Such expressions as $7a+3b-c$, $5ab-2cd$, where the letters (or combinations of letters) are different in different terms, cannot be simplified unless we know the numerical values of the letters.

57. *To prove that* (i) $(a+b)(c+d)=ac+bc+ad+bd$;

(ii) $(a+b)^2=a^2+2ab+b^2$;

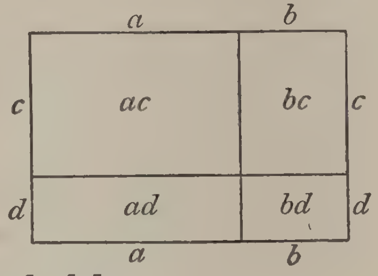
(iii) $(a+b)(a-b)=a^2-b^2$.

(i) We have seen that $m(c+d)=cm+dm$. (Art. 55.)

Write $a+b$ in the place of m, then

$$\begin{aligned}(a+b)(c+d)&=c(a+b)+d(a+b)\\&=ac+bc+ad+bd.\end{aligned}$$

This may also be shewn graphically. In the following diagram we have a rectangle whose adjacent sides are $a+b$ and $c+d$ units respectively.

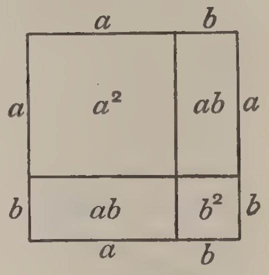


The whole area $=(a+b)(c+d)$ square units, and it is made up of four smaller rectangles whose areas are ac, bc, ad, bd, square units respectively.

Thus $\qquad (a+b)(c+d)=ac+bc+ad+bd.$

Similarly $\qquad (a+b)(c-d)=ac+bc-ad-bd.$

(ii)
$$\begin{aligned}(a+b)^2&=(a+b)(a+b)\\&=a^2+ab+ab+b^2\\&=a^2+2ab+b^2.\end{aligned}$$



Similarly $\qquad (a-b)^2=a^2-2ab+b^2.$

(iii) $\qquad (a+b)(a-b)=a^2+ab-ab-b^2=a^2-b^2.$

EXAMPLES. (i)
$$\begin{aligned}103\times97&=(100+3)(100-3)=100^2-3^2\\&=10000-9=9991.\end{aligned}$$

(ii)
$$\begin{aligned}339^2-319^2&=(339+319)(339-319)\\&=658\times20=13160.\end{aligned}$$

58. In such expressions as $(9+6-5)\div2$, the brackets shew that the group of terms $9+6-5$, taken as a whole, namely 10, is to be divided by 2, giving the result 5.

The expression $(9+6-5)\div2$ may also be written $\dfrac{9+6-5}{2}$, in which form the line acts as a vinculum to shew that the terms $9+6-5$ are to be taken as a whole.

Example. *Distinguish between*

(i) $13 - 5 \times 2$ *and* $(13 - 5) \times 2$;
(ii) $(16 - 4 + 9) \div 3$ *and* $16 - 4 + 9 \div 3$;
(iii) $(24 - 12) \div (6 - 3)$ *and* $24 - 12 \div 6 - 3$.

(i) $13 - 5 \times 2$ means that the product 5×2 has to be subtracted from 13; thus the result is $13 - 10$, or 3.

$(13 - 5) \times 2$ means that $13 - 5$, taken as a whole, has to be multiplied by 2; thus the result is 8×2, or 16.

(ii) $(16 - 4 + 9) \div 3$ means that $16 - 4 + 9$, taken as a whole, has to be divided by 3; thus the result is $21 \div 3$, or 7.

$16 - 4 + 9 \div 3$ means $12 + \frac{9}{3}$; thus the result is $12 + 3$, or 15.

(iii) $(24 - 12) \div (6 - 3)$ means that the difference between 24 and 12 has to be divided by the difference between 6 and 3; thus the result is $12 \div 3$, or 4.

$24 - 12 \div 6 - 3$ means $24 - \frac{12}{6} - 3$; thus the result is $24 - 2 - 3$, or 19.

From these cases it appears that in an expression which involves the signs $+$, $-$, $\times$, $\div$, the order of the operations is important.

Operations within brackets must be performed first; when no brackets are used, all operations of multiplication and division must be performed before those of addition and subtraction.

EXAMPLES III. c.

In Examples 1–12 read off the expressions when the brackets have been removed.

1. $8 + (5 - 3)$. **2.** $10 - (5 + 3)$. **3.** $(9 - 2) + 7$.
4. $13 - (8 - 3)$. **5.** $7 + (3 + 2)$. **6.** $(11 - 3) - (8 - 2)$.
7. $a + (b - c)$. **8.** $a - (b + c)$. **9.** $(a - b) + c$.
10. $a - (b - c)$. **11.** $b + (c + a)$. **12.** $(a - b) - (c - d)$.

Simplify

13. $15 - 3 \times 2$, $(15 - 3) \times 2$. **14.** $(18 - 6) \div 3$, $18 - 6 \div 3$.
15. $15 + 4 \times 3 + 6$. **16.** $(15 + 4) \times (3 + 6)$.
17. $32 - 8 \div 4 + 2$. **18.** $(32 - 8) \div (4 + 2)$.
19. $45 + 3 \times 5 - \frac{12}{3}$. **20.** $45 - (3 + 5) \div \frac{12}{3}$.
21. $(27 - 5) - 2(16 - 10)$. **22.** $(27 - 5) - \frac{16 - 10}{2}$.

Simplify, by removing brackets and collecting coefficients,

23. $7a + 3a - 4a$. **24.** $10b - 3b + 8b$. **25.** $10x - (8x - 3x)$.
26. $14m - m(6 + 2)$. **27.** $(15 - 3)y - 6y$. **28.** $(12 - 7)x - (4 - 3)x$.

59. Illustrations leading up to Equations. The value of an unknown number can be found if we are told how it is connected with one or more known numbers. For instance

(i) *A certain number when added to* 5 *gives* 11 *as the sum.* Here the number in question is evidently 6.

(ii) *If a certain number is taken from* 20, *the remainder is* 13. Here the number is clearly 7.

(iii) *A certain number multiplied by* 8 *gives* 32 *as the product.* We see at once that the number is 4.

(iv) *A certain number divided by* 9 *gives* 5 *as the quotient.* Here the required number is evidently 45.

EXAMPLES III. d. (*Oral.*)

1. What number when added to 8 gives 20 as the sum?

2. What number must be taken from 16 to make the remainder 7?

3. If from a certain number 11 is taken, the remainder is 24. What must the number be?

4. Two numbers added together make a sum of 27. One is 18; what is the other?

5. The difference between two numbers is 14, and the greater is 30; what is the less?

6. What number when multiplied by 7 gives 56?

7. The product of two numbers is 84. One of them is 12; what is the other?

8. Three numbers multiplied together give 72. Two of them are known to be 9 and 4; what is the third?

9. What number divided by 11 gives 12 as the quotient?

10. In a division sum the dividend is 96, the quotient 8, and there is no remainder. What is the divisor?

If x denotes an unknown number, state its value when

11. $x+7=13$. **12.** $8+x=21$.

13. $x-4=11$. **14.** $19-x=12$.

15. $18=x+9$. **16.** $14=x-7$.

17. $x\times 3=9$. **18.** $x\times 5=20$. **19.** $4\times x=28$.

Give the value of y when

20. $y+3=4+7$. **21.** $28-y=17$. **22.** $y-3=10-3$.

23. $7y=35$. **24.** $48=8y$. **25.** $11y=44$.

26. $y\div 4=7$. **27.** $8=y\div 5$. **28.** $\frac{y}{3}=6$.

60. Equations. An **equation** is a statement that two algebraical expressions are equal.

Thus (i) $x+1=5$,

(ii) $3x+2=14-x$

are equations.

The parts of an equation separated by the sign of equality are called **sides** of the equation, and are distinguished as the *right side* and the *left side.*

It will be found on trial that there is only one numerical value of x which makes each of the equations (i) and (ii) a true arithmetical statement.

Thus $x+1=5$ is only true when $x=4$,

and $3x+2=14-x$ is only true when $x=3$.

These values are said to **satisfy** the equations.

The object of the present section is to shew how to find the values which satisfy equations of the simpler kinds.

61. The symbol whose value it is required to find in any equation is called the **unknown quantity.** The process of finding its value is called **solving the equation.** The value so found is called the **root** of the equation.

62. In the solution of the equations with which we are concerned the process depends only on the following **axioms.**

1. If to equals we add equals the sums are equal.
2. If from equals we take equals the remainders are equal.
3. If equals are multiplied by equals the products are equal.
4. If equals are divided by equals the quotients are equal.

EXAMPLE 1. *Solve the equation* $7x=14$.

Dividing both sides by 7 (Axiom 4), we get

$$x=2.$$

EXAMPLE 2. *Solve the equation* $\frac{x}{2}=5$.

Multiplying both sides by 2 (Axiom 3), we get

$$x=10.$$

EXAMPLE 3. *Solve the equation* $7x-2x=11+2-3$.

First collect the terms on each side; thus

$$5x=10.$$

Dividing by 5 (Axiom 4), we get

$$x=2.$$

Example 4. *Solve the equation* $3(x-2)-2=x+12$.

Removing the brackets, we have

$$3x-6-2=x+12. \qquad \text{..........................(i)}$$

Subtracting x from each side, we get

$$3x-x-8=12.$$

Adding 8 to each side, we have

$$3x-x=12+8\,; \qquad \text{..................................(ii)}$$

$$\therefore\ 2x=20\,;$$

dividing by 2,

$$x=10.$$

Note. By carefully examining the lines marked (i) and (ii) it will be seen that *any term may be removed from one side and transferred to the other by merely changing its sign.*

63. Beginners should **verify**, that is, prove the correctness of their solutions by substituting, in both sides, the value obtained for the unknown quantity.

In the last equation $\quad 3(x-2)-2=x+12,$

if

$$x=10,$$

$$\text{the left side}=3\times 8-2=22,$$

and

$$\text{the right side}=10+12=22.$$

Since these two results are equal the solution is correct.

64. From the above examples it will be seen that the object in each case is to reduce the equation, by means of the axioms, until it consists of *a single term containing the unknown quantity on one side, and a single known term on the other.* The root is then found by dividing each side by the coefficient of the unknown quantity.

EXAMPLES III. e.

(*Examples* 1–17 *may be taken orally.*)

Find the values which satisfy the following equations:

1. $3x=12$.
2. $4x=16$.
3. $6x=18$.
4. $7x=21$.
5. $6=2x$.
6. $11x=33$.
7. $9x=54$.
8. $52=13x$.
9. $\frac{x}{2}=5$.
10. $\frac{x}{3}=4$.
11. $\frac{x}{4}=30$.
12. $\frac{x}{5}=16$.
13. $x+2x=10-7$.
14. $3x+5x=28-4$.
15. $8x-4x=24$.
16. $3x-x=10$.
17. $17x-4x=26$.

Find the values which satisfy the following equations :

18. $7x-4=17$. **19.** $3x-5=10$. **20.** $2x+15=23$.
21. $4x-3=3x+4$. **22.** $8x-9=39-4x$. **23.** $5x+3=15-x$.
24. $3x+4=5(x-2)$. **25.** $2x+3=16-(2x-3)$.
26. $2(x+7)=30-(x-2)$. **27.** $15x-1=17+3(6-x)$.
28. $48-x=7(18-2x)$. **29.** $23(3x-6)=3(5x+8)$.

65. Simple Applications. We shall now shew how arithmetical examples may be solved by the aid of equations. The process consists of expressing the verbal statement of the question by means of signs and symbols ; an equation is thus formed which can be solved by the methods already explained. After the first example the solution will be left as an exercise for the pupil.

EXAMPLE 1. *The difference between two numbers is* 8 ; *if* 1 *be added to five times the smaller, the result will be twice the greater : find the numbers.*

Let x be used to denote the smaller number ; then $x+8$ denotes the greater.

Five times the smaller, increased by 1, is denoted by $5x+1$;

and twice the greater is denoted by $2(x+8)$;

hence, by the conditions of the question,

$$\begin{aligned}5x+1&=2(x+8)\\&=2x+16.\end{aligned}$$

Subtracting $2x+1$ from each side, we get

$$3x=15 ;$$

$$\therefore\ x=5,$$

$$\text{and } x+8=13.$$

Thus the two numbers are 5 and 13.

EXAMPLE 2. *Divide* £47 *between* A, B, *and* C, *so that* A *may have* £10 *more than* B, *and* B £8 *more than* C.

Let x represent the *number* of pounds that C has, then B has $x+8$ pounds, and A has $x+8+10$ pounds.

Hence $$x+(x+8)+(x+8+10)=47 ;$$

whence it will be found that $x=7$;

so that C has £7, B £15, A £25.

NOTE. The symbol x represents a *number*, and such loose and inexact expressions as "*Let x equal what C has,*" or "*Let x equal C's money,*" must never be used.

Example 3. *A sum of £8. 17s. is made up of 124 coins which are either florins or shillings: how many are there of each?*

Let x denote the number of florins, then $124-x$ = the number of shillings.

But x florins + $(124-x)$ shillings make up £8. 17*s.*;

∴ expressing all in shillings, we have the equation

$$2x+124-x=177;$$

whence $x=53$. Thus there were 53 florins and 71 shillings.

Note. It is important to express all the quantities in the same denomination: shillings are here selected as being the most convenient.

EXAMPLES III. f.

1. Find two numbers whose sum is 28, and whose difference is 4.

2. Divide 45 into two parts so that twice the greater equals three times the less.

3. If double a number is increased by 3, and the result multiplied by 4, the product is 52; find the number.

4. Divide 60 into two parts, so that three times the greater may exceed 100 by as much as eight times the less falls short of 200.

5. Divide £67 between A, B, and C, so that A may have £15 more than B, and B £8 more than C.

6. Divide £66 between A, B, and C, so that B's share may be double of A's, and C may have £4 less than B.

7. If a sum of £85 is divided between A, B, and C, so that A has £10 less than B, and C has three times as much as A; find the share of each.

8. Divide £188 between A, B, and C, so that A may have £37 less than B, and C's share may be £11 more than twice A's share.

9. A purse contains £2. 13*s.* 6*d.* in shillings and sixpences. The number of coins is 67; how many are there of each kind?

10. A and B have £12 between them; A receives £1. 5*s.* from B and finds that he has seven times as much money as B: how much had each at first?

11. A has three times as much money as B; after giving B ten shillings he has only twice as much: what had each at first?

12. Two boys together have £1. 10*s.*; if one had 6*s.* less and the other 9*s.* more, the former's money would be one-half that of the latter: what has each of them?

CHAPTER IV.

FACTORS. PRIME NUMBERS. HIGHEST COMMON FACTOR. LOWEST COMMON MULTIPLE.

66. ONE number is a **factor** (or *measure*) of another when it divides the other without remainder.

Thus 3 and 5 are factors of 15; 2, 3, 4, 6 are factors of 12.

The word *measure* is going out of use, but it is convenient to retain it in connection with concrete quantities.

Thus a foot, a yard, and a furlong are *measures* of a mile. So also, a shilling, a florin, and a half-crown are *measures* of a sovereign.

67. A number which is exactly divisible by another number is called a **multiple** of it.

Thus 63 is a multiple of 7, and of 9.

And, if m stands for any whole number, mx is a multiple of x.

68. All multiples of 2 are called **even** numbers.

Thus 2, 4, 6, 8, 10, ... are even numbers.

Numbers which are not multiples of 2 are called **odd** numbers.

Thus 1, 3, 5, 7, 9, ... are odd numbers.

And generally, if n stands for any whole number, then $2n$ is the symbol for any *even* number, and $2n+1$ represents any *odd* number.

69. A **prime number** (or a **prime**) is one which has no factors except itself and 1.

Thus 2, 3, 5, 7, 11, 13, 17, 19, ... are primes.

70. A **composite** number is one which has other factors besides itself and 1.

Thus 6, 15, 27 are composite numbers.

71. Two numbers are said to be **prime to each other** when their only common factor is 1.

Thus 10 and 13, 15 and 17, 40 and 49 are pairs of numbers prime to each other. It is to be noticed that the numbers themselves are not necessarily prime numbers.

72. It is important to notice the way in which any whole number can be expressed in terms of powers of 10. For example,

$$4693 = 4 \times 1000 + 6 \times 100 + 9 \times 10 + 3$$
$$= 4 \times 10^3 + 6 \times 10^2 + 9 \times 10 + 3.$$

Similarly, any number N consisting of four digits a, b, c, d, in order from left to right, may be expressed as follows:

$$N = a \times 10^3 + b \times 10^2 + c \times 10 + d.$$

Tests of Divisibility.

73. In the present section the word *divisible* is to be understood as *divisible without remainder.*

To find when a number is divisible by 2 *or* 5.

Any number which ends with a cipher is a multiple of 10, and therefore divisible by 2 and by 5.

Any number greater than 10, which does not end in a cipher is made up of a multiple of 10 together with the number expressed by the units' digit. Hence *if this last digit is divisible by* 2 *or* 5 *the whole number is divisible by these factors respectively.*

EXAMPLES. (i) $38416 = 38410 + 6 =$ a multiple of $10 + 6$;
(ii) $38415 = 38410 + 5 =$ a multiple of $10 + 5$.

Thus (i) is a multiple of 2; (ii) is a multiple of 5.

NOTE. All even numbers must end in 2, 4, 6, 8, or 0;
All odd numbers ,, ,, 1, 3, 5, 7, or 9.

74. *To find when a number is divisible by* 4 *or* 25.

Any number ending in two ciphers is a multiple of 100, and therefore divisible by 4 and by 25.

Any number greater than 100 which does not end with two ciphers is made up of a multiple of 100 together with the number formed by the last two digits. Hence *if this last number is divisible by* 4 *or* 25 *the whole number is divisible by these factors respectively.*

EXAMPLES. (i) $34736 = 34700 + 36 =$ a multiple of $100 + 36$;
(ii) $34775 = 34700 + 75 =$ a multiple of $100 + 75$.

Thus (i) is divisible by 4; (ii) is divisible by 25.

75. *To find when a number is divisible by* 8 *or* 125.

Proceeding as before we may shew that *a number is divisible by* 8 *or* 125 *when the number formed by its last three digits is divisible by these factors respectively.*

EXAMPLES. (i) 43136 and 43560 are divisible by 8;
(ii) 43625 and 43500 are divisible by 125.

76. *To find when a number is divisible by* 9 *or* 3.

Take any number such as 58476.

Now $58476 = 50000 + 8000 + 400 + 70 + 6$.

Also
$$\begin{aligned}
50000 &= 5 \times 10000 = 5(9999+1) = (\text{a mult. of } 9) + 5, \\
8000 &= 8 \times 1000 = 8(999+1) = (\text{a mult. of } 9) + 8, \\
400 &= 4 \times 100 = 4(99+1) = (\text{a mult. of } 9) + 4, \\
70 &= 7 \times 10 = 7(9+1) = (\text{a mult. of } 9) + 7, \\
6 &= 6.
\end{aligned}$$

Hence, by addition,

$$58476 = \text{a multiple of } 9 + \text{the sum of the digits } 6,\ 7,\ 4,\ 8,\ 5.$$

In the same way it may be shewn that *any* number greater than 10 is a multiple of 9 + the sum of its digits; hence *a number is divisible by* 9 *only when the sum of its digits is so divisible.*

Again since 9 is a multiple of 3,

$$\text{any number} = \text{a multiple of } 3 + \text{the sum of its digits};$$

that is, *a number is divisible by* 3 *when the sum of its digits is so divisible.*

NOTES. (i) A number is divisible by 6 when it is even and also satisfies the test for divisibility by 3.

(ii) When a number is divided by 9 or 3 the remainder is the same as when the sum of the digits is divided by 9 or 3.

Hence also the remainder on dividing a number by 9 may be obtained by "casting out the nines" (Art. 72) from the sum of the digits.

EXAMPLE. *Find the remainders when* (i) 8145, (ii) 5807613 *are divided by* 9.

(i) The sum of the digits = 18, which is a multiple of 9, thus the number is divisible by 9.

(ii) Casting out nines from the sum of the digits, as explained in Art. 12, we have

4, 10, **1**, 8, 16, **7**, 12, **3**.

Thus the remainder is 3.

77. *To find when a number is divisible by* 11.

We first note the following results:

$$\begin{aligned}
10 &= 11 - 1, \\
100 &= 99 + 1 = (\text{a multiple of } 11) + 1, \\
1000 &= 990 + 11 - 1 = (\text{a multiple of } 11) - 1, \\
10000 &= 9999 + 1 = (\text{a multiple of } 11) + 1, \\
100000 &= 99990 + 11 - 1 = (\text{a multiple of } 11) - 1
\end{aligned}$$

and so on.

Now take any number such as 763425.

Then $763425=5+20+400+3000+60000+700000$.

Also

$$\begin{aligned} 5 &= && 5,\\ 20 &= 2\times 10 &&= (\text{a multiple of } 11)-2,\\ 400 &= 4\times 100 &&= (\text{a multiple of } 11)+4,\\ 3000 &= 3\times 1000 &&= (\text{a multiple of } 11)-3,\\ 60000 &= 6\times 10000 &&= (\text{a multiple of } 11)+6,\\ 700000 &= 7\times 100000 &&= (\text{a multiple of } 11)-7. \end{aligned}$$

Hence, by addition,

$$\begin{aligned} 763425 &= (\text{a multiple of } 11)+5-2+4-3+6-7\\ &= \text{a multiple of } 11+(5+4+6)-(2+3+7). \end{aligned}$$

Now $5+4+6$ = the sum of the digits in the 1st, 3rd, 5th (or *odd*) places, and $2+3+7$ = the sum of the digits in the 2nd, 4th, 6th (or *even*) places.

Since the reasoning is quite general we conclude that *a number is divisible by* 11 *provided that the difference between the sum of the digits in the even places and the sum of the digits in the odd places is divisible by* 11, *or equal to* 0.

78. With the exception of divisibility by 7 (for which there is no useful test) we have now tests for divisibility by all numbers up to 12.

They may be summarised as follows:

A number is divisible by

2, if it ends in 0, or in a digit which is a multiple of 2;

3, if the sum of the digits is divisible by 3;

4, if it ends in 00, or in two digits which form a number divisible by 4;

5, if it ends in 0 or 5;

6, if it is even and also satisfies the test for divisibility by 3;

8, if it ends in 000, or in three digits which form a number divisible by 8;

9, if the sum of the digits is divisible by 9;

10, if it ends in 0;

11, if the difference between the sums of the digits in the even and odd places is 0 or a multiple of 11;

12, if it is divisible by 4 and by 3.

79. By means of Art. 76 we may give the reason for the Test of Multiplication by "casting out nines" to which reference was made in Art. 12.

When division is not exact we know that

$$\textit{dividend} = (\textit{divisor} \times \textit{quotient}) + \textit{remainder}.$$

Let a and b denote two numbers which when divided by 9 give quotients c and d, and remainders e and f respectively. That is, e and f are the remainders after casting out nines from the sums of the digits of the two numbers a and b.

Then $$a = 9c + e, \quad b = 9d + f.$$

$$\begin{aligned} \therefore \text{ product of } a \text{ and } b &= \text{product of } 9c+e \text{ and } 9d+f \\ &= (9c+e) \times (9d+f) \\ &= 9c \,.\, 9d + 9de + 9cf + ef. \end{aligned}$$

$$\therefore \text{ product of } a \text{ and } b = \text{a multiple of } 9 + \text{product of } e \text{ and } f.$$

Hence the remainder left after casting out nines from the product of the two numbers a and b is equal to the remainder left after casting out nines from the product of e and f. See Art. 12.

Resolution into Prime Factors.

80. Suppose it is required to find all the prime numbers less than 100.

We first write down all the numbers in their natural order up to 100.

1, 2, 3, ~~4~~, 5, ~~6~~, 7, ~~8~~, ~~9~~, ~~10~~,
11, ~~12~~, 13, ~~14~~, ~~15~~, ~~16~~, 17, ~~18~~, 19, ~~20~~,
~~21~~, ~~22~~, 23, ~~24~~, ~~25~~, ~~26~~, ~~27~~, ~~28~~, 29, ~~30~~,

and so on.

Beginning from 2, we strike out every second number, 4, 6, 8, ..., thus rejecting multiples of 2. The next prime number is 3; we therefore strike out every third number 6, 9, 12, ..., thus rejecting multiples of 3. Some of these being also multiples of 2 will have been rejected already.

From 5 we strike out every fifth number (not already struck out) and thus reject all multiples of 5. Proceeding in this way we eventually reject all multiples of the prime numbers 2, 3, 5, 7, ... Hence when the process is complete the numbers left are prime.

This method was discovered by Eratosthenes in the third century B.C., and is referred to as the **Sieve of Eratosthenes**.

It will be found that the prime numbers less than 100 are

1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37,
41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

81. A number is said to be resolved into prime factors when it is expressed in the form of a product only of prime numbers or their powers.

Thus $700 = 7 \times 10 \times 10 = 7 \times 2 \times 5 \times 2 \times 5 = 7 \times 2^2 \times 5^2$.

82. A composite number can often be expressed in factors in different ways, but there is *only one way of resolution into prime factors.*

Thus $144 = 12 \times 12 = 9 \times 16 = 4 \times 36$,

but $3 \times 3 \times 2 \times 2 \times 2 \times 2$, or $3^2 \times 2^4$ is the only way of writing it in prime factors.

EXAMPLE 1. *Resolve* 7020 *into prime factors.*

$$\begin{array}{r|l} 10 & 7020 \\ \hline 9 & 702 \\ \hline 4 & 78 \\ \hline & 13 \end{array}$$

The number is obviously divisible by 10; the first quotient satisfies the test for 9, and the next quotient that for 4. The quotient 13 is prime.

Thus the number $= 10 \times 9 \times 4 \times 13 = 2 \times 5 \times 3^2 \times 2^2 \times 13$
$= 2^3 \times 3^2 \times 5 \times 13$.

When divisors are not evident the prime numbers should be tried in succession.

EXAMPLE 2. *Resolve* 9163 *into prime factors.*

$$\begin{array}{r|l} 7 & 9163 \\ \hline 7 & 1309 \\ \hline 11 & 187 \\ \hline & 17 \end{array}$$

Taking the primes in order we find that the tests for 2, 3, 5 are not satisfied. We next try 7 as often as possible, and then 11. The quotient 17 is prime.

$\therefore$ the number $= 7 \times 7 \times 11 \times 17 = 7^2 \times 11 \times 17$.

EXAMPLE 3. *To find whether* 467 *is prime or not.*

We find by trial that 467 is not divisible by any of the primes 2, 3, 5, 7, ... 19, 23. But on dividing by 23 we get a quotient *less than* 23, and we need go no further. For if 467 contained a prime factor beyond 23, the quotient would be less than 23, and must have been revealed as a factor by the former trials. Hence 467 has no prime factor greater or less than 23; that is, it is a prime number.

EXAMPLES IV. a.

Without actual division find which of the following numbers are divisible by one or more of the factors 3, 4, 5, 8, 9, 11.

1. 204. **2.** 171. **3.** 153. **4.** 315. **5.** 341. **6.** 730.
7. 2211. **8.** 1009. **9.** 8712. **10.** 10967. **11.** 36324. **12.** 924.

13. Without division find the remainder when each of the following numbers is divided by 9:

218, 3407, 276381, 4327065, 12345678.

14. In the following numbers supply the missing digit so as to make each a multiple of 9:

30*4, 573*1, *6708, 836*57.

15. In the following numbers supply the missing digit so as to make each a multiple of 11:

148*3, 7*563, 8276*845.

16. What is the smallest number which must be added to 803642 in order to obtain (i) a multiple of 9, (ii) a multiple of 11?

Separate into prime factors

17. 45. **18.** 84. **19.** 132. **20.** 105. **21.** 135.
22. 288. **23.** 405. **24.** 462. **25.** 980. **26.** 1386.
27. 3150. **28.** 9009. **29.** 3332. **30.** 5382.
31. 6120. **32.** 7245. **33.** 19206. **34.** 249984.

In Examples 35–44 find which of the numbers are prime. Express in prime factors those which are not prime.

35. 67. **36.** 303. **37.** 113. **38.** 679. **39.** 197.
40. 712. **41.** 223. **42.** 317. **43.** 667. **44.** 10681.

45. Write down the numbers (i) 7543 and (ii) 8029 in their complete form as the sum of powers of 10 with numerical coefficients.

46. How would you express the number whose digits, in order from left to right, are p, q, and r? Why may not such a number be expressed by pqr?

47. Write down a number whose digits are a, b, c; and state the condition that it may be divisible (i) by 2, (ii) by 5.

48. Express in symbols a number whose digits are a, b, c, d.

Prove that the number is divisible by 9, if $a+b+c+d$ is so divisible.

49. Write down any *two* numbers whose digits are l, m, n (by taking the digits in different orders). Shew that the difference between two such numbers is always divisible by 9.

83. The **square root** of a number is that number whose square is equal to the given number.

Thus the square root of 49 is 7, because $7^2=49$.

The **cube root** of a number is that number whose cube is equal to the given number.

Thus the cube root of 64 is 4, because $4^3=64$.

NOTE. The square and cube root of 1 are each equal to 1.

The root of a number is denoted by the symbol $\sqrt{\ }$ with a small figure prefixed to indicate the *order* of the root. In the case of the square root the small figure is usually omitted.

Thus the square root of 9 is denoted by $\sqrt[2]{9}$ or more simply by $\sqrt{9}$,
the cube root of 64 .... $\sqrt[3]{64}$.

The symbol $\sqrt{\ }$ is sometimes referred to as the **radical sign.**

84. The square of a composite number must contain the square of every factor of that number. For example

$$6=2\times3;\qquad 6^2=2\times3\times2\times3\ =2^2\times3^2,$$
$$56=2^3\times7;\quad 56^2=2^3\times7\times2^3\times7=2^6\times7^2.$$

It will be noticed that each prime factor of the number is repeated an even number of times in the square of the number. Conversely, when a square number has been expressed in prime factors, its square root can be written down at once by simply halving the index of the power of each prime factor.

EXAMPLE 1. *Find the square root of the square number* 213444.

Here
$$\begin{aligned}213444&=4\times9\times11^2\times49\\&=2^2\times3^2\times11^2\times7^2;\\ \therefore\ \text{sq. root of } 213444&=2\times3\times11\times7\\&=462.\end{aligned}$$

$$\begin{array}{r|l}4 & 213444\\ \hline 9 & 53361\\ \hline 11 & 5929\\ \hline 11 & 539\\ \hline & 49\end{array}$$

EXAMPLE 2. *By means of factors find whether* 5292 *is a square number or not.*

Here
$$\begin{aligned}5292&=4\times9\times7^2\times3\\&=2^2\times3^3\times7^2.\end{aligned}$$

And since 3 occurs to an *odd* power, 5292 is not a square number.

$$\begin{array}{r|l}4 & 5292\\ \hline 9 & 1323\\ \hline 7 & 147\\ \hline 7 & 21\\ \hline & 3\end{array}$$

85. When a number is expressed in factors its cube contains each factor repeated 3 times.

Thus $10=2\times5;\ 10^3=2\times5\times2\times5\times2\times5\ =2^3\times5^3$, ... (i)
$18=2\times3^2;\ 18^3=2\times3^2\times2\times3^2\times2\times3^2=2^3\times3^6$. ....(ii)

In (i) each of the factors 2 and 5 occurs 3 times in the cube.

In (ii) ,, ,, 2 and 3^2 ,, ,, ,,

It should also be noticed that the cube of every even number must contain the factor 2^3, or 8.

Conversely, when a cube number has been expressed in prime factors, its cube root can be written down by taking the product of the prime factors each with an index which is one-third of the corresponding index in the cube number.

Moreover any even cube number must be divisible by 8. Hence the method of the last article may be very quickly applied.

EXAMPLE. *Find the cube root of the cube number* 3652264.

Here $3652264 = 8 \times 7^3 \times 1331$

$= 2^3 \times 7^3 \times 11^3$.

$\therefore$ cube root of $3652264 = 2 \times 7 \times 11$

$= 154$.

$$\begin{array}{r|l} 8 & 3652264 \\ \hline 7 & 456533 \\ \hline 7 & 65219 \\ \hline 7 & 9317 \\ \hline & 1331 \end{array}$$

We first use the factor 8 since the number is even.

Then on finding that 7 is a factor we divide by it three times, and 1331 is obviously the cube of 11.

EXAMPLES IV. b.

(*Examples* 1 *to* 17 *and* 33 *may be taken orally.*)

Read off the square roots of

1. 4, 9, 16, 25, 36, 49, 64, 81, 100, 144, 169.

2. $2^2 \times 5^2$. **3.** $3^2 \times 7^2$. **4.** $2^4 \times 3^2$. **5.** $2^6 \times 5^2$.

6. $3^2 \times 13^2$. **7.** $5^4 \times 2^2$. **8.** $3^2 \times 2^4 \times 5^4$. **9.** $2^6 \times 7^2 \times 5^6$.

10. a^2b^2. **11.** $25a^2b^4$. **12.** $49m^2n^2p^4$. **13.** $81a^4c^2$.

Read off the values of

14. $\sqrt{5^2 \times 7^2}$. **15.** $\sqrt{5^2 . 2^2 . 7^2}$. **16.** $\sqrt{16a^2b^2c^4}$. **17.** $\sqrt{144m^4n^6}$.

By separating into prime factors determine which of the following numbers are squares, and find their square roots:

18. 324. **19.** 576. **20.** 628. **21.** 1323. **22.** 2304.

23. 10082. **24.** 1936. **25.** 3136. **26.** 6294. **27.** 7056.

Find the least factor by which the following numbers must be multiplied so that in each case the product may be a square number:

28. 392. **29.** 605. **30.** 1584. **31.** 1183. **32.** 19845.

33. Read off the cube roots of

8, 27, 64, 125, 216, 343, 512, 729, 1000, 1331, 1728.

The following numbers are perfect cubes. Find their cube roots by means of factors.

34. 10648. **35.** 21952. **36.** 46656. **37.** 91125.

38. Find the least square number that is divisible by 7, 8, and 9.

39. Find a square number between 1000 and 2000, which is divisible by 13.

40. If $a=3$, $b=4$, $c=27$, find the value of

(i) $\sqrt{abc}$. (ii) $\sqrt{\frac{b^2c}{a}}$. (iii) $\sqrt{7(a+b)}$. (iv) $\sqrt{25(a^2+b^2)}$.

Highest Common Factor.

86. A factor which divides two or more numbers is called a **common factor** (or common *measure*).

Thus 5 is a common factor of 15, 35, and 45,

and x is a common factor of ax, bx, and cx.

87. The **Highest Common Factor** of two or more numbers is the highest number which divides each of them exactly. The abbreviation H.C.F. is used for the words *highest common factor*.

The terms *highest common divisor* (H.C.D.) and *greatest common measure* (G.C.M.) are sometimes used instead of highest common factor.

88. In the case of numbers which have been expressed in prime factors the H.C.F. can be written down by inspection.

Example. *Find the* H.C.F. *of* 126, 132, *and* 360.

Expressing the numbers in prime factors it will be found that

$126=2\times3^2\times7$,

$132=2^2\times3^2\times11$,

$360=2^3\times3^3\times5$.

The highest power of 2 which will divide 2, 2^2, and 2^3 is 2; the highest power of 3 which will divide 3^2, 3^3, is 3^2, and there are no other *common* factors.

Thus the H.C.F. is 2×3^2, or 18.

89. Sometimes the H.C.F. may be found without expressing all the numbers in prime factors.

Example. *Find the* H.C.F. *of* 440, 1800, 2800.

$440=10\times11\times4$, and of these three factors 10 and 4 divide all three numbers, 11 does not.

$\therefore$ the H.C.F. $=10\times4=40$.

EXAMPLES IV. c.

(*Examples* 1 *to* 14 *may be taken orally.*)

Read off the H.C.F. of

1. 9, 15. **2.** 12, 18. **3.** 6, 24. **4.** 16, 20.
5. 27, 45. **6.** 28, 49. **7.** 42, 60. **8.** 35, 56.
9. 12, 72. **10.** 30, 45. **11.** 4, 6, 12.
12. 14, 21, 28. **13.** 24, 30, 36. **14.** 15, 60, 75.

Find by resolution into prime factors the H.C.F. of

15. 144, 180. **16.** 46, 138. **17.** 136, 153.
18. 135, 225. **19.** 198, 308. **20.** 240, 512.
21. 384, 1296. **22.** 891, 231. **23.** 45, 120, 180.
24. 24, 72, 108. **25.** 51, 68, 153. **26.** 36, 108, 216.
27. 111, 74, 185. **28.** 140, 210, 315. **29.** 350, 600, 550.
30. 234, 288, 270. **31.** 540, 945, 1215. **32.** 660, 1617, 4235.

Write down the H.C.F. of

33. $3xy^2, 6x^2y$. **34.** $8ab^2c, 12a^2bc^2$. **35.** $20p^3q, 15p^2q^2r$.
36. $16a^3b, 24ac^3, 32a^2b^2c^2$. **37.** $21axy, 14a^2xy^2, 6x^2y^2$.

90. When the numbers are not easily separated into prime factors a different process has to be used.

The method depends on the following principles :

(i) If a number contains a certain factor, any multiple of the number is divisible by that factor.

Thus 3 is a factor of 15, and therefore of $15 \times n$, where n stands for any whole number.

(ii) If two numbers have a common factor, it will divide their sum and their difference ; and also the sum and difference of any multiples of them.

Thus 3 being a common factor of 27 and 15, is also a factor of $27 + 15$, and of $27 - 15$.

Again 3 will divide $27m + 15n$ and $27m - 15n$, where m and n stand for any whole numbers.

Example. *Find the* H.C.F. *of* 506 *and* 1863.

$$\begin{array}{rrrrr}
506\,)\,1863\,(\,3 & & & & \\
\underline{1518} & & & & \\
 & 345\,)\,506\,(\,1 & & & \\
 & \underline{345} & & & \\
 & & 161\,)\,345\,(\,2 & & \\
 & & \underline{322} & & \\
 & & & 23\,)\,161\,(\,7 & \\
 & & & \underline{161} &
\end{array}$$

Divide the greater number by the less; divide the less by the remainder: divide the first remainder by the new remainder, and so on till there is no remainder. The last divisor is the H.C.F. required.

For the H.C.F. of 1863 and 506 is also a factor of $1863 - 506 \times 3$, or 345; therefore a common factor of 506 and 345.

Again the H.C.F. of these numbers is a factor of their difference 161, and therefore a common factor of 345 and 161. Therefore also of $345 - 161 \times 2$, or 23. Therefore also of 161 and 23. And the *highest* common factor of these is obviously 23.

From this example it is seen that the method succeeds because at any stage every common factor of the original numbers is a factor of the dividend and divisor at that stage. Hence the H.C.F. will be the same as that for the *last* divisor and dividend; that is the last divisor is the H.C.F.

If in any example by this method the last divisor is 1, the numbers have no common factor except unity, that is they are prime to each other.

The work may be arranged more compactly in parallel columns, placing the quotients to right and left alternately. Or more briefly still by employing the Italian method of division, but as it is difficult to detect a mistake made in the course of the work this method is not recommended to beginners.

$$\begin{array}{r|r|r|l}
1 & 506 & 1863 & 3 \\
 & \underline{345} & \underline{1518} & \\
7 & 161 & 345 & 2 \\
 & \underline{161} & \underline{322} & \\
 & & 23 &
\end{array}$$

By the Italian Method.

$$\begin{array}{r|r|r|l}
1 & 506 & 1863 & 3 \\
7 & 161 & 345 & 2 \\
 & & 23 &
\end{array}$$

91. The work may often be shortened by the following devices:

(i) Any obvious factors may be removed from the numbers before the application of the rule. The H.C.F. of these factors, if any, must be reserved and multiplied into the H.C.F. given by the rule.

(ii) Since every remainder contains the H.C.F. as a factor, we may throw out any factor of the remainder at any stage if it is not a factor common to the numbers whose H.C.F. we are finding.

The following example illustrates the method above described. For comparison we also give the full working by the method of Art. 90.

Example. *Find the* H.C.F. *of* 55224 *and* 122012.

55224 = 8 × 9 × 767,

122012 = 4 × 11 × 2773.

We may reject 9 and 11 which are not common factors. The H.C.F. of 8 and 4 is 4, which must be reserved as a factor of the H.C.F.

$$\begin{array}{r|r|r|l}
4 & 55224 & 122012 & 2 \\
 & 46256 & 110448 & \\
\hline
3 & 8968 & 11564 & 1 \\
 & 7788 & 8968 & \\
\hline
5 & 1180 & 2596 & 2 \\
 & 1180 & 2360 & \\
\hline
 & & 236 &
\end{array}$$

H.C.F. = 236.

$$\begin{array}{r|r|r|l}
13 & 767 & 2773 & 3 \\
 & 59 & 2301 & \\
\hline
 & 177 & 8\,|\,472 & \\
 & 177 & 59 &
\end{array}$$

The factor 8 is rejected because it is not a common factor of 767 and 2773.

Thus the H.C.F. = 4 × 59
= 236.

92. When there are more than two numbers, not easily put into factors, we find the H.C.F. of two; then the H.C.F. of the result and a third number, and so on. The final H.C.F. is that required.

Example 1. *Find the* H.C.F. *of* 806, 663, 377.

We may first reject 2 from the first, and 3 from the second number, and proceed with 403, 221, 377.

The rule gives 13 as the H.C.F. of the first two, and since 13 divides 377 it is the required H.C.F.

Example 2. *What is the greatest length which can be used to measure exactly the following lengths:* 20 ft., 13 ft. 9 in., 17 ft. 6 in., 21 ft. 3 in.?

We must express these lengths in the same denomination and find their greatest common measure (Art. 66).

Expressed in inches we have 240 in., 165 in., 210 in., 255 in., and the G.C.M. of these is 15 in., or 1 ft. 3 in.

EXAMPLES IV. d.

Find the H.C.F. of

1. 117, 221. **2.** 203, 319. **3.** 559, 817. **4.** 644, 532.

5. 255, 391. **6.** 329, 799. **7.** 527, 1147. **8.** 623, 833.

9. 348, 1024. **10.** 1702, 1998. **11.** 3451, 9367.

12. 1379, 2401. **13.** 4199, 5083. **14.** 2893, 10520.

15. 13547, 17081. **16.** 20677, 31279. **17.** 10549, 13563.

18. 16984, 5404. **19.** 16995, 64890. **20.** 94248, 504900.

21. 183, 793, 976. **22.** 658, 940, 1128. **23.** 403, 744, 1023.

24. 42237, 75582, 8892. **25.** 3456, 26244, 99225.

26. Find the greatest number which will divide 14490 and 31530 so as to leave the remainder 6 in each case.

27. Find the greatest number which will divide 11296 and 13528 so as to leave remainders 11 and 23 respectively.

28. Shew that 53477 and 68401 are prime to one another.

29. If two sums of £5 and 5 guineas are to be paid in coins all of one kind, what is the largest coin that can be used?

30. What is the length of the largest square tile that can be used in paving a hall 18 ft. 8 in. by 15 ft. 9 in.?

31. In finding the H.C.F. of two numbers the last remainder is 35, and the quotients in order are 1, 2, 1, 3. Find the numbers.

32. Find the largest number which is such that when 142408, 153599, and 166402 are divided by it, the remainders are all the same.

33. In a long division sum the quotient consists of two figures; if the dividend is 40051, and the two remainders are 173 and 294, find the divisor and quotient.

Lowest Common Multiple.

93. A number which is exactly divisible by two or more numbers is called a **common multiple** of those numbers.

Thus 42 is a common multiple of 3 and 7,

and mxy is a common multiple of x and y.

94. The **lowest common multiple** of two or more numbers is the *lowest* number which contains each of them as a factor.

Thus each of the numbers 12, 24, 48 is a common multiple of 2, 3, and 4, but their *lowest* common multiple is 12.

The term *least* common multiple is also used, and the letters L.C.M. are used as an abbreviation for either term.

95. When numbers can be expressed in prime factors their L.C.M. can be written down by inspection.

EXAMPLE. *Find the* L.C.M. *of* 18, 28, 108, *and* 105.

$18 = 2 \times 3^2$,

$28 = 2^2 \times 7$,

$108 = 2^2 \times 3^3$,

$105 = 3 \times 5 \times 7$.

The L.C.M. must contain every prime factor of each of the numbers, and moreover it must contain the highest power of each factor which appears in any one of them.

Thus it must contain 2^2 or it would not be a multiple of 28; and it must contain 3^3 or it would not be a multiple of 108. The other factors are 5 and 7, which do not occur more than once in any of the given numbers.

Thus the L.C.M. $= 2^2 \times 3^3 \times 5 \times 7 = 3780$.

NOTE. We might have struck out 18 at once, since any multiple of 108 must contain 18 as a factor.

96. The prime factors of the L.C.M. may often be obtained more quickly by the following method. It will be seen that the work admits of more than one arrangement.

$$\begin{array}{rrrrr}
7\,| & \cancel{18}, & 28, & 108, & 105 \\ \cline{2-5}
 & & 3\,|\,\cancel{4}, & 108, & 15 \\ \cline{3-5}
 & & & 36, & 5
\end{array}$$

L.C.M. $= 7 \times 3 \times 5 \times 36$
$= 3780$.

$$\begin{array}{rrrrr}
2\,| & \cancel{18}, & 28, & 108, & 105 \\ \cline{2-5}
 & 2\,| & 14, & 54, & 105 \\ \cline{3-5}
 & & 3\,|\,\cancel{7}, & 27, & 105 \\ \cline{3-5}
 & & & 3\,|\,9, & 105 \\ \cline{4-5}
 & & & 3, & 35
\end{array}$$

L.C.M. $= 2 \times 2 \times 3 \times 3 \times 3 \times 35 = 3780$.

Explanation. After striking out any number which is a factor of any other, divide the remaining numbers by any **prime factor** which is a factor of two of them at least, bringing down into the line of quotients any number which is not divisible by the prime. Repeat this process till the line of quotients consists entirely of numbers prime to each other. Then the product of the successive divisors and the final quotients is the L.C.M. required. It is important to observe that in the rule thus stated the divisors should be *prime* numbers.

EXAMPLES IV. e.

(*Examples* 1–30 *may be taken orally.*)

Read off the L.C.M. of

1. 2, 6.
2. 5, 10.
3. 15, 10.
4. 3, 8.
5. 6, 8.
6. 14, 21.
7. 28, 21.
8. 21, 35.
9. 15, 25.
10. 18, 24.
11. 2, 4, 6.
12. 3, 5, 7.
13. 5, 9, 10.
14. 6, 12, 18.
15. 5, 45, 30.

Read off as the product of prime factors the L.C.M. of

16. $2^2\times 3$, $3^3\times 2$. **17.** $2\times 3^2\times 5$, $2^3\times 5^2$.
18. $2\times 3\times 7$, $3\times 7\times 5$. **19.** $3^2.7$, 5.7^2, 2.5^2.
20. 3^4, $2^2.3$, 2.5^2. **21.** $3^2.7$, 11, $7^2.2$.
22. ab, bc, ca. **23.** $2xy$, $5yz$, $4zx$. **24.** $8x^2$, $4xy$, $12y^2$.

Read off (as the product of prime factors) both the H.C.F. and the L.C.M. of

25. 5.3^2, $5^2.3$, $5^3.3^2$. **26.** $2^4.7$, 2.7^3, $2^2.7^2$.
27. 3^4, $2^3.3^2$, $2^4.3$, 2.3^3. **28.** ab^2, a^2b, a^3b^2.
29. $12ab$, $6bc$, $4abc$. **30.** $9xyz$, $3y^2z$, xyz.

Find by the method of prime factors the L.C.M. of

31. 6, 8, 10. **32.** 6, 12, 15. **33.** 18, 24, 36.
34. 4, 12, 16, 20. **35.** 18, 27, 36, 28.
36. 16, 24, 48, 60. **37.** 22, 33, 121, 132.
38. 14, 49, 84, 63. **39.** 120, 144, 96, 100.
40. 12, 18, 30, 48, 60. **41.** 63, 12, 84, 28, 70.
42. 16, 42, 64, 70, 80, 112. **43.** 135, 126, 90, 255.
44. 80, 108, 64, 720, 864. **45.** 60, 36, 65, 78, 208.

Find by the method of prime factors both the H.C.F. and the L.C.M. of

46. 45, 75, 225. **47.** 28, 42, 98. **48.** 165, 231, 550.

97. When the numbers are not easily separated into prime factors we proceed as follows.

Let A and B be two numbers, and X their H.C.F.

Let A and B contain X exactly m and n times respectively;

$$\text{then } A=mX, \text{ and } B=nX;$$

where m and n have no common factor.

$$\therefore\ \text{L.C.M. of } A \text{ and } B=mnX=\frac{mX\,.\,nX}{X}=\frac{A\,.\,B}{X}.$$

Hence the L.C.M. of two numbers may be found by dividing their product by their H.C.F.

Also we conclude that *the product of two numbers is the same as the product of their* H.C.F. *and* L.C.M.

In the same way it may be shewn that if A, B, C are any three numbers whose H.C.F. is X, their L.C.M. $=\dfrac{A\,.\,B\,.\,C}{X^2}$.

EXAMPLE. *Find the* L.C.M. *of* 299, 221, *and* 759.

The H.C.F. of 299 and 221 is 13. By division we find that

$$299 = 13 \times 23, \text{ and } 221 = 13 \times 17.$$

Since all these factors must be factors of the L.C.M. we make trial of the factors 17 and 23 applied to the number 759. Thus we find

$$759 = 23 \times 3 \times 11.$$

Thus the L.C.M. required $= 13 \times 23 \times 17 \times 3 \times 11 = 167739$.

EXAMPLES IV. f.

Find the L.C.M. of

1. 221, 533. **2.** 667, 437. **3.** 1397, 3683. **4.** 817, 2021.

5. 9669, 16115. **6.** 3024, 4752, 7488. **7.** 959, 3973, 2329.

8. What is the least sum of money that can be distributed exactly either in half-crowns or half-guineas?

9. What is the greatest number that will divide 3104, 20832, 34144?

10. Find the smallest number which contains 156, 168, 208, and 432 as divisors.

11. What is the lowest number which, when divided separately by 15, 20, 48, and 36, will in each case leave 9 as remainder?

12. Find the H.C.F. and the L.C.M. of 161, 253, 299, 322.

13. Four cisterns are capable of holding 72, 24, 56, and 120 gallons respectively. What is the capacity of the greatest vessel which can be used to fill them exactly?

14. Determine the smallest sum of money out of which a number of men, women, and children respectively may receive £1. 7*s.* 6*d.*, 18*s.* 9*d.*, and 8*s.* 3*d.* each.

15. Four men can walk 105, 112, 126, and 168 miles in a week respectively. What is the least distance they can all walk in an exact number of weeks?

16. Three bells toll at intervals of 18, 24, 32 seconds: if they begin to toll together, what length of time will elapse before they toll together again?

17. *A*, *B*, and *C* start at the same time, and in the same direction, to run round a circular course. If *A* makes the circuit in 252 seconds, *B* in 308, and *C* in 198, and they all start from the same point, when will they next be all at the starting point together?

18. The H.C.F. of two numbers is 119; their L.C.M. is 11781; one of the numbers is 1071. Find the other.

CHAPTER V.

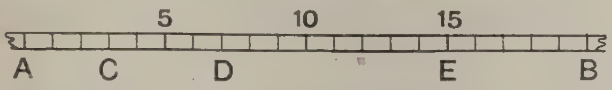
FRACTIONS.

98. WHEN a number expresses an *exact* number of units of any kind it is called a **whole number** or an **integer.**

Thus if £1 is the unit, £3, £5, £12 are respectively *three-times*, *five-times*, and *twelve-times* the unit. Each is an **integral multiple** of the unit.

99. When the unit is supposed to be divided into any number of equal parts, and one or more of those parts are taken, the result is called a **fraction.**

Let the unit be the line AB, divided into 20 equal parts, so that each part is *one-twentieth* of the unit.



Then AC, AD, AE respectively contain 3, 7, 15 parts, and represent 3 *twentieths*, 7 *twentieths*, and 15 *twentieths* of the unit.

Again if £1 is the unit, and we suppose the unit to be divided into 20 equal parts, each part will be 1 shilling, and 3*s.*, 7*s.*, 15*s.* will be respectively *three-twentieths*, *eight-twentieths*, and *fifteen-twentieths* of the unit.

These fractions are written $\frac{3}{20}$, $\frac{7}{20}$, $\frac{15}{20}$. Thus a fraction is expressed by two numbers one over the other, separated by a horizontal bar.

The lower number, which expresses the number of equal parts into which the unit is divided, is called the **denominator.** The upper number which expresses the number of parts taken in any fraction is called the **numerator.** The numerator and denominator are sometimes called the **terms** of a fraction.

NOTE. When the numerator and denominator of a fraction are equal its value is unity.

Thus in the above illustrations 20 twentieths of the line make up AB, which is the whole unit.

And 20 twentieths of £1, that is 20 shillings, make up the whole pound. Thus $\frac{20}{20}=1$.

Similarly $\frac{4}{4}$, $\frac{5}{5}$, $\frac{12}{12}$ are each equal to 1.

100. Fractions expressed in this manner are known as **Vulgar** (or Common) Fractions to distinguish them from *Decimal Fractions* (or Decimals) which will be explained in Chap. VIII.

The particular unit supposed to be divided is not necessarily expressed, but the notation is the same.

The fractions $\frac{1}{2}$, $\frac{2}{3}$, $\frac{4}{5}$, $\frac{5}{16}$, ... are read as follows:

one-half, two-thirds, four-fifths, five-sixteenths, ...,

and apply to the subdivisions of any kind of unit.

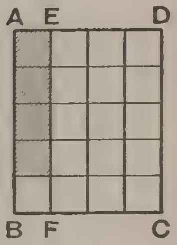
101. A fraction may also be defined as **the result of dividing the numerator by the denominator**. For, by the former definition,

$\frac{4}{5}$ denotes *four times the fifth part of the unit.*

But if we divide 4 units by 5 we get a result which is four times as great as the fifth part of 1 unit.

Hence the fraction $\frac{4}{5}$ is the result of dividing 4 units by 5.

This may be proved graphically as follows:



In the adjoining diagram let each of the vertical strips, such as ABFE, denote the unit; then the fig. ABCD represents 4 units. Also it contains 20 squares;

$\therefore$ 4 units $\div 5 = \frac{20}{5}$, or 4 squares.

Again the fraction $\frac{4}{5} = \frac{4}{5}$ of the unit strip ABFE

$= 4$ squares.

Thus the fraction $\frac{4}{5}$ is the result of dividing 4 units by 5.

Hence there are two ways in which any fraction may be regarded.

(i) The fraction $\frac{4}{5} = 4$ parts each of which is *one-fifth* of the unit.

(ii) The fraction $\frac{4}{5} =$ *one-fifth* part of 4 units

$=$ the quotient when 4 is divided by 5.

102. Another mode of reading fractions is sometimes used:

thus $\frac{5}{8}$ is read *five over eight*,

$\frac{11}{13}$ „ *eleven over thirteen*, and so on.

NOTE. Instead of using the horizontal line to separate numerator and denominator, it is sometimes convenient to write fractions in the following form

1/2, 2/3, 4/5, 5/16.

Verbally they are read as above explained in Art. 100.

EXAMPLES V. a. (*Oral.*)

Read off the value of

1. Five-twelfths of 1*s*.

2. Nine-twelfths of 1*s*.

3. Six-twelfths of 1*s*.

4. One-half of 1*s*.

5. Two-thirds of 1*s*.

6. Five-sixths of 1*s*.

7. Seven-twentieths of £1.

8. Fifteen-twentieths of £1.

9. Three-tenths of £1.

10. Three-fourths of £1.

11. Seven-eighths of £1.

12. Nine-fortieths of £1.

13. Three-fourths of 1 cwt.

14. Seven-twentieths of 1 ton.

15. One-twelfth of 1 ft.

16. Two-thirds of 1 yd.

17. Five-sixteenths of 1 lb.

18. Nineteen-sixtieths of 1 hr.

19. $\frac{11}{20}$ of £1.

20. $\frac{15}{20}$ of £1.

21. $\frac{3}{4}$ of £1.

22. $\frac{5}{12}$ of 1*s*.

23. $\frac{1}{40}$ of £1.

24. $\frac{6}{240}$ of £1.

25. $\frac{7}{10}$ of £1.

26. $\frac{14}{20}$ of £1.

27. $\frac{13}{36}$ of 1 yd.

28. $\frac{7}{12}$ of 1 ft.

29. $\frac{3}{100}$ of 1 metre.

30. $\frac{7}{1000}$ of 1 metre.

31. $\frac{17}{1000}$ of 1 kilometre.

32. $\frac{29}{100}$ of 1 kilogram.

33. $\frac{19}{100}$ of 1 franc.

34. $\frac{37}{1760}$ of 1 mile.

35. $\frac{10}{220}$ of 1 furlong.

36. $\frac{11}{60}$ of 1 hr.

37. $\frac{5}{24}$ of 1 day.

38. $\frac{59}{112}$ of 1 cwt.

Read off the number of

39. Pence in $\frac{1}{2}$, $\frac{1}{6}$, $\frac{1}{3}$, $\frac{3}{4}$ of a shilling.

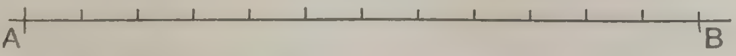
40. Shillings in $\frac{1}{10}$, $\frac{7}{10}$, $\frac{1}{20}$, $\frac{9}{20}$ of a sovereign.

41. Inches in $\frac{1}{36}$, $\frac{5}{36}$, $\frac{1}{12}$, $\frac{7}{12}$ of a yard.

42. Hours in $\frac{1}{24}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{12}$, $\frac{5}{6}$, $\frac{7}{8}$ of a day.

43. Ounces in $\frac{1}{16}$, $\frac{9}{16}$, $\frac{1}{8}$, $\frac{5}{8}$ of a lb.

44. Pounds in $\frac{1}{112}$, $\frac{5}{112}$, $\frac{3}{56}$, $\frac{7}{56}$ of a cwt.



45. The line AB is divided into 12 equal parts. Copy this on squared paper, and place letters P, Q, R, X, Y, Z so that

AP may represent $\frac{7}{12}$ of AB ; AQ may represent $\frac{5}{12}$ of AB ;
BR .... $\frac{11}{12}$ of AB ; BX .... $\frac{1}{2}$ of AB ;
AY .... $\frac{1}{3}$ of AB ; BZ .... $\frac{1}{4}$ of AB.

MS - Vch 4.1

46. Each line of your squared paper is divided into inches, and each inch into *tenths* of an inch. Rule lines to represent $\frac{7}{10}$ of an inch, $\frac{9}{10}$ of an inch, 2 inches and $\frac{3}{10}$ of an inch, 3 inches and $\frac{4}{10}$ of an inch, 1 inch and $\frac{8}{10}$ inch.

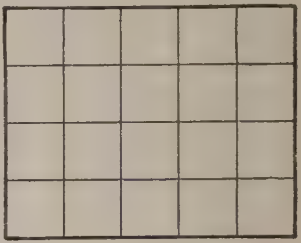
[N.B. 2 inches and $\frac{3}{10}$ of an inch may be written $2\frac{3}{10}$ inch; and so on.]

47. A line AB is divided into n equal parts, and X is so placed that AX contains m of them. What fraction is the line AX of the line AB?

Illustrate this on squared paper, making

$$AB = 4\tfrac{1}{2} \text{ inches}, \; n = 15, \text{ and } m = 5.$$

48. Copy this figure on squared paper, and by means of letters name portions that will shew $\frac{1}{20}$, $\frac{3}{20}$, $\frac{1}{5}$, $\frac{1}{4}$ of the whole area.



EXAMPLES V. b. (*Oral.*)

1. What fractions of 1 shilling are 5 pence, 7 pence, 11 pence?

2. What fractions of £1 are 3 shillings, 7 shillings, 13 shillings?

3. What fractions of 1 foot are 1 inch, 5 inches, x inches?

4. Express 3 oz., 9 oz., 17 oz. as fractions of 1 lb.

5. Express 13 centimetres, 21 cm., y cm. as fractions of 1 metre.

6. Express 17 centimes, 41 centimes, z centimes as fractions of 1 franc.

7. How many sums each of 4 shillings are contained in £1? What fractions of £1 are 4 shillings, 8 shillings, 12*s.*, 16*s.*?

8. How many lengths each of 4 inches are there in 1 yard? Hence express the following lengths as fractions of a yard: 4 inches, 8 inches, 16 inches, 20 inches, 28 inches, 32 inches.

9. How many sixpences are there in £1? Hence express 18 pence, 42 pence, 66 pence, $6x$ pence as fractions of £1.

10. What fractions of 1 foot are 3 inches, 4 inches, 6 inches?

11. What fractions

of a franc are 10 centimes, 30 centimes, 70 centimes?
.... 20 centimes, 40 centimes, 60 centimes?
.... 25 centimes, 50 centimes, 75 centimes?

12. Express 9 metres, 9 decametres, 9 hectometres as fractions of 1 kilometre.

103. From the foregoing examples it will have been noticed that, when the unit is the same, there are different ways of expressing the same fraction.

For instance,
$$\left.\begin{array}{l}\frac{3}{4}\text{ of }1s.=3\textit{ threepences}=9\text{ pence,}\\ \frac{9}{12}\text{ of }1s.=9\text{ pence;}\end{array}\right\}$$
so that $\frac{3}{4}=\frac{9}{12}$.

Again,
$$\left.\begin{array}{l}\frac{14}{20}\text{ of £}1=14\text{ shillings,}\\ \frac{7}{10}\text{ of £}1=7\textit{ florins}=14\text{ shillings;}\end{array}\right\}$$
so that $\frac{14}{20}=\frac{7}{10}$.

These are particular illustrations of the following general principle which is applicable to all fractions.

104. *The value of a fraction is not altered if we multiply or divide the numerator and denominator by the same quantity.*

To prove that $\frac{3}{4}=\frac{3\times 2}{4\times 2}=\frac{6}{8}$.

By $\frac{3}{4}$ we mean 3 equal parts, 4 of which make up the unit .... (i);
by $\frac{6}{8}$ .... 6 .... 8 .... (ii).

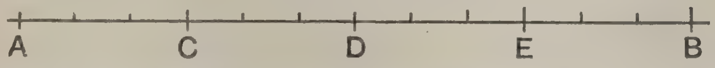
Since 4 parts in (i) = 8 parts in (ii);
$\therefore$ 1 part ...... = 2 .... ;
$\therefore$ 3 parts ...... = 6 ....

That is, $\frac{3}{4}=\frac{6}{8}$.

We have thus shewn that by *multiplying* each term of the fraction $\frac{3}{4}$ by 2 its value is unaltered.

Or we may give a graphical proof as follows:

To prove that $\frac{9}{12}=\frac{9\div 3}{12\div 3}=\frac{3}{4}$.



Let the unit be represented by the straight line AB, and let it be divided into 4 equal parts at the points C, D, E, so that $\frac{3}{4}$ of AB = AE.

Again let each of the parts AC, CD, DE, EB be subdivided into 3 equal parts, then the whole line AB contains 12 of these parts. Also AE contains 9 of such parts;

hence $\frac{9}{12}$ of AB = AE = $\frac{3}{4}$ of AB.

That is, $\frac{9}{12}=\frac{3}{4}$.

We have thus shewn that by *dividing* each term of the fraction $\frac{9}{12}$ by 3 its value is unaltered.

Similar reasoning will shew the truth of the *general principle*, which may be expressed thus

$$\frac{a}{b}=\frac{ma}{mb},$$

where a, b, and m stand for any whole numbers whatever.

105. Consider the fraction $\frac{60}{75}$.

By the last article, $\frac{60}{75}=\frac{12}{15}=\frac{4}{5}$, dividing the terms successively by 5 and 3. Hence it appears that by removing equal factors from the terms of a fraction it may be expressed in a simpler form without altering its value.

106. When a fraction has been so simplified that its numerator and denominator have no common factor it is said to be **reduced to lowest terms.**

EXAMPLE 1. *Reduce* $\frac{30}{105}$ *to its lowest terms.*

$$\frac{30}{105}=\frac{\cancel{5}\times 6}{\cancel{5}\times 21}=\frac{\cancel{3}\times 2}{\cancel{3}\times 7}$$
$$=\frac{2}{7}.$$

It is usual to strike out or 'cancel' like factors from numerator and denominator.

EXAMPLE 2. *Reduce* $\frac{609}{1595}$ *to its lowest terms.*

By trial, $\frac{609}{1595}=\frac{3\times 203}{5\times 319}$

$$=\frac{3\times 7\times \cancel{29}}{5\times 11\times \cancel{29}}$$
$$=\frac{21}{55}.$$

Here no *common* factor is at once evident, but we easily recognise the prime factors 3, 7, 5, and 11, and thus disclose the common factor 29.

NOTE. In working examples in fractions results must always be reduced to lowest terms.

EXAMPLES V. c. (*Oral.*)

1. Express by the simplest fraction the share of each person when
(i) 5 things are divided amongst 6, 20, 30.
(ii) 8 .... 12, 16, 40.

2. What is the share of each when 12 buns are divided between 18, 24, 31 boys?

Express as a fraction of a shilling:

3. The cost of 1 lb. of raisins if 12 lbs. cost 8*s*.

4. The cost of 1 photograph if 30 cost 25*s*.

5. The cost of 1 egg at 10*s*. a gross.

6. The cost of 1 dozen oranges at 8*s*. a gross.

7. The cost of 1 lb. of potatoes at 5*s*. for 100 lbs.

8. What is the cost of (i) an apple at 15 for 1*s.*; (ii) an egg at 9*d.* a dozen; (iii) a bun at 6*s.* 8*d.* a hundred?

9. Express each of the fractions $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$ with denominator 12.

10. Express each of the fractions $\frac{3}{5}$, $\frac{6}{7}$, $\frac{12}{13}$ with numerator 24.

11. Read off each of the fractions $\frac{5}{10}$, $\frac{3}{12}$, $\frac{8}{12}$, $\frac{5}{15}$, $\frac{6}{18}$, $\frac{14}{21}$, $\frac{35}{56}$, $\frac{28}{63}$, $\frac{22}{121}$ in its simplest form.

EXAMPLES V. d.

(*Examples* 1–10 *should be taken orally.*)

Read off the following fractions in their lowest terms:

1. $\frac{6}{8}$, $\frac{6}{9}$, $\frac{8}{12}$, $\frac{9}{12}$, $\frac{2}{10}$, $\frac{5}{10}$. 2. $\frac{10}{15}$, $\frac{15}{10}$, $\frac{3}{15}$, $\frac{15}{3}$, $\frac{5}{15}$, $\frac{15}{5}$.

3. $\frac{6}{15}$, $\frac{10}{25}$, $\frac{25}{25}$, $\frac{4}{20}$, $\frac{5}{20}$, $\frac{20}{4}$. 4. $\frac{20}{30}$, $\frac{30}{20}$, $\frac{15}{35}$, $\frac{14}{63}$, $\frac{36}{63}$, $\frac{63}{63}$.

5. $\frac{21}{77}$. 6. $\frac{30}{42}$. 7. $\frac{33}{88}$. 8. $\frac{56}{72}$. 9. $\frac{72}{72}$.

10. By expressing 2 feet and 24 inches as fractions of 1 yard, shew that $\frac{2}{3}=\frac{24}{36}$.

Reduce the following fractions to their lowest terms:

11. $\frac{39}{195}$. 12. $\frac{315}{420}$. 13. $\frac{288}{504}$. 14. $\frac{224}{352}$. 15. $\frac{468}{504}$.

16. $\frac{231}{3003}$. 17. $\frac{476}{532}$. 18. $\frac{84}{1932}$. 19. $\frac{132}{2244}$. 20. $\frac{2508}{3036}$.

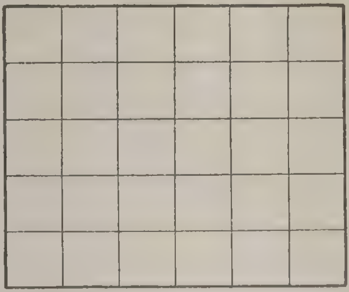
21. By expressing 7 shillings and 84 pence as fractions of £1, shew that $\frac{7}{20}=\frac{84}{240}$.

22. Shew *graphically*, after the manner of Art. 104, that

(i) $\frac{6}{9}=\frac{2}{3}$; (ii) $\frac{6}{15}=\frac{2}{5}$; (iii) $\frac{15}{20}=\frac{3}{4}$.

23. Shew by first principles, following the argument of Art. 104, that

(i) $\frac{5}{7}=\frac{15}{21}$; (ii) $\frac{x}{y}=\frac{mx}{my}$.



24. Use the adjoining diagram to shew that

(i) $\frac{5}{30}=\frac{1}{6}$; (ii) $\frac{6}{30}=\frac{1}{5}$;

(iii) $\frac{10}{30}=\frac{1}{3}$; (iv) $\frac{15}{30}=\frac{1}{2}$.

25. Reduce the following fractions to their lowest terms:

(i) $\frac{x\times a\times y}{y\times x\times b}$. (ii) $\frac{abc}{xca}$. (iii) $\frac{4\times a^2\times b^2}{6\times b^2\times a}$. (iv) $\frac{12max^2}{18a^2mx}$.

107. When common factors of numerator and denominator are not easily found by inspection we must use the rule for finding their H.C.F.

EXAMPLE 1. *Reduce* $\frac{247}{323}$ *to its lowest terms.*

Here $\frac{247}{323}=\frac{13\times\cancel{19}}{17\times\cancel{19}}$
$=\frac{13}{17}.$

As no factors are obvious, the common factor 19 must be found by the rule for finding the H.C.F.

EXAMPLE 2. *Reduce* $\frac{2277}{2369}$ *to its lowest terms.*

$\frac{2277}{2369}=\frac{11\times9\times\cancel{23}}{\cancel{23}\times103}$
$=\frac{99}{103}.$

By trial the numerator is divisible by 9 and 11, *neither of which will divide the denominator.* Hence if the fraction admits of reduction, 23 must be a factor of the denominator. On division we obtain the other factor 103.

EXAMPLES V. d. (*Continued.*)

Simplify

26. $\frac{221}{247}$. 27. $\frac{391}{408}$. 28. $\frac{377}{403}$. 29. $\frac{437}{551}$. 30. $\frac{299}{368}$.
31. $\frac{1265}{2185}$. 32. $\frac{837}{927}$. 33. $\frac{1395}{2232}$. 34. $\frac{572}{1287}$. 35. $\frac{954}{2544}$.

108. Any whole number can be expressed in a fractional form in an unlimited number of ways.

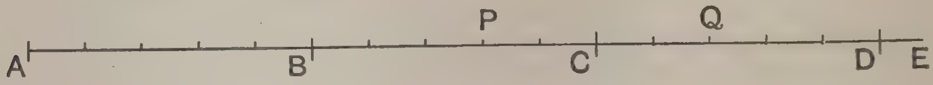
Thus $3=\frac{3}{1}=\frac{3\times4}{4}=\frac{12}{4}$; $3=\frac{3}{1}=\frac{3\times7}{7}=\frac{21}{7}$;
$1=\frac{5}{5}=\frac{9}{9}=\frac{12}{12}=\frac{29}{29}$, and so on.

109. When the numerator is less than the denominator, a fraction is called a **proper fraction.**

Thus $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{9}{10}$ are proper fractions, and each is less than 1.

110. When the numerator is greater than the denominator, a fraction is called an **improper fraction.**

Let the unit be one inch, and let the line AE be marked in inches at B, C, and D, and let each inch be divided into 5 equal parts, or *fifths* of an inch.



Then $\frac{8}{5}$ of an inch $=AP=AB+BP$
$=1$ inch $+\frac{3}{5}$ of an inch.

And $\frac{12}{5}$ of an inch $=AQ=AC+CQ$
$=2$ inches $+\frac{2}{5}$ of an inch.

Or by Art. 99,

$$\begin{aligned} \tfrac{8}{5} &= 8 \textit{ fifths} \text{ of the unit} = (5+3) \textit{ fifths} \\ &= 5 \textit{ fifths} + 3 \textit{ fifths} \\ &= 1 + \tfrac{3}{5}. \end{aligned}$$

$$\tfrac{12}{5} = (10+2) \textit{ fifths} = \tfrac{10}{5} + \tfrac{2}{5} = 2 + \tfrac{2}{5}.$$

Thus an improper fraction is made up of one or more units together with a proper fraction. When expressed in this way it is called a **mixed number.** The plus sign is usually omitted and the above results are written $\frac{8}{5} = 1\frac{3}{5}$, $\frac{12}{5} = 2\frac{2}{5}$.

EXAMPLE 1. *Express* (i) $\frac{11}{6}$; (ii) $\frac{29}{8}$; (iii) $\frac{316}{17}$ *as mixed numbers.*

(i) $\frac{11}{6} = \frac{6+5}{6} = 1\frac{5}{6}$;

(ii) $\frac{27}{8} = \frac{24+3}{8} = 3\frac{3}{8}$;

(iii) $\frac{316}{17} = 18\frac{10}{17}$.

$$\begin{array}{r} 17\,)\,316\,(\,18 \\ \underline{17} \\ 146 \\ \underline{136} \\ 10 \end{array}$$

Here we divide the numerator by the denominator. In each case the quotient gives the number of units in the whole number, and the remainder (or number of *parts* over) gives the numerator of the proper fraction. The intermediate step of the written work may be dispensed with after a little practice.

EXAMPLE 2. *Express* $7\frac{3}{11}$ *as an improper fraction.*

$$\begin{aligned} 7\tfrac{3}{11} &= \tfrac{7 \times 11 + 3}{7} \\ &= \tfrac{77+3}{11} = \tfrac{80}{11}. \end{aligned}$$

Here we reverse the process of Ex. 1. Since each unit contains 11 *elevenths*, 7 units = 77 *elevenths.* To this product we add 3 to obtain the numerator of the improper fraction.

EXAMPLES V. e.

(*Examples* 1 *and* 15 *should be taken orally.*)

Express as mixed numbers, or integers:

1. $\frac{5}{2}$, $\frac{7}{3}$, $\frac{9}{4}$, $\frac{15}{8}$, $\frac{12}{3}$, $\frac{14}{3}$, $\frac{16}{13}$, $\frac{25}{11}$, $\frac{36}{9}$, $\frac{45}{10}$, $\frac{39}{26}$.

2. $\frac{59}{16}$. **3.** $\frac{52}{19}$. **4.** $\frac{128}{15}$. **5.** $\frac{73}{17}$. **6.** $\frac{96}{19}$. **7.** $\frac{90}{12}$.

8. $\frac{282}{30}$. **9.** $\frac{174}{20}$. **10.** $\frac{172}{23}$. **11.** $\frac{113}{8}$. **12.** $\frac{315}{51}$. **13.** $\frac{339}{21}$.

14. Explain why (i) $\frac{17}{10} = 1 + \frac{7}{10}$; (ii) $\frac{29}{10} = 2\frac{9}{10}$;

and in each case illustrate your answer graphically by a line ruled on squared paper.

Express as improper fractions:

15. $3\frac{1}{2}$, $2\frac{3}{8}$, $1\frac{5}{9}$, $4\frac{7}{12}$, $5\frac{1}{5}$, $10\frac{2}{7}$, $11\frac{1}{9}$, $6\frac{13}{20}$, $8\frac{17}{100}$.

16. $5\frac{11}{13}$. 17. $12\frac{9}{13}$. 18. $15\frac{11}{14}$. 19. $20\frac{5}{23}$. 20. $11\frac{13}{25}$.

21. $40\frac{15}{19}$. 22. $18\frac{6}{49}$. 23. $27\frac{8}{21}$. 24. $53\frac{10}{23}$. 25. $29\frac{8}{57}$.

111. Comparison of Fractions. Since a fraction is the result of dividing the numerator by the denominator, when two fractions have the same denominator, the greater is that which has the greater numerator. And when two fractions have the same numerator, the *greater* is that which has the *less* denominator.

Thus $\frac{7}{12}$ is greater than $\frac{5}{12}$, and $\frac{7}{8}$ is greater than $\frac{7}{12}$.

112. By Art. 104, it is always possible to express a fraction in a new form by taking any number which is an exact multiple of its denominator for the new denominator.

Thus $\frac{5}{8}=\frac{5\times4}{8\times4}=\frac{5\times7}{8\times7}=\frac{5\times11}{8\times11}$, and so on.

EXAMPLE. *Express the fractions $\frac{3}{10}$ and $\frac{5}{14}$ with denominator* 70 *and find which of them is the greater.*

$$\frac{3}{10}=\frac{3\times\mathbf{7}}{10\times\mathbf{7}}=\frac{21}{70};$$

$$\frac{5}{14}=\frac{5\times\mathbf{5}}{14\times\mathbf{5}}=\frac{25}{70}.$$

The multipliers 7 and 5 are found by dividing the new denominator 70 by the denominators 10 and 14 in turn. As the fractions now have the same denominator the greater of the two is that with the greater numerator.

Thus $\frac{5}{14}$ is greater than $\frac{3}{10}$.

113. If it is required to compare two or more fractions with different denominators, it is usually best to replace them by equivalent fractions with the same denominator for all. The work will be simplified if we choose for the common denominator the L.C.M. of the denominators of the given fractions.

EXAMPLE. *Arrange the fractions $\frac{2}{15}$, $\frac{3}{10}$, $\frac{5}{21}$ in order of magnitude.*

$$\frac{2}{15}=\frac{2\times\mathbf{14}}{15\times\mathbf{14}}=\frac{28}{210}, \;\ldots\ldots(1)$$

$$\frac{3}{10}=\frac{3\times\mathbf{21}}{10\times\mathbf{21}}=\frac{63}{210}, \;\ldots\ldots(3)$$

$$\frac{5}{21}=\frac{5\times\mathbf{10}}{21\times\mathbf{10}}=\frac{50}{210}. \;\ldots\ldots(2)$$

$$\begin{array}{r|l} 5 & 15,\ 10,\ 21 \\ \hline & \;\;3,\;\;\ 2,\ 21 \end{array}$$

L.C.M. of 15, 10, 21 = 5 × 2 × 21 = 210.

The factors 14, 21, and 10 are obtained by dividing 210 by the denominators of the given fractions in turn. By comparing the numerators we see that $\frac{28}{210}$ is the least and $\frac{63}{210}$ the greatest of the given fractions.

Thus $\frac{2}{15}$, $\frac{5}{21}$, $\frac{3}{10}$ are the original fractions when arranged in *ascending* order.

NOTE. The L.C.M. of the denominators of a series of fractions is called their **least common denominator** (L.C.D.). It is usually best to keep the L.C.D. in factors, then the multipliers required to reduce the given fractions to their L.C.D. are found by *mentally* crossing out the factors of the given denominators. Thus in the above example,

$$\cancel{5}\times 2\times \cancel{3}\times 7=14,\quad \cancel{5}\times \cancel{2}\times 3\times 7=21,\quad 5\times 2\times \cancel{3}\times \cancel{7}=10.$$

EXAMPLES V. f.

(*Examples* 1–6 *may be taken orally.*)

Express the following pairs of fractions with their L.C.D.:

1. $\frac{1}{2}, \frac{1}{3}$; $\frac{1}{3}, \frac{1}{5}$; $\frac{1}{2}, \frac{3}{4}$; $\frac{2}{5}, \frac{1}{10}$; $\frac{5}{6}, \frac{1}{12}$; $\frac{7}{24}, \frac{3}{8}$.

2. $\frac{3}{4}, \frac{7}{12}$; $\frac{3}{4}, \frac{1}{6}$; $\frac{2}{15}, \frac{3}{10}$; $\frac{1}{6}, \frac{2}{9}$; $\frac{1}{15}, \frac{1}{12}$; $\frac{1}{14}, \frac{1}{21}$.

In the following pairs of fractions which is the greater?

3. $\frac{3}{5}, \frac{4}{5}$; $\frac{4}{7}, \frac{5}{7}$; $\frac{4}{7}, \frac{4}{9}$; $\frac{1}{5}, \frac{1}{6}$; $\frac{3}{11}, \frac{6}{11}$; $\frac{8}{21}, \frac{8}{23}$.

4. $\frac{3}{4}, \frac{7}{12}$; $\frac{7}{8}, \frac{9}{16}$; $\frac{3}{7}, \frac{11}{14}$; $\frac{5}{6}, \frac{7}{9}$; $\frac{9}{24}, \frac{5}{8}$; $\frac{10}{12}, \frac{25}{30}$.

Read off the following sets of fractions expressed with their L.C.D.:

5. $\frac{1}{2}, \frac{2}{3}, \frac{1}{6}$; $\frac{1}{2}, \frac{1}{4}, \frac{1}{3}$; $\frac{2}{5}, \frac{3}{10}, \frac{1}{2}$; $\frac{3}{4}, \frac{1}{8}, \frac{5}{12}$.

6. $\frac{2}{5}, \frac{3}{10}, \frac{7}{30}$; $\frac{4}{15}, \frac{3}{5}, \frac{7}{30}$; $\frac{5}{7}, \frac{2}{3}, \frac{3}{14}$; $\frac{7}{18}, \frac{5}{12}, \frac{11}{24}$.

Arrange in ascending order of magnitude:

7. $\frac{1}{2}, \frac{4}{7}, \frac{2}{3}$. **8.** $\frac{7}{18}, \frac{5}{12}, \frac{1}{3}$. **9.** $\frac{1}{5}, \frac{1}{4}, \frac{1}{10}, \frac{1}{2}$.

In the following pairs of fractions which is the greater, and by how much?

10. $\frac{40}{56}, \frac{56}{72}$. **11.** $\frac{15}{35}, \frac{33}{88}$. **12.** $\frac{7}{36}, \frac{13}{48}$. **13.** $\frac{24}{25}, \frac{97}{100}$.

Addition and Subtraction of Fractions.

114. When the fractions are of the same kind (*i.e.* all *ninths*, or all *elevenths*, and so on), we have only to take the sum or difference of the numerators, retaining the *common denominator*.

For just as $5\ \textit{feet}+2\ \textit{feet}=7\ \textit{feet}$,

so $\frac{5}{9}+\frac{2}{9}=5\ \textit{ninths}+2\ \textit{ninths}=7\ \textit{ninths}$;

and $\frac{5}{9}-\frac{2}{9}=5\ \textit{ninths}-2\ \textit{ninths}=3\ \textit{ninths}$.

Again $\frac{16}{25}+\frac{7}{25}+\frac{3}{25}=\frac{16+7+3}{25}=\frac{26}{25}=1\frac{1}{25}$.

And $\frac{16}{25}+\frac{7}{25}-\frac{3}{25}=\frac{16+7-3}{25}=\frac{20}{25}=\frac{4}{5}$.

NOTE. Results should be brought to their *lowest terms*, and improper fractions should be expressed as mixed numbers.

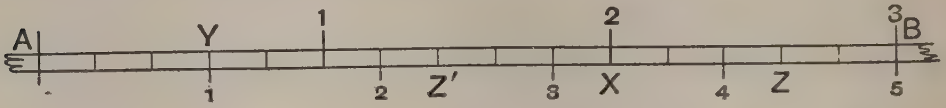
BANGALORE-1

115. When the fractions have different denominators they must first be expressed with a *common* denominator; and the work will be simplified by taking the *least* common denominator.

For instance $\frac{2}{3}+\frac{1}{5}=\frac{10}{15}+\frac{3}{15}=\frac{13}{15}$.

$\frac{2}{3}-\frac{1}{5}=\frac{10}{15}-\frac{3}{15}=\frac{7}{15}$.

This may be illustrated graphically as follows:



Take the line AB as the unit, and suppose it divided into *thirds* and also into *fifths*, as shewn in the diagram.

Place the letters X and Y, so that AX may represent $\frac{2}{3}$ of AB, and AY $\frac{1}{5}$ of AB.

Now subdivide AB into 15 equal parts, and take XZ and XZ′ each equal to AY on opposite sides of X.

Then AX contains 10 *fifteenths* of AB;

and AY .... 3 *fifteenths* ....

Now AX + AY = AX + XZ = AZ;

that is, $\frac{2}{3}+\frac{1}{5}$ = 10 *fifteenths* + 3 *fifteenths* = 13 *fifteenths*.

Similarly AX − AY = AX − XZ′ = AZ′;

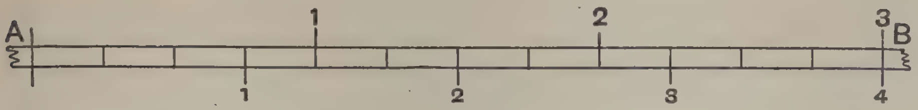
that is, $\frac{2}{3}-\frac{1}{5}$ = 10 *fifteenths* − 3 *fifteenths* = 7 *fifteenths*.

EXAMPLES V. g.

(*Examples* 1–32 *should be taken orally.*)

Find the value of

1. $\frac{3}{7}+\frac{2}{7}$.
2. $\frac{3}{5}+\frac{1}{5}$.
3. $\frac{1}{4}+\frac{3}{4}$.
4. $\frac{3}{8}+\frac{5}{8}$.
5. $\frac{1}{2}+\frac{1}{3}$.
6. $\frac{1}{3}+\frac{1}{5}$.
7. $\frac{1}{3}+\frac{1}{4}$.
8. $\frac{1}{4}+\frac{1}{5}$.
9. $\frac{1}{3}+\frac{1}{6}$.
10. $\frac{1}{8}+\frac{3}{4}$.
11. $\frac{3}{5}+\frac{1}{10}$.
12. $\frac{3}{14}+\frac{2}{7}$.
13. $\frac{3}{8}+\frac{5}{16}$.
14. $\frac{1}{9}+\frac{1}{6}$.
15. $\frac{5}{6}-\frac{1}{6}$.
16. $\frac{13}{15}-\frac{4}{15}$.
17. $\frac{13}{10}-\frac{3}{10}$.
18. $\frac{2}{3}-\frac{1}{6}$.
19. $\frac{5}{18}-\frac{2}{9}$.
20. $\frac{8}{9}-\frac{2}{27}$.
21. $\frac{16}{25}-\frac{3}{5}$.
22. $\frac{3}{10}+\frac{4}{15}$.
23. $\frac{7}{18}+\frac{5}{12}$.
24. $\frac{5}{15}+\frac{4}{20}$.
25. $\frac{7}{12}-\frac{11}{24}$.
26. $\frac{3}{5}-\frac{1}{4}$.
27. $\frac{2}{3}-\frac{1}{8}$.
28. $\frac{1}{4}-\frac{1}{9}$.
29. $\frac{1}{2}+\frac{1}{3}+\frac{1}{6}$.
30. $\frac{1}{2}+\frac{1}{5}+\frac{1}{10}$.
31. $\frac{1}{3}+\frac{3}{4}-\frac{5}{12}$.
32. $\frac{5}{18}+\frac{2}{9}-\frac{1}{2}$.



33. Use the above diagram to prove that

(i) $\frac{2}{3}+\frac{1}{4}=\frac{11}{12}$. (ii) $\frac{5}{12}-\frac{1}{4}=\frac{2}{12}$. (iii) $\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$.

34. Construct a diagram to illustrate graphically the following equivalents:

(i) $\frac{2}{5}+\frac{1}{3}=\frac{11}{15}$. (ii) $\frac{1}{5}+\frac{2}{15}=\frac{1}{3}$. (iii) $\frac{2}{5}+\frac{4}{15}=\frac{2}{3}$.

Find the value of

35. $\frac{a}{4}+\frac{a}{5}$. **36.** $\frac{5m}{6}+\frac{m}{12}$. **37.** $\frac{p}{6}-\frac{p}{9}$.

38. $\frac{1}{a}+\frac{1}{b}$. **39.** $\frac{2}{3a}-\frac{1}{2a}$. **40.** $\frac{2}{m}+\frac{3}{n}$.

Verify these results by substituting $a=3$, $b=4$, $p=5$, $m=1$, $n=7$.

EXAMPLE. *Find the sum of* $\frac{2}{3}$, $\frac{5}{12}$, $\frac{11}{20}$.

The L.C.D. of 3, 12, $20=3\times4\times5=60$.

$$\therefore \text{ the required sum}=\frac{40+25+33}{60}=\frac{98}{60}=\frac{49}{30}=1\tfrac{19}{30}.$$

$$\frac{2}{3}=\frac{2\times20}{3\times20}=\frac{40}{60},\quad \frac{5}{12}=\frac{5\times5}{12\times5}=\frac{25}{60},\quad \frac{11}{20}=\frac{11\times3}{20\times3}=\frac{33}{60}.$$

After a little practice the work given in smaller type may be omitted.

NOTE. If any of the given fractions are not in their lowest terms they should be reduced before finding the L.C.D.

EXAMPLES V. g. (*Continued.*)

Find the value of

41. $\frac{8}{21}+\frac{2}{7}+\frac{1}{3}$. **42.** $\frac{3}{7}+\frac{2}{5}+\frac{2}{35}$. **43.** $\frac{4}{21}+\frac{5}{6}+\frac{3}{14}$.

44. $\frac{7}{12}+\frac{11}{20}+\frac{13}{15}$. **45.** $\frac{4}{24}+\frac{2}{9}+\frac{5}{18}$. **46.** $\frac{6}{8}+\frac{20}{24}+\frac{1}{12}$.

47. $\frac{1}{39}+\frac{7}{65}-\frac{1}{15}$. **48.** $\frac{12}{35}-\frac{4}{21}+\frac{7}{15}$. **49.** $\frac{11}{12}-\frac{5}{36}-\frac{2}{27}$.

50. I inherit $\frac{2}{3}$ of a business, and then purchase another $\frac{1}{4}$ of it. What part of the business must I now acquire if I wish to possess the whole?

116. In adding a series of fractions, some of which are mixed numbers, it is convenient to add the integral and fractional parts separately. Any improper fractions should first be expressed as mixed numbers.

EXAMPLE 1. *Find the value of* $3\frac{5}{6}+1\frac{2}{9}+\frac{31}{12}+\frac{7}{30}$.

$$\begin{aligned}\text{The required sum} &= 3\tfrac{5}{6}+1\tfrac{2}{9}+2\tfrac{7}{12}+\tfrac{7}{30}\\ &= 3+1+2+\tfrac{5}{6}+\tfrac{2}{9}+\tfrac{7}{12}+\tfrac{7}{30}\\ &= 6+\frac{150+40+105+42}{180}\\ &= 6+\tfrac{337}{180}=6+1\tfrac{157}{180}\\ &= 7\tfrac{157}{180}.\end{aligned}$$

First write

$\frac{31}{12}=2\frac{7}{12}$.

Next collect the integral parts and the fractional separately.

The L.C.D. $=180$.

EXAMPLE 2. *Find the value of* $5\frac{9}{28}-2\frac{1}{35}$.

The L.C.D. $=7\times4\times5=140$. The integral and fractional parts are taken separately.

$$\begin{aligned}\text{Thus } 5\tfrac{9}{28}-2\tfrac{1}{35} &= 5-2+\tfrac{9}{28}-\tfrac{1}{35}=3+\frac{45-4}{140}\\ &= 3\tfrac{41}{140}.\end{aligned}$$

EXAMPLES V. h.

Find the sum of

1. $3\frac{2}{5}$, $1\frac{1}{6}$, $\frac{4}{15}$.
2. $2\frac{1}{8}$, $4\frac{3}{16}$, $\frac{7}{40}$.
3. $6\frac{3}{5}$, $4\frac{2}{7}$, $1\frac{3}{10}$.
4. $\frac{11}{15}$, $8\frac{4}{5}$, $\frac{33}{20}$.
5. $4\frac{5}{36}$, $2\frac{1}{9}$, $3\frac{3}{4}$.
6. $\frac{40}{39}$, $\frac{32}{15}$, $\frac{7}{65}$.

Find the difference between

7. $2\frac{3}{5}$ and $1\frac{1}{5}$.
8. $2\frac{5}{21}$ and $1\frac{1}{7}$.
9. $6\frac{7}{8}$ and $2\frac{3}{4}$.
10. $1\frac{4}{9}$ and $\frac{1}{5}$.
11. $\frac{13}{15}$ and $\frac{7}{10}$.
12. $\frac{11}{12}$ and $\frac{13}{16}$.

Find the value of

13. $2\frac{3}{5}+3\frac{9}{10}+5\frac{7}{12}+4$.
14. $5\frac{6}{7}+1\frac{1}{15}+3\frac{3}{5}+2\frac{10}{21}$.
15. $\frac{7}{48}+3\frac{5}{12}+2\frac{13}{16}+5\frac{7}{24}$.
16. $6\frac{1}{90}+\frac{4}{15}+3\frac{7}{24}+1\frac{11}{36}$.
17. $\frac{19}{39}+\frac{7}{60}+\frac{57}{52}+\frac{11}{20}$.
18. $\frac{9}{77}+2\frac{7}{33}+\frac{48}{63}+7\frac{10}{11}$.
19. $\frac{7}{26}-\frac{4}{39}+\frac{1}{6}$.
20. $5\frac{2}{21}-2\frac{3}{56}-\frac{1}{24}$.
21. $8\frac{2}{5}-5\frac{7}{55}-3\frac{3}{11}$.
22. $2\frac{5}{77}-1\frac{2}{55}-\frac{1}{35}$.
23. $3+\frac{7}{10}+\frac{8}{100}+\frac{9}{1000}$.
24. $5+\dfrac{1}{10}+\dfrac{3}{10^2}+\dfrac{7}{10^3}$.

117. In the following examples some modification of the work will be required before subtraction can be performed.

EXAMPLE 1. *Find the value of* (i) $1-\frac{5}{12}$; (ii) $3-\frac{7}{16}$.

(i) $1-\frac{5}{12}=\frac{12}{12}-\frac{5}{12}=\frac{7}{12}$;

(ii) $3-\frac{7}{16}=2+\frac{16}{16}-\frac{7}{16}$

$=2\frac{9}{16}$.

In (i) the unit is replaced by $\frac{12}{12}$.

In (ii) we take 1 unit out of the 3 units and replace it by the equivalent fraction $\frac{16}{16}$.

EXAMPLE 2. *Subtract* $1\frac{11}{24}$ *from* $4\frac{2}{9}$.

The L.C.D. $=3\times3\times8=72$.

$$4\frac{2}{9}-1\frac{11}{24}=4-1+\frac{2}{9}-\frac{11}{24}$$
$$=3+\frac{16-33}{72}$$
$$=2+\frac{72+16-33}{72}$$
$$=2\frac{55}{72}.$$

Here, at the second step, as we cannot subtract 33 from 16, one of the three units is replaced by the equivalent fraction $\frac{72}{72}$.

NOTE. Before subtraction improper fractions should be replaced by mixed numbers.

EXAMPLES V. k.

(*Examples* 1–25 *should be taken orally.*)

Find the value of

1. $1-\frac{2}{5}$. **2.** $1-\frac{3}{8}$. **3.** $1-\frac{5}{6}$. **4.** $1-\frac{1}{12}$. **5.** $1-\frac{5}{20}$.

6. $2-\frac{1}{3}$. **7.** $2-\frac{5}{8}$. **8.** $3-\frac{1}{10}$. **9.** $2-\frac{3}{12}$.

10. $4-\frac{2}{7}$. **11.** $6-3\frac{1}{5}$. **12.** $5-2\frac{5}{7}$. **13.** $9-\frac{6}{8}$.

14. $10-4\frac{1}{9}$. **15.** $2-1\frac{1}{2}$. **16.** $3-2\frac{1}{3}$. **17.** $2\frac{1}{2}-\frac{1}{4}$.

18. $1\frac{5}{9}-\frac{7}{9}$. **19.** $2\frac{1}{16}-\frac{15}{16}$. **20.** $5\frac{13}{15}-\frac{3}{5}$. **21.** $7\frac{5}{10}-\frac{1}{2}$.

22. $8\frac{7}{14}-\frac{3}{7}$. **23.** $9\frac{1}{12}-\frac{3}{4}$. **24.** $5\frac{1}{5}-\frac{7}{15}$. **25.** $10\frac{1}{2}-\frac{19}{30}$.

Simplify

26. $9\frac{3}{11}-2\frac{9}{22}$. **27.** $8\frac{3}{7}-2\frac{7}{9}$. **28.** $5\frac{17}{28}-\frac{27}{35}$.

29. $\frac{47}{20}-\frac{57}{40}$. **30.** $19\frac{25}{63}-15\frac{26}{27}$. **31.** $21\frac{4}{105}-20\frac{59}{60}$.

32. $\frac{346}{23}-\frac{20}{21}$. **33.** $7\frac{41}{45}-\frac{503}{72}$.

34. Read off the integers nearest to the following mixed numbers, and in each case state by how much the nearest integer differs from the mixed number:

$5\frac{1}{4}$, $5\frac{3}{4}$, $3\frac{3}{8}$, $3\frac{5}{8}$, $8\frac{4}{10}$, $8\frac{6}{10}$, $1\frac{7}{12}$, $1\frac{5}{12}$,
$7\frac{17}{36}$, $7\frac{19}{36}$, $11\frac{49}{100}$, $11\frac{51}{100}$.

35. What is the excess of $3\frac{1}{8}$ over $2\frac{1}{4}$?

36. What must be added to $3\frac{3}{11}$ to make the total 5?

37. By how much does $\frac{3}{8}+\frac{7}{12}$ fall short of unity?

38. By how much does 3 exceed $2\frac{2}{9}-\frac{5}{6}$?

39. Which is the greater $3\frac{1}{8}-1\frac{1}{6}$ or $2-\frac{1}{12}$, and by how much?

40. How many shillings must be added to £$\frac{1}{4}$ + £$\frac{3}{5}$ to make up £1?

41. Add to $2\frac{1}{2}-\frac{3}{5}+\frac{9}{10}$ the smallest fraction which will make the result a whole number.

42. Subtract $\frac{11}{3}-\frac{5}{7}$ from 10. How much does the result differ from the nearest integer?

43. What fractions are represented by x in the following statements?

(i) $6\frac{2}{9}-x=1\frac{7}{18}$; (ii) $x+1\frac{1}{8}-\frac{7}{120}=1\frac{3}{5}$.

118. We conclude this section with a few harder examples of Addition and Subtraction. The terms may be taken in any order, and as before the integral and fractional parts should be dealt with separately. Moreover labour may often be saved by grouping together two or more fractions which easily combine to give a simple L.C.D. In this case care should be taken to adjust the signs when enclosing two or more terms in brackets, or combining them with a single vinculum.

EXAMPLE 1. *Simplify* $7-1\frac{3}{4}-4\frac{7}{8}$.

$$\begin{aligned}\text{The expression} &= 2-\tfrac{3}{4}-\tfrac{7}{8}\\ &= 2-\left(\tfrac{3}{4}+\tfrac{7}{8}\right)\\ &= 2-\tfrac{6+7}{8}\\ &= 2-1\tfrac{5}{8}\\ &= \tfrac{3}{8}.\end{aligned}$$

Or thus

$$\begin{aligned}\text{The expression} &= \left(2-\tfrac{3}{4}\right)-\tfrac{7}{8}\\ &= 1\tfrac{1}{4}-\tfrac{7}{8}\\ &= \tfrac{10}{8}-\tfrac{7}{8}\\ &= \tfrac{3}{8}.\end{aligned}$$

EXAMPLE 2. *Simplify* $8\frac{13}{15}+\frac{4}{39}-3\frac{7}{10}-2\frac{7}{26}$.

The expression $=3+\frac{13}{15}-\frac{7}{10}+\frac{4}{39}-\frac{7}{26}$

$\doteq 3+\frac{5}{30}+\frac{8-21}{13\times3\times2}$

$=3+\frac{1}{6}-\frac{1}{6}$

$=3.$

In the first line of work, after collecting the whole numbers, the first and third fractions have been brought together and the second and fourth.

EXAMPLES V. 1.

Simplify the following expressions:

1. $\frac{7}{8}-\frac{1}{2}+\frac{3}{16}$.
2. $\frac{7}{8}-(\frac{1}{2}+\frac{3}{16})$.
3. $14-1\frac{3}{4}+6\frac{2}{3}$.
4. $14-(1\frac{3}{4}+6\frac{2}{3})$.
5. $11-3\frac{2}{3}-2\frac{3}{5}$.
6. $11-(3\frac{2}{3}-2\frac{3}{5})$.
7. $\frac{3}{11}+\frac{5}{22}-\frac{1}{15}-\frac{3}{10}$.
8. $\frac{5}{13}+\frac{3}{26}-\frac{1}{7}-\frac{4}{21}$.
9. $5\frac{11}{15}-1\frac{2}{5}-2\frac{7}{9}-\frac{5}{36}$.
10. $3\frac{7}{26}-\frac{4}{39}+2\frac{2}{21}-5\frac{3}{36}$.

[In Examples 11 and 12, group together first and third terms and second and fourth.]

11. $3\frac{10}{11}+\frac{17}{5}-1\frac{9}{22}-4\frac{9}{10}$.
12. $12\frac{1}{2}-5\frac{13}{72}+1\frac{3}{8}-\frac{1}{36}$.
13. $3\frac{8}{45}-10\frac{1}{7}+5\frac{2}{35}+2\frac{11}{63}$.
14. $9\frac{5}{51}-3\frac{2}{9}+5\frac{7}{18}-10\frac{9}{34}$.
15. $6\frac{2}{3}-3\frac{3}{4}+\frac{29}{6}-1\frac{33}{44}$.
16. $3\frac{11}{24}-\frac{19}{9}+6\frac{3}{8}+\frac{1}{2}-\frac{5}{12}+\frac{13}{36}$.

17. Subtract $\frac{4}{7}-\frac{8}{21}$ from the sum of $\frac{3}{4}$, $\frac{6}{7}$ and $\frac{7}{12}$.

18. From $8\frac{5}{51}-3\frac{2}{9}$ take the difference between $10\frac{9}{34}$ and $5\frac{17}{18}$.

[Write the expression in the form $8\frac{5}{51}-3\frac{2}{9}-(10\frac{9}{34}-5\frac{17}{18})$; remove brackets, and group the terms suitably.]

19. By how much does $2\frac{2}{55}-\frac{4}{35}$ exceed the difference between $1\frac{8}{105}$ and $\frac{17}{77}$?

20. If I receive £$2\frac{3}{20}$ and £$1\frac{3}{5}$ and then spend £$3\frac{1}{6}$ and £$\frac{1}{12}$, what have I left?

21. From a purse containing £10 a man pays sums equivalent to £$2\frac{1}{8}$, £$4\frac{3}{16}$, £$\frac{17}{40}$. How much has he left?

22. *A*, *B*, and *C* have to collect a certain sum for a charity; if they collect $\frac{5}{24}$, $\frac{3}{10}$, and $\frac{1}{5}$ of the whole sum respectively, what fraction of the whole has still to be collected? If the whole sum is £240, what are the four separate sums?

23. *A* owes *B* the following sums of money: 1*s.* $3\frac{1}{4}d.$, 2*s.* $6\frac{1}{2}d.$, $6\frac{1}{2}d.$ and $3\frac{1}{2}d.$ *A* has only shillings in his pocket, and *B* has only pence. How can they *most nearly* settle the matter?

24. The three joints of a fishing-rod measure $40\frac{5}{8}$ inches, $40\frac{3}{8}$ inches, and $38\frac{1}{4}$ inches; and when the rod is put together $5\frac{1}{2}$ inches are lost by overlapping of the joints. Express the total length as nearly as possible *in feet and an exact number of inches.*

25. A man left $\frac{1}{3}$ of his property to his eldest son, $\frac{1}{4}$ to his second son, and $\frac{1}{5}$ to his third; while legal and other expenses absorbed $\frac{2}{15}$. The residue was left to a charity, which thus obtained £1200. What was the whole amount of the property?

26. In a South American town of 350,000 inhabitants $\frac{2}{5}$ of the population are of Spanish origin, $\frac{1}{5}$ of it are Italians, $\frac{1}{10}$ British, $\frac{1}{12}$ German, and $\frac{1}{20}$ are French. The rest are returned as "various." How many persons approximately are there in this last class?

27. Prove that

(i) $\dfrac{a+b}{a}+\dfrac{a+b}{b}-2=\dfrac{a^2+b^2}{ab}$. (ii) $\dfrac{a-b}{a^2b^2}-\dfrac{1}{ab^2}+\dfrac{1}{a^2b}=0.$

And in each case verify your result by substituting $a=2$ and $b=3$.

Multiplication and Division by an Integer.

119. Multiplication. Just as in multiplying 2 *feet* by 4, we take each foot four times, so in multiplying 2 *fifteenths* by 4 we take each *fifteenth* four times.

So that $$\frac{2}{15}\times 4=\frac{2\times 4}{15}=\frac{8}{15}.$$

Similarly $$\frac{a}{b}\times m=\frac{a\times m}{b}=\frac{ma}{b}.$$

Hence *to multiply a fraction by an integer we have only to* **multiply its numerator by that integer.**

EXAMPLES. (i) $\frac{3}{11}\times 3=\frac{3\times 3}{11}=\frac{9}{11}.$

(ii) $\frac{5}{16}\times 6=\frac{5\times \overset{3}{\cancel{6}}}{\underset{8}{\cancel{16}}}=\frac{15}{8}=1\frac{7}{8}.$

(iii) $\frac{5}{8}\times 16=\frac{5\times \overset{2}{\cancel{16}}}{\cancel{8}}=5\times 2=10.$

(iv) $\frac{2}{15}\times 5=\frac{2\times \cancel{5}}{\underset{3}{\cancel{15}}}=\frac{2}{3}.$

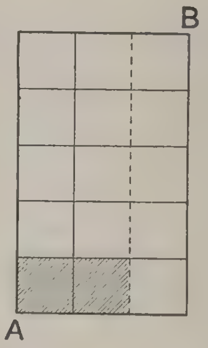
From (iv) it is seen that *when the multiplier is a factor of the denominator, the result of multiplication may be obtained by* **dividing the denominator by the integer.**

Thus in $\frac{2}{15}\times 5=\frac{2}{3}$, instead of taking each fifteenth five times, we may convert each *fifteenth* into a part *five times as great*, namely into a *third.* Thus 2 *fifteenths* × 5 = 2 *thirds.*

Graphical illustration. Here the unit is represented by the rectangle AB, of which the height and breadth are divided into 5 and 3 equal parts respectively; so that the whole contains 15 equal squares representing *fifteenths* of the unit.

Hence $\frac{2}{15}$ of the unit is represented by 2 squares (shaded); and to multiply by 5 converts each of these squares into a *column* of squares, each column being one-*third* of the unit rectangle.

That is, $\frac{2}{15}\times 5=\frac{2}{3}$.



120. In multiplying a mixed number by an *integer*, the mixed number need not first be expressed as an improper fraction.

EXAMPLE. *Multiply* (i) $7\frac{8}{25}$ *by* 2; (ii) $9\frac{11}{36}$ *by* 6.

(i) $7\frac{8}{25}\times 2=$ *twice* 7 together with *twice* $\frac{8}{25}=14+\frac{16}{25}=14\frac{16}{25}$.

(ii) $9\frac{11}{36}\times 6=9\times 6+\frac{11}{36}\times 6=54+\frac{11}{6}=55\frac{5}{6}$.

121. Division by an integer. Just as 6 *feet* divided by 3 gives 2 *feet*, so 6 *sevenths* divided by 3 gives 2 *sevenths*;

that is, $\frac{6}{7}\div 3=\frac{2}{7}$, or $\frac{6\div 3}{7}$.

Now suppose $\frac{6}{7}$ is to be divided by **5**.

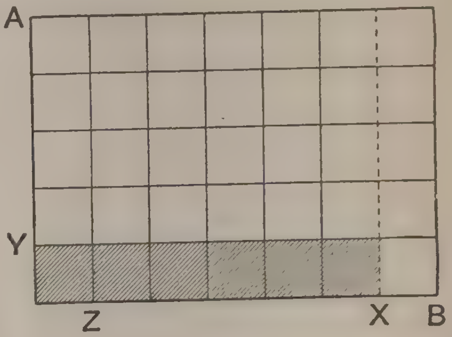
Here since we cannot divide the *number* of sevenths, we must break up each seventh into 5 equal subdivisions of which the unit therefore contains 35, and the given fraction 30.

Then $\frac{6}{7}\div 5=\frac{30}{35}\div 5=\frac{6}{35}$, or $\frac{6}{7\times 5}$.

And generally, $\frac{a}{b}\div m=\frac{a}{b\times m}$.

Hence *to divide a fraction by an integer, we either* **divide the numerator** (*if the numerator is a multiple of the integer*) or **multiply the denominator** *by the integer.*

Graphical illustration. The second case only need be illustrated.



To shew that $\frac{6}{7} \div 5 = \frac{6}{35}$.

Suppose the unit represented by a rectangle AB, whose length contains 7 equal parts (denominator), and breadth 5 equal parts (divisor). Then the unit rectangle contains 35 equal squares representing *thirty-fifths.*

Now $\frac{1}{7}$ of the unit is represented by 1 column of squares, namely the fig. AZ.

$\therefore$ $\frac{6}{7}$ of the unit is represented by 6 columns, namely the fig. AX (which contains 5 rows).

$\therefore$ $\frac{6}{7} \div 5$ is represented by *one* such row YX, which is seen to contain **6** squares, or 6 *thirty-fifths.*

That is, $$\frac{6}{7} \div 5 = \frac{6}{35}.$$

EXAMPLE 1. *Divide* (i) $\frac{4}{5}$ *by* 2; (ii) $\frac{3}{8}$ *by* 5.

$$\text{(i)}\ \frac{4}{5} \div 2 = \frac{4 \div 2}{5} = \frac{2}{5}. \qquad \text{(ii)}\ \frac{3}{8} \div 5 = \frac{3}{8 \times 5} = \frac{3}{40}.$$

EXAMPLE 2. *Find the value of* $3\frac{5}{12} \div 8$.

$$3\frac{5}{12} \div 8 = \frac{41}{12} \div 8 = \frac{41}{12 \times 8} = \frac{41}{96}.$$

122. When the dividend is a mixed number, and the integral part is greater than the divisor, we proceed as follows:

EXAMPLE 1. *Divide* $13\frac{3}{8}$ *by* 4.

$$13\frac{3}{8} \div 4 = \left(13 + \frac{3}{8}\right) \div 4 = \left(12 + 1\frac{3}{8}\right) \div 4$$
$$= 3 + \left(\frac{11}{8} \div 4\right) = 3 + \frac{11}{32} = 3\frac{11}{32}.$$

EXAMPLE 2. *Divide* $301\frac{1}{5}$ *by* 24.

$$\begin{array}{r|l} 4 & 301\frac{1}{5} \\ \hline 6 & \ \ 75\frac{3}{10} \ldots \text{ for remainder} = 1\frac{1}{5}\text{; and } 1\frac{1}{5} \div 4 = \frac{6}{20} = \frac{3}{10}. \\ \hline & \ \ 12\frac{11}{10} \ldots \text{ for remainder} = 3\frac{3}{10}\text{; and } 3\frac{3}{10} \div 6 = \frac{33}{60} = \frac{11}{20}. \end{array}$$

NOTE. We have seen that a fraction is the result of dividing one number by another (Art. 101). Hence in any case where the division is not exact, the **complete quotient** can be expressed as a mixed number.

Thus $$20 \div 7 = \frac{20}{7} = 2\frac{6}{7}.$$

EXAMPLES V. m.

(*Examples* 1–12 *should be taken orally.*)

Multiply

1. $\frac{3}{7}$ by 2, $\frac{4}{9}$ by 2, $\frac{2}{11}$ by 3, $\frac{3}{16}$ by 5, $\frac{4}{15}$ by 5.

2. $\frac{5}{12}$ by 6, $\frac{7}{16}$ by 4, $\frac{3}{13}$ by 4, $\frac{3}{40}$ by 8, $\frac{7}{12}$ by 6.

3. $\frac{2}{15}$ by 3, 5, 10, 12, 30.

4. $\frac{17}{24}$ by 4, 6, 8, 12, 24.

5. $\frac{11}{36}$ by 4, 9, 12, 18, 36.

6. $\frac{8}{25}$ by 5, 10, 15, 50, 200.

7. Find the value of £$1\frac{2}{9} \times 3$, $3\frac{5}{8}$ ft. × 8, $1\frac{7}{12}$ cwt. × 6.

8. What is the cost of 25 tons of coal at £$1\frac{1}{5}$ a ton?

Divide

9. $\frac{6}{7}$ by 2, $\frac{12}{17}$ by 3, $\frac{10}{13}$ by 5, $\frac{8}{9}$ by 4, $\frac{3}{5}$ by 5.

10. $\frac{2}{3}$ by 8, $\frac{16}{17}$ by 8, $\frac{14}{19}$ by 14, $\frac{1}{5}$ by 9, $\frac{7}{9}$ by 14.

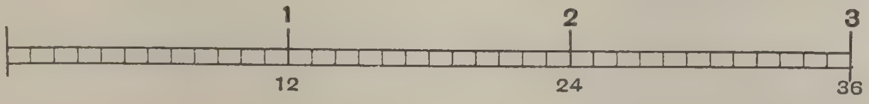
11. $\frac{30}{31}$ by 2, 5, 6, 15, 60.

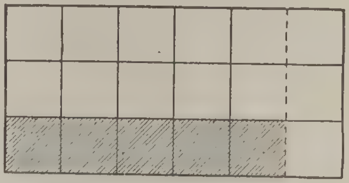
12. $\frac{48}{49}$ by 16, 24, 8, 4, 3.

13. Explain carefully from first principles, following the argument of Art. 119, why

$$\text{(i)}\ \frac{5}{12} \times 7 = \frac{35}{12} = 2\frac{11}{12}; \quad \text{(ii)}\ \frac{5}{12} \times 6 = \frac{5}{2} = 2\frac{1}{2}.$$

And illustrate the truth of each statement from the following diagram.



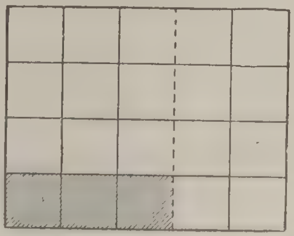


14. Use the adjoining diagram to shew that $\frac{5}{18} \times 3 = \frac{5}{6}$. By extending the columns standing on the shaded squares, shew that $\frac{5}{18} \times 6 = 1\frac{2}{3}$.

Draw similar diagrams on squared paper to illustrate

$$\text{(i)}\ \frac{3}{16} \times 4 = \frac{3}{4}; \quad \text{(ii)}\ \frac{3}{16} \times 8 = 1\frac{1}{2}.$$

15. Explain carefully why a fraction is divided by an integer *by multiplying its denominator by that integer.*



Use the adjoining diagram to illustrate the truth of

$$\frac{3}{5} \div 4 = \frac{3}{20}.$$

And draw on your squared paper similar diagrams to illustrate

$$\text{(i)}\ \frac{5}{6} \div 3 = \frac{5}{18}; \quad \text{(ii)}\ 2\frac{5}{6} \div 3 = \frac{17}{18}.$$

Find the value of

16. $7\frac{3}{14}\times 7$. **17.** $9\frac{5}{16}\times 8$. **18.** $4\frac{5}{12}\times 36$. **19.** $1\frac{3}{8}\times 20$.
20. $16\frac{7}{8}\div 15$. **21.** $14\frac{5}{8}\div 13$. **22.** $9\frac{7}{9}\div 16$. **23.** $20\frac{3}{10}\div 29$.

24. Write down the quotient, remainder, and *complete quotient*, when (i) 112 is divided by 9; (ii) 127 by 17; (iii) 200 by 13.

25. If £$2\frac{5}{8}$ is divided among 7 persons what is the share of each?

26. What is the circumference of a wheel which revolves 10 times in $9\frac{1}{11}$ yds.?

27. If $7\frac{3}{5}$ tons of coal are carried in 19 trucks, what is the load carried by each?

28. Find the value of 18 lbs. at $7\frac{5}{12}$ shillings a pound.

29. Find the total length of 100 rails, each $5\frac{7}{10}$ metres long.

30. If 31 yards of cloth cost $103\frac{1}{3}$ shillings, what is the cost of 1 yard?

31. Eleven posts are fixed at equal intervals in a straight line. The distance between the first and last is $141\frac{2}{3}$ yards. What is the distance between any pair of consecutive posts?

Given the following approximate equivalents:

1 *kilometre* = $\frac{5}{8}$ *mile*; 1 *kilogram* = $2\frac{1}{5}$ *lbs.*;
1 *litre* = $1\frac{3}{4}$ *pints*; 1 *hectare* = $2\frac{1}{2}$ *acres*;

find roughly:

32. How many miles there are in 16 kilometres; in 22 Km.; in 100 Km.

33. How many pounds in 10 kilograms; in 35 Kg.; in 73 Kg.

34. How many pints in 26 litres; in 42 litres; in 100 litres.

35. How many acres in 2 hectares; in 100 hectares; and by how much does an area of 73 hectares differ from 182 acres?

Multiplication by a Fraction. Compound Fractions.

123. **Fractions of Simple Quantities.** In finding the value of a fraction of any quantity we treat the quantity just as we treat the *unit*, when we form the given fraction. That is to say, we divide the quantity into the number of equal parts indicated by the denominator, and of these we take the number indicated by the numerator.

EXAMPLE. *Find the value of* (i) $\frac{7}{9}$ of £45; (ii) $\frac{5}{8}$ of 3 inches.

(i) *One* ninth of £45 $=$ £45 $\div 9 =$ £5;

$\therefore$ 7 ninths ...... $=$ £5 $\times 7 =$ £35.

Or $\frac{7}{9}$ of £45 $=$ £$\frac{\overset{5}{\cancel{45}}\times 7}{\cancel{9}} =$ £35.

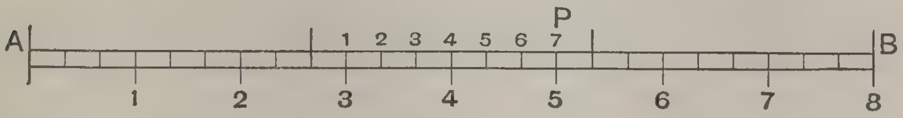
(ii) $\frac{1}{8}$ of 3 inches $=(3\div 8)$ inches $=\frac{3}{8}$ inch;

$\therefore$ $\frac{5}{8}$ .... $=\frac{3\times 5}{8}$ inches $=1\frac{7}{8}$ inches.

Similarly $\frac{a}{b}$ of $m=\frac{m\times a}{b}$.

Graphical illustration. *To prove that* $\frac{5}{8}$ *of* 3 *inches* $=1\frac{7}{8}$ *inches.*

Take a line AB 3 inches long. Divide AB into 8 equal parts, and also divide *each inch* into 8 equal parts. (The former of these subdivisions are shewn by the figures below the line, while the figures above the line shew the subdivisions of the second inch.)



Then AB must contain 24 eighths of 1 inch;

$\therefore$ each eighth of AB contains **3** eighths of 1 inch.

Accordingly, AP, representing 5 eighths of AB, contains 15 eighths of 1 inch, or, as is seen from the diagram, $1\frac{7}{8}$ inch.

That is, $\frac{5}{8}$ of 3 inches $=1\frac{7}{8}$ inch.

EXAMPLES V. n.

(*Examples* 1–6 *should be taken orally.*)

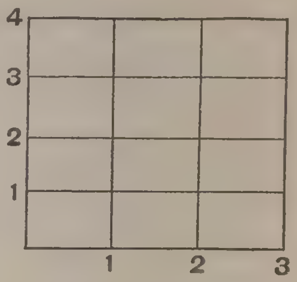
Read off the values of

1. $\frac{1}{4}$ of £20; $\frac{3}{4}$ of £20; $\frac{1}{5}$ of £15; $\frac{4}{5}$ of £15.
2. $\frac{1}{6}$ of 18 cwt.; $\frac{5}{6}$ of 18 cwt.; $\frac{1}{10}$ of 50 Km.; $\frac{7}{10}$ of 50 Km.
3. $\frac{1}{4}$ of 5 yds.; $\frac{3}{4}$ of 5 yds.; $\frac{1}{5}$ of £8; $\frac{4}{5}$ of £8.
4. $\frac{1}{10}$ of 15 francs; $\frac{3}{10}$ of 15 fr.; $\frac{7}{10}$ of 15 fr.; $\frac{9}{10}$ of 15 fr.
5. $\frac{1}{12}$ of 20 inches; $\frac{5}{12}$ of 20 in.; $\frac{7}{12}$ of 20 in.; $\frac{11}{12}$ of 3 in.
6. $1\frac{1}{2}$ of £4; $2\frac{1}{3}$ of 6 cwt.; $1\frac{3}{4}$ of 10 Km.; $1\frac{3}{8}$ of 12 ft.

7. Draw a line 3 inches long. Divide it into 4 equal parts, and also each inch into 4 equal parts. Hence shew that

$$\tfrac{1}{4} \text{ of } 3 = \tfrac{3}{4} \text{ of } 1.$$

Shew how the same conclusion may be drawn from the adjoining diagram. [Let each *column* represent the unit.]



8. Draw on squared paper rectangular diagrams (as in Ex. 7) to illustrate the following:

(i) $\frac{2}{3}$ of $5 = \frac{10}{3}$. (ii) $\frac{3}{5}$ of $4 = \frac{12}{5}$.

Find the value of

9. $\frac{13}{16}$ of 244. 10. $4\frac{3}{14}$ of 21. 11. $6\frac{5}{8}$ of 100. 12. $8\frac{2}{5}$ of 40.

13. Simplify (i) $\dfrac{m}{n}$ of x; (ii) $\dfrac{m}{n}$ of nx; (iii) $\dfrac{a}{bc}$ of ac.

14. I am entitled to $\frac{7}{15}$ of the proceeds of a business making an average annual profit of £9325. What is my income from this source?

15. A man leaves $\frac{5}{8}$ of his estate to his son and the rest to his daughter. The estate is worth £5650. What is the daughter's share worth?

16. A railway 896 kilometres in length is under construction. When $\frac{11}{16}$ of the distance is in running order, how many kilometres remain to complete the work?

17. A firm's capital consists of 12 shares, of which I possess 7. Out of this property I give my son £2500. If the whole capital is worth £30,000, how much remains to me?

18. A steamer whose bunkers carry 1620 tons of coal, burns $\frac{13}{18}$ of this amount on a passage. For how many more days could she run, burning 90 tons a day?

19. A firm's profits for the year are £10,043. Of this sum the senior partner takes $\frac{4}{11}$, the second $\frac{3}{11}$, and the third $\frac{2}{11}$. The rest is added to the reserve fund. How much does each partner get, and how much goes to the reserve?

20. On discharging a cargo of West Indian oranges, it was found that $\frac{3}{8}$ had gone bad on the passage. The rest made a profit of 2 shillings per hundred oranges. What profit was made on a consignment of 12,000?

124. Multiplication by a Fraction. We have now to ascertain in what sense we are to understand *multiplication by a* **fraction.**

When we multiply £5 by 4, we give to each unit in the multiplier (4) the value of the multiplicand (£5). Thus £5 × 4 may be expressed by the words 4 **of** £5. In this sense we may interpret £5 *multiplied by* $\frac{7}{9}$, or £5 × $\frac{7}{9}$, as meaning $\frac{7}{9}$ **of** £5.

And generally we accept $m \times \frac{a}{b}$ as meaning $\frac{a}{b}$ **of** m.

Thus (i) 728 miles × $\frac{11}{16} = \frac{11}{16}$ of 728 miles $= \frac{\overset{91}{\cancel{728}} \times 11}{\underset{2}{\cancel{16}}} = 500\frac{1}{2}$ miles.

(ii) 68 tons × $4\frac{2}{7} = 4\frac{2}{7}$ of 68 tons $= \frac{68 \times 30}{7} = 291\frac{3}{7}$ tons.

125. A Fraction of a Fraction. To find the value of $\frac{4}{5}$ of a *unit* we divide the unit by 5 and multiply the result by 4.

Similarly, to find the value of $\frac{4}{5}$ of $\frac{2}{3}$, we divide $\frac{2}{3}$ by 5, and multiply the result by 4.

The first operation gives $\frac{2}{3 \times 5}$, and the second gives $\frac{2 \times 4}{3 \times 5}$.

Now we have seen that multiplying any quantity by $\frac{4}{5}$ is the same as taking $\frac{4}{5}$ of that quantity;

that is, $\frac{2}{3} \times \frac{4}{5} = \frac{4}{5}$ of $\frac{2}{3} = \frac{2 \times 4}{3 \times 5}$.

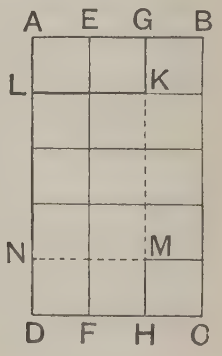
Similarly $\frac{a}{b} \times \frac{m}{n} = \frac{m}{n}$ of $\frac{a}{b} = \frac{a \times m}{b \times n}$.

Multiplication of Fractions. Hence *to multiply two fractions, we* **multiply the numerators together** *to form the numerator of the product, and* **multiply the denominators together** *to form the denominator of the product.*

126. An expression like $\frac{4}{5}$ of $\frac{2}{3}$ is called a **compound fraction,** while $\frac{4}{5}$ of unity is a **simple fraction.**

The result of the last article may be obtained graphically as follows.

Let the fig. ABCD be taken as the unit. It consists of 3 columns such as AEFD, each containing 5 squares. Thus the unit consists of 15 squares.



The fig. AEFD represents $\frac{1}{3}$, and AGHD represents $\frac{2}{3}$ of the unit.

Now $\frac{1}{5}$ of $\frac{2}{3}$ = the row AGKL;

∴ $\frac{4}{5}$ of $\frac{2}{3}$ = 4 of such rows

= the fig. AGMN

= 8 squares

= $\frac{8}{15}$ of the unit.

That is, $\frac{4}{5}$ of $\frac{2}{3} = \frac{8}{15} = \frac{2 \times 4}{3 \times 5}$.

EXAMPLE 1. *Multiply* $\frac{20}{39}$ *by* $\frac{26}{45}$.

$$\text{The product} = \frac{\overset{4}{\cancel{20}} \times \overset{2}{\cancel{26}}}{\underset{3}{\cancel{39}} \times \underset{9}{\cancel{45}}} = \frac{4 \times 2}{3 \times 9} = \frac{8}{27}.$$

EXAMPLE 2. *Find the value of* $2\frac{5}{14} \times \frac{7}{66}$.

$$\text{The product} = \frac{\cancel{33}}{\underset{2}{\cancel{14}}} \times \frac{\cancel{7}}{\underset{2}{\cancel{66}}} = \frac{1}{2 \times 2}$$

$$= \frac{1}{4}.$$

Here we first express $2\frac{5}{14}$ as an improper fraction.

After removing the factors 7 and 33 from numerator and denominator the resulting numerator is 1.

NOTE. It is a common mistake for beginners to overlook the unit numerator and to carelessly write down the result as 2 × 2, or 4.

EXAMPLE 3. *Find the value of* $7\frac{1}{8} \times \frac{5}{76}$ of $2\frac{2}{15}$.

$$\text{The product} = \frac{\overset{\cancel{3}}{\cancel{57}}}{\cancel{8}} \times \frac{\cancel{5}}{\underset{\cancel{4}}{\cancel{76}}} \times \frac{\cancel{32}}{\cancel{15}}$$

$$= 1.$$

Here we first replace 'of' by the sign × ; 19 divides 57 and 76, leaving quotients 3 and 4; the 3 and 5 in numerator cancel with 15 in the denominator, and the 8 and 4 in denominator cancel with 32 in numerator. The only factors in numerator and denominator now left are units, which are not expressed. Thus the product is 1.

NOTE. When all the factors of numerator and denominator cancel each other, it is a common mistake with beginners to give the result as **0**. If the unit factors were expressed, the result would be $\frac{1 \times 1 \times 1}{1 \times 1 \times 1}$, which of course cannot be zero. A little reflection will shew that *the result of such a multiplication can never be zero.*

127. When the product of *two* numbers is unity each is called the **reciprocal** of the other.

Thus $\frac{3}{5}$ and $\frac{5}{3}$ are reciprocals, for $\frac{3}{5} \times \frac{5}{3} = 1$;

$6\frac{3}{4}$ and $\frac{4}{27}$ ...., for $\frac{27}{4} \times \frac{4}{27} = 1$.

To obtain the reciprocal of any fraction we have simply to make numerator and denominator change places.

EXAMPLES V. p.

(*Examples* 1–11 *should be taken orally.*)

Read off the values of the following:

1. £25 × $\frac{3}{10}$; 56 feet × $1\frac{5}{7}$; 40 tons × $2\frac{3}{4}$; 21 litres × $1\frac{5}{6}$.

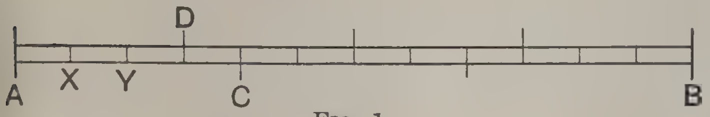


FIG. 1.

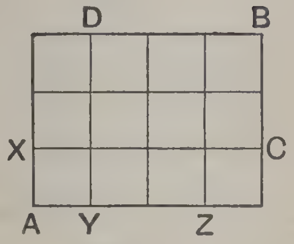


FIG. 2.

2. Taking in turn the whole line in Fig. 1 and the whole rectangle in Fig. 2, to represent the unit, point out in each figure what represents

(i) $\frac{1}{3}$; (ii) $\frac{1}{4}$; (iii) $\frac{1}{3}$ of $\frac{1}{4}$; (iv) $\frac{1}{4}$ of $\frac{1}{3}$;
(v) $\frac{2}{3}$ of $\frac{1}{4}$; (vi) $\frac{3}{4}$ of $\frac{1}{3}$; (vii) $\frac{2}{3}$ of $\frac{3}{4}$.

Read off the following products in their lowest terms:

3. $\frac{1}{3}$ of $\frac{2}{5}$, $\frac{2}{3}$ of $\frac{1}{7}$, $\frac{3}{4}$ of $\frac{1}{5}$, $\frac{1}{8}\times\frac{4}{9}$, $\frac{2}{3}\times\frac{3}{5}$.

4. $\frac{5}{6}\times\frac{3}{8}$, $\frac{4}{5}$ of $\frac{10}{13}$, $\frac{7}{9}\times\frac{5}{14}$, $\frac{5}{8}$ of $\frac{11}{15}$, $\frac{5}{24}$ of $\frac{12}{35}$.

5. $1\frac{4}{7}$ of $\frac{7}{11}$, $\frac{11}{18}\times\frac{9}{22}$, $\frac{16}{21}$ of $\frac{7}{8}$, $3\frac{2}{11}\times\frac{11}{35}$, $5\frac{4}{5}$ of $\frac{10}{29}$.

Read off the values of the following:

6. $\frac{1}{2}$ of $\frac{3}{4}$ of £24; $\frac{2}{3}$ of $\frac{4}{5}$ of 30 cwt.; $\frac{5}{6}$ of $\frac{1}{2}$ of 72 miles.

7. 10 yards × $\frac{3}{4}\times 2\frac{2}{3}$; $10\frac{1}{2}$ tons × $\frac{2}{3}\times 5$; $12\frac{1}{2}$ feet × $\frac{4}{5}\times\frac{3}{4}$.

Read off the reciprocals of

8. $\frac{11}{12}$, $1\frac{2}{3}$, $\frac{1}{5}$, 5, $2\frac{1}{8}$, $\frac{13}{15}$, $2\frac{3}{11}$, $1\frac{4}{25}$.

9. Multiply $\frac{48}{53}$ by the reciprocals of 2, 3, 4, 8, 12.

10. Multiply the reciprocals of $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{9}$, $\frac{1}{18}$ by $\frac{1}{36}$.

Read off in their simplest form the values of

11. $\frac{a}{b}$ of $\frac{x}{y}$; $\frac{mc}{a}\times\frac{1}{m}$; $\frac{a}{bx}$ of $\frac{x}{b}$; $\frac{a}{b}\times\frac{b}{c}\times\frac{c}{a}$.

12. Draw on squared paper a rectangular diagram to illustrate the following:

(i) $\frac{1}{5}$ of $\frac{3}{4}=\frac{3}{20}$. (ii) $\frac{1}{5}$ of $\frac{3}{4}=\frac{3}{4}$ of $\frac{1}{5}$. (iii) $\frac{4}{5}$ of $\frac{3}{4}=\frac{3}{5}$.

Illustrate the following identities by means of diagrams drawn on squared paper.

13. (i) $\frac{2}{3}\times\frac{4}{5}=\frac{8}{15}$. (ii) $\frac{2}{3}\times\frac{3}{5}=\frac{2}{5}$. (iii) $\frac{1}{3}+\frac{1}{5}=\frac{8}{15}$.

14. (i) $\frac{5}{6}\times\frac{3}{4}=\frac{15}{24}$. (ii) $\frac{5}{6}=\frac{20}{24}$. (iii) $\frac{3}{4}-\frac{1}{6}=\frac{7}{12}$.

15. (i) $\frac{5}{7}\times\frac{3}{4}=\frac{15}{28}$. (ii) $\frac{2}{7}$ of $1\frac{3}{4}=\frac{1}{2}$. (iii) $\frac{3}{4}\times\frac{4}{3}=1$.

16. Prove from first principles (following the reasoning of Art. 125) that $\frac{3}{11}$ of $6\frac{1}{9}=1\frac{2}{3}$.

Find the value of

17. $6\frac{3}{8}$ of $1\frac{11}{17}$. **18.** $2\frac{1}{10}\times 4\frac{2}{7}$. **19.** $1\frac{29}{36}$ of $1\frac{1}{26}$.

20. $2\frac{37}{42}\times 9\frac{9}{55}$. **21.** $\frac{3}{8}\times\frac{16}{21}\times\frac{35}{64}$. **22.** $1\frac{1}{9}\times\frac{3}{8}$ of $2\frac{2}{5}$.

23. $\frac{5}{9}$ of $4\frac{1}{20}\times\frac{16}{63}$. **24.** $2\frac{5}{6}\times\frac{4}{15}\times 1\frac{11}{34}$.

25. $7\frac{1}{8}$ of $\frac{5}{76}$ of $4\frac{4}{15}$. **26.** $\frac{4}{11}$ of $4\frac{1}{8}$ of £15.

27. $55\frac{2}{3}$ miles $\times 1\frac{7}{8}\times 1\frac{1}{9}$. **28.** $14\frac{2}{7}$ cwt. $\times 2\frac{1}{10}\times 3\times 2\frac{2}{9}$.

29. $\dfrac{a}{xc}\times\dfrac{3cy}{ab}\times\dfrac{x}{12}$. **30.** $\dfrac{5m^2}{n^2}\times\dfrac{n}{m}\times\dfrac{1}{15}$.

31. Given that 1 kilometre $=\frac{5}{8}$ mile (nearly), find roughly the number of miles in (i) $17\frac{1}{3}$ Km.; (ii) $27\frac{1}{5}$ Km.; (iii) $102\frac{2}{9}$ Km.

32. Taking $2\frac{1}{5}$ lbs. as a rough equivalent of 1 kilogram, estimate the number of pounds in (i) $12\frac{1}{2}$ Kg.; (ii) $18\frac{3}{4}$ Kg.; (iii) $\frac{7}{10}$ of a quintal (100 kilograms).

Find the value of

33. $4\frac{1}{20}$ tons of coal at $18\frac{1}{3}$ shillings per ton.

34. $4\frac{1}{8}$ yards of cloth at 3*s.* 4*d.* per yard.

35. $14\frac{2}{3}$ cwt. of copper at £5. 5*s.* per cwt.

36. A Captain's regimental pay (Royal Artillery) is $\frac{23}{32}$ of a Major's pay; and a Major's is $\frac{8}{9}$ of a Colonel's. A Colonel draws 18 shillings a day. What does a Captain draw?

37. *A*'s age is $\frac{5}{4}$ of *B*'s, and *B*'s age is $\frac{4}{3}$ of *C*'s; while *M*'s age is equal to the united ages of *A*, *B*, and *C*. If *C* is 15, how old are *A* and *M*?

38. The value of an estate is gradually declining in such a way that at the end of each year it is worth only $\frac{5}{6}$ of its value at the beginning. It was worth £12,000 in January 1905. What was it worth in December 1906?

39. The length and breadth of a room appear on measurement with a foot-rule to be 24 feet and 20 feet respectively; but it is found that each foot of the rule is $\frac{1}{12}$ foot too short. What are the real dimensions of the room? Had the foot-rule been too *long* by the same amount, what would the dimensions have appeared to be?

40. If $\frac{7}{15}$ of a stick is cut off, and then $\frac{3}{16}$ of the remainder, what fraction of the whole is left?

41. To reach a certain town a man travels $\frac{18}{25}$ of the distance by train, $\frac{3}{14}$ of the remainder by coach, and the rest on foot. What fraction of the whole distance does he walk?

42. A sum of £2. 8*s*. is divided among four persons; the first having $\frac{7}{24}$; the second $\frac{8}{17}$ of the remainder; and the third $\frac{2}{9}$ of the sum still remaining. Find the share of the fourth.

Division by a Fraction.

128. When a quantity a is divided by the quantity b, the **quotient** is defined to be that which when multiplied by b produces a. This operation of division is denoted by $a \div b$, $\frac{a}{b}$ or a/b; where a is the **dividend,** and b the **divisor.**

Division is thus the inverse of multiplication, and

$$(a \div b) \times b = a.$$

This statement may also be expressed verbally as follows:

$$\textit{quotient} \times \textit{divisor} = \textit{dividend}.$$

129. *To divide any quantity* P (integral or fractional) by $\frac{2}{3}$.

We have
$$\begin{aligned} \textit{quotient} \times \tfrac{2}{3} &= \textit{dividend} \\ &= \mathrm{P}. \end{aligned}$$

Multiplying both sides by $\frac{3}{2}$,

$$\textit{quotient} \times \tfrac{2}{3} \times \tfrac{3}{2} = \mathrm{P} \times \tfrac{3}{2};$$

$$\text{but } \tfrac{2}{3} \times \tfrac{3}{2} = 1;$$

$$\therefore \ \textit{quotient} = \mathrm{P} \times \tfrac{3}{2},$$

$$\text{that is, } \quad \mathbf{P \div \tfrac{2}{3} = P \times \tfrac{3}{2}}.$$

Also $\frac{3}{2}$ is the *reciprocal* of the given divisor $\frac{2}{3}$; hence *to divide by a fraction*, **multiply by its reciprocal.**

We may arrive at the same rule another way. Suppose, as before, it is required to divide P by $\frac{2}{3}$.

Now since $\frac{2}{3}$ is *one-third* of 2, if we were to divide by 2, our divisor would be 3 times too great, and consequently our quotient 3 times too small. Hence to get the correct quotient we *divide by the numerator* 2 and *multiply by the denominator* 3.

That is, $$P \div \frac{2}{3} = P \times \frac{3}{2}.$$

And generally, $$P \div \frac{m}{n} = P \times \frac{n}{m}.$$

EXAMPLE. *Divide* (i) $\frac{5}{28}$ *by* $\frac{2}{7}$; (ii) 16 *by* $3\frac{3}{7}$.

$$\text{(i)}\ \frac{5}{28} \div \frac{2}{7} = \frac{5}{\underset{4}{\cancel{28}}} \times \frac{\cancel{7}}{2}; \qquad \text{(ii)}\ 16 \div 3\tfrac{3}{7} = 16 \div \frac{24}{7}$$

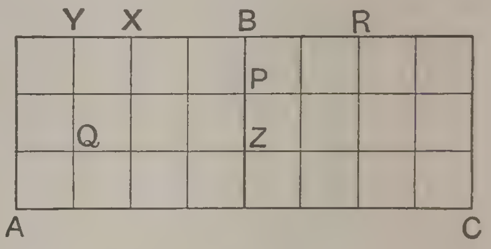
$$= \frac{5}{8}. \qquad\qquad = \overset{2}{\cancel{16}} \times \frac{7}{\underset{3}{\cancel{24}}} = \frac{14}{3} = 4\tfrac{2}{3}.$$

NOTE. Since $1 \div \frac{a}{b} = 1 \times \frac{b}{a} = \frac{b}{a}$, we see that when unity is divided by a fraction the quotient is the *reciprocal* of that fraction.

EXAMPLES V. q.

(*Examples* 1–10 *should be taken orally.*)

1. In the adjoining diagram each of the rectangles AB, BC represents the unit. Use the diagram to illustrate



(i) $2 \div \frac{1}{2} = 4$. (ii) $2 \div \frac{1}{4} = 8$. (iii) $2 \div \frac{1}{3} = 6$.

(iv) $2 \div \frac{1}{12} = 24$. (v) $2 \div \frac{2}{3} = 3$. (vi) $1\frac{1}{2} \div \frac{1}{2} = 3$.

Read off the quotients in the following divisions:

2. $\frac{2}{3} \div \frac{1}{3}$; $\frac{3}{5} \div \frac{2}{5}$; $1 \div \frac{2}{5}$; $2 \div \frac{2}{5}$; $\frac{2}{3} \div \frac{2}{5}$.

3. $3 \div \frac{1}{3}$; $3 \div \frac{1}{6}$; $3 \div \frac{2}{3}$; $\frac{5}{6} \div \frac{5}{6}$; $5 \div \frac{5}{6}$.

4. $1\frac{1}{2} \div \frac{1}{2}$; $2\frac{3}{4} \div \frac{3}{4}$; $1\frac{5}{6} \div \frac{11}{2}$; $4\frac{1}{2} \div \frac{9}{4}$; $3\frac{1}{2} \div 2\frac{1}{3}$.

5. How often is $\frac{1}{2}$*d.* contained in $7\frac{1}{2}$*d.*?

6. How many times does $7\frac{1}{2}$ ft. contain 6 inches?

7. How often does 33 tons contain $1\frac{5}{6}$ tons?

8. The circumference of a wheel is $2\frac{2}{5}$ metres in length; how many times will it revolve in running 30 metres?

9. How many carts, each containing $3\frac{3}{8}$ tons, will be required to carry 27 tons?

10. How many strips of cloth, $\frac{3}{5}$ yd. in length, can be cut from 15 yds.? And how many $2\frac{1}{3}$ yds. long from 21 yds.?

11. Illustrate the following results by means of divided lines or rectangles drawn on squared paper.

(i) $\frac{2}{3}\div\frac{1}{6}=4$. (ii) $4\div 1\frac{1}{3}=3$. (iii) $\frac{2}{3}\div\frac{3}{4}=\frac{8}{9}$. (iv) $2\frac{1}{3}\div 1\frac{1}{3}=1\frac{3}{4}$.

12. Following the argument of Art. 129, prove that

$$a\div\frac{x}{y}=a\times\frac{y}{x}.$$

Find the value of

13. $21\div 1\frac{5}{9}$. **14.** $3\frac{3}{8}\div 2\frac{13}{16}$. **15.** $4\frac{3}{4}\div 7\frac{1}{8}$. **16.** $1\frac{19}{58}\div 1\frac{15}{29}$.

17. $\frac{25}{27}\div\frac{40}{81}$. **18.** $14\frac{17}{20}\div 3\frac{3}{5}$. **19.** $85\div 6\frac{4}{5}$. **20.** $4\frac{3}{8}\div 1\frac{29}{36}$.

21. $ma\div\frac{a}{b}$. **22.** $\frac{x}{y}\div\frac{m}{n}$. **23.** $\frac{mx}{y}\div nx$. **24.** $\frac{ax}{b}\div\frac{x}{mb}$.

25. By what fraction must $4\frac{7}{8}$ be multiplied to give the product $4\frac{1}{2}$?

26. By what mixed number must $4\frac{7}{12}$ be divided to give the quotient $1\frac{1}{3}$?

27. How many times is $4\frac{1}{8}$ of $\frac{4}{11}$ contained in 15?

28. Divide $10\frac{2}{3}$ by the product of $2\frac{2}{27}$ and $2\frac{4}{7}$.

Given the following approximate equivalents:

1 kilometre $=\frac{5}{8}$ *mile;* *1 metre* $=39\frac{3}{8}$ *inches;*
1 kilogram $=2\frac{1}{5}$ *lbs.;* *1 litre* $=1\frac{3}{4}$ *pints;*

find roughly the value of

29. (i) 1 mile in Km. (ii) 1 mile in metres.
(iii) 100 miles in Km. (iv) 1 yard in metres.
(v) 1 inch in centimetres. (vi) 147 feet in metres.

30. (i) 1 lb. in Kg. (ii) 1 cwt. in Kg.
(iii) 1 pint in litres. (iv) 1 gallon in litres.
(v) 55 Kg. in lbs. (vi) 1 kilolitre in gallons.

And (vii) find the number of grains in 1 gram, having given 1 lb. = 7000 grains.

Combined Processes. Complex Fractions.

130. When several fractions are connected by the signs +, −, ×, ÷, some care is needed as to the order of operations. To avoid confusion it is agreed that:

(i) *Operations of multiplication and division must be performed in order from left to right.*

(ii) *All operations of multiplication and division must be performed before those of addition and subtraction.*

Note. In all cases an expression within brackets must be regarded as a single quantity. A sign preceding a bracket operates upon the enclosed expression taken as a whole.

Example 1. *Simplify the following expressions:*

(i) $\frac{2}{5}\times\frac{6}{7}\div 1\frac{1}{3}$; (ii) $\frac{2}{5}\div\frac{6}{7}\times 1\frac{1}{3}$; (iii) $\frac{2}{5}\div\frac{6}{7}$ of $1\frac{1}{3}$.

(i) $\frac{2}{5}\times\frac{6}{7}\div 1\frac{1}{3}=\frac{2}{5}\times\frac{6}{7}\times\frac{3}{4}=\frac{9}{35}$;

(ii) $\frac{2}{5}\div\frac{6}{7}\times 1\frac{1}{3}=\frac{2}{5}\times\frac{7}{6}\times\frac{4}{3}=\frac{28}{45}$;

(iii) Here the sign "of" connects the fractions $\frac{6}{7}$ and $1\frac{1}{3}$, and the expression $\frac{6}{7}$ of $1\frac{1}{3}$ must be regarded as a single quantity.

Thus $\frac{2}{5}\div\frac{6}{7}$ of $1\frac{1}{3}=\frac{2}{5}\div\left(\frac{6}{7}\times\frac{4}{3}\right)=\frac{2}{5}\div\frac{8}{7}=\frac{2}{5}\times\frac{7}{8}=\frac{7}{20}$.

In cases like this the use of brackets is recommended. The sign 'of' between two quantities invariably has the same effect as if they were enclosed in brackets.

Example 2. *Simplify* $1\frac{13}{20}$ of $\frac{7}{11}-2\frac{11}{15}+1\frac{13}{19}\div\frac{80}{171}$.

Here the expression must be regarded as consisting of three *terms* separated by the signs − and +, each term being first reduced by itself.

Using brackets we have

the expression

$$=\left(\frac{\overset{3}{\cancel{33}}}{20}\times\frac{7}{\cancel{11}}\right)-2\frac{11}{15}+\left(\frac{\overset{2}{\cancel{32}}}{\cancel{19}}\times\frac{\overset{9}{\cancel{171}}}{\underset{5}{\cancel{80}}}\right)$$

$$=\frac{21}{20}-2\frac{11}{15}+\frac{18}{5}=1\frac{1}{20}+3\frac{3}{5}-2\frac{11}{15}$$

$$=2\frac{3+36-44}{60}=1\frac{60+39-44}{60}$$

$$=1\frac{55}{60}=1\frac{11}{12}.$$

EXAMPLES V. r.

Simplify the following expressions:

1. $3\frac{1}{2} \times \frac{2}{5} \div \frac{1}{3}$. **2.** $3\frac{1}{2} \div \frac{2}{5} \times \frac{1}{3}$. **3.** $3\frac{1}{2} \div \frac{2}{5}$ of $\frac{1}{3}$.

4. $3\frac{1}{2} + \frac{2}{5} \div \frac{1}{3}$. **5.** $3\frac{1}{2} \div \frac{2}{5} + \frac{1}{3}$. **6.** $3\frac{1}{2} + \frac{2}{5}$ of $\frac{1}{3}$.

7. $3\frac{1}{2} \div \left(\frac{2}{5} + \frac{1}{3}\right)$. **8.** $3\frac{1}{2}$ of $\left(\frac{2}{5} + \frac{1}{3}\right)$. **9.** $3\frac{1}{2} \times \frac{2}{5} + \frac{1}{3}$.

10. $4\frac{4}{5} - \frac{3}{5}$ of $\frac{1}{2}$. **11.** $4\frac{4}{5}$ of $\frac{3}{5} - \frac{1}{2}$. **12.** $4\frac{4}{5} - \frac{3}{5} \div \frac{1}{2}$.

13. $4\frac{4}{5} \div \frac{3}{5} - \frac{1}{2}$. **14.** $\left(4\frac{4}{5} + \frac{3}{5}\right) \times \frac{1}{2}$. **15.** $4\frac{4}{5} + \left(\frac{3}{5} \times \frac{1}{2}\right)$.

16. $\left(\frac{1}{5} + \frac{3}{8}\right) \times \left(\frac{1}{3} + \frac{3}{11}\right)$. **17.** $\frac{1}{5} + \frac{3}{8} \times \frac{1}{3} + \frac{3}{11}$.

18. $\frac{1}{5} + \frac{3}{8} \div \frac{3}{11} + \frac{1}{3}$. **19.** $\left(\frac{1}{5} + \frac{3}{8}\right) \div \left(\frac{3}{11} + \frac{1}{3}\right)$.

20. $\left(\frac{7}{18} - \frac{5}{27}\right) \div 6\frac{1}{9}$. **21.** $\left(\frac{3}{7} + \frac{1}{9}\right)$ of $\left(8\frac{16}{17} - 5\frac{31}{34}\right)$.

22. $\left(5\frac{7}{26} - 1\frac{4}{39}\right) \div \left(6\frac{2}{9} - 4\frac{5}{6}\right)$. **23.** $\left(3\frac{1}{4} \times 4\frac{1}{3}\right) \div \left(2\frac{1}{2} - \frac{1}{3}\right) \times \left(3\frac{1}{2} - \frac{1}{4}\right)$.

24. $\left(\frac{1}{4} + \frac{3}{5}\right)\left(\frac{5}{6} - \frac{5}{7}\right) \div \left(\frac{7}{8} - \frac{7}{9}\right)$. **25.** $\left(\frac{2}{19} + \frac{1}{3}\right) \div \left(3 - \frac{1}{3}\right) \times \left(\frac{1}{3} + \frac{1}{5}\right)$.

Find the value of

26. $\frac{4}{5} + \frac{5}{8}$ of $\frac{16}{25} + \frac{9}{17}$ of $\frac{34}{63}$. **27.** $\frac{5}{13}$ of $1\frac{14}{25} + 2\frac{3}{10} - \frac{7}{18}$ of $1\frac{1}{35}$.

28. $5\frac{1}{3}$ of $4\frac{1}{2} - 3\frac{1}{4} \div \frac{5}{16} - 1\frac{3}{5}$. **29.** $2\frac{1}{3} + \frac{4}{5}$ of $3\frac{1}{3}$ of $\frac{3}{4} - 2\frac{1}{2} + \frac{5}{11}$.

30. $\frac{29}{105} + \frac{4}{35}$ of $1\frac{3}{4} - \frac{7}{11}$ of $3\frac{2}{3} \div 8\frac{1}{6}$.

31. Divide $\left(2\frac{1}{4} - 1\frac{3}{8}\right)\left(4\frac{1}{2} + 3\frac{1}{16}\right)$ by $2\frac{3}{16} - 1\frac{1}{24}$.

32. Find the difference between $2\frac{1}{9} + 6\frac{1}{4}$ of $1\frac{3}{5}$ and $2\frac{1}{2} \times 3\frac{2}{5} \div 1\frac{2}{15}$.

33. Subtract $2\frac{1}{7}$ of $9\frac{1}{2} - 5\frac{1}{14}$ from $7\frac{1}{2}$ of $\left(9\frac{1}{2} - 5\frac{1}{14}\right)$.

If $a=2$, $b=3$, $c=4$, find the value of

34. $\frac{1}{a+b+c}$. **35.** $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$. **36.** $\frac{a}{b} \times \frac{b}{c} \times \frac{c}{a}$.

37. $\frac{a}{b} + \frac{b}{c} + \frac{c}{a}$. **38.** $\frac{(a+b)(b+c)(c+a)}{a+b+c}$. **39.** $\frac{(a^2+1)(b^2+1)}{c^2-1}$.

131. Complex Fractions. A fraction which has a fraction in numerator or denominator or both is called a **complex fraction**.

Thus $$\frac{\frac{5}{8}}{4},\quad \frac{7}{\frac{3}{4}},\quad \frac{\frac{3}{4}}{\frac{5}{6}},\quad \frac{2\frac{5}{8}}{3\frac{1}{6}},\quad \frac{2\frac{1}{2}+\frac{7}{8}}{5\frac{2}{3}-1\frac{1}{2}},$$

are complex fractions. The thicker line is used to distinguish the numerator and denominator clearly. For convenience of printing such fractions are sometimes written

$$\tfrac{5}{8}/4,\quad 7/\tfrac{3}{4},\quad \tfrac{3}{4}/\tfrac{5}{6},\quad 2\tfrac{5}{8}/3\tfrac{1}{6},\quad (2\tfrac{1}{2}+\tfrac{7}{8})/(5\tfrac{2}{3}-1\tfrac{1}{2}),$$

where the numerator and denominator are separated by a slanting line. It will be noticed that in the last case both numerator and denominator must be enclosed in brackets.

Since a fraction has been defined as the result of dividing the numerator by the denominator, the simplification of complex fractions depends upon principles already explained.

EXAMPLE 1. *Simplify* (i) $\dfrac{2\frac{5}{8}}{3\frac{1}{6}}$; (ii) $\dfrac{3\frac{5}{6}+2\frac{1}{2}}{5\frac{2}{3}-1\frac{1}{4}}$.

(i) *First Method.* $\dfrac{2\frac{5}{8}}{3\frac{1}{6}} = 2\frac{5}{8} \div 3\frac{1}{6} = \dfrac{21}{\underset{4}{\cancel{8}}} \times \dfrac{\overset{3}{\cancel{6}}}{19} = \dfrac{63}{76}$.

(i) *Second Method.* Multiply the numerator and denominator by 24, *the* L.C.M. *of the denominators* 8 *and* 6.

Then $$\frac{2\frac{5}{8}}{3\frac{1}{6}} = \frac{48+15}{72+4} = \frac{63}{76}.$$

(ii) Using the second method, we multiply numerator and denominator by 12, which is the L.C.M. of 6, 2, 3, and 4.

Thus the fraction $= \dfrac{46+30}{68-15} = \dfrac{76}{53} = 1\frac{23}{53}$.

NOTE. The second method is usually to be preferred, but it can only be used (i) when the complex fraction consists of a single fraction (or integer) in numerator and denominator; (ii) when the fractions in numerator and denominator are connected solely by the signs + and −.

EXAMPLE 2. *Simplify* (i) $\dfrac{5\frac{5}{9} \times \frac{18}{25}}{3\frac{1}{7} \div \frac{11}{14}}$; (ii) $\dfrac{3\frac{3}{4} - \frac{11}{12}}{\frac{7}{15} \times 1\frac{7}{8} \text{ of } \frac{8}{21}}$.

Here we must use the first method in each case.

(i) Numerator $= \dfrac{\overset{2}{\cancel{50}}}{\cancel{9}} \times \dfrac{\overset{2}{\cancel{18}}}{\cancel{25}} = 4$. Denominator $= \dfrac{\overset{2}{\cancel{22}}}{\cancel{7}} \times \dfrac{\overset{2}{\cancel{14}}}{\cancel{11}} = 4$.

$\therefore$ the whole fraction $= \frac{4}{4} = 1$.

(ii) Numerator $= 3\frac{9-11}{12} = 2\frac{21-11}{12} = 2\frac{10}{12} = 2\frac{5}{6}$.

Denominator $= \frac{7}{15} \times \frac{15}{8} \times \frac{8}{21} = \frac{1}{3}$.

$\therefore$ the whole fraction $= 2\frac{5}{6} \div \frac{1}{3} = \frac{17}{6} \times 3 = 8\frac{1}{2}$.

EXAMPLES V. s.

(*Examples* 1–15 *may be taken orally.*)

1. Read off in the simplest form the values of the following fractions:

$$\frac{1}{\frac{1}{2}},\quad \frac{1}{\frac{5}{6}},\quad \frac{2}{\frac{1}{3}},\quad \frac{3}{\frac{1}{4}},\quad \frac{\frac{1}{2}}{2},\quad \frac{\frac{2}{5}}{3},\quad \frac{\frac{4}{5}}{2},\quad \frac{3}{\frac{3}{5}},\quad \frac{\frac{1}{2}}{\frac{1}{3}},\quad \frac{\frac{1}{4}}{\frac{1}{8}}.$$

Simplify

2. $\dfrac{1}{1\frac{1}{2}}$. 3. $\dfrac{2}{2\frac{1}{2}}$. 4. $\dfrac{1\frac{1}{2}}{3}$. 5. $\dfrac{1\frac{2}{3}}{1\frac{5}{6}}$. 6. $\dfrac{3\frac{1}{3}}{4\frac{1}{2}}$.

7. $\frac{1}{3}/\frac{1}{5}$. 8. $1/\frac{5}{6}$. 9. $2/\frac{3}{8}$. 10. $1\frac{1}{3}/1\frac{1}{2}$. 11. $3\frac{1}{3}/2$.

12. $\dfrac{\frac{1}{2}-\frac{1}{3}}{\frac{1}{2}+\frac{1}{3}}$. 13. $\dfrac{\frac{3}{4}-\frac{1}{2}}{\frac{1}{4}}$. 14. $\dfrac{\frac{1}{3}-\frac{1}{6}}{\frac{1}{3}+\frac{1}{6}}$. 15. $\dfrac{1\frac{1}{2}+1\frac{1}{4}}{2\frac{1}{2}+\frac{1}{4}}$.

Find the value of

16. $\dfrac{\frac{3}{8}}{\frac{1}{4}+\frac{1}{8}}$. 17. $\dfrac{\frac{3}{7}-\frac{1}{5}}{\frac{4}{7}+\frac{1}{5}}$. 18. $\dfrac{2\frac{1}{4}+\frac{1}{8}}{3\frac{1}{8}-\frac{3}{4}}$. 19. $\dfrac{\frac{4}{5}-\frac{3}{4}}{\frac{3}{5}+\frac{1}{4}}$.

20. $\dfrac{21\frac{3}{7}}{15\frac{5}{8}}$. 21. $\dfrac{7\frac{11}{23}}{4\frac{68}{69}}$. 22. $\dfrac{3\frac{1}{2}-1\frac{7}{8}}{3\frac{1}{4}\times 2\frac{1}{2}}$. 23. $\dfrac{\frac{6}{7}\text{ of }1\frac{1}{20}}{10\frac{4}{5}\div 27}$.

24. $\dfrac{1\frac{11}{24}\times 4\frac{4}{5}}{8\frac{3}{4}\div\frac{5}{8}}$. 25. $\dfrac{\frac{9}{20}\text{ of }8\frac{1}{3}}{3\frac{1}{4}-\frac{11}{16}}$. 26. $\dfrac{1\frac{1}{2}}{22\frac{1}{2}}\div\dfrac{21\frac{1}{3}}{1\frac{1}{3}}$. 27. $\dfrac{1\frac{1}{4}\div 1\frac{1}{2}}{\frac{1}{15}+(1-\frac{9}{10})}$.

28. $\dfrac{\frac{1}{2}+\frac{3}{4}+\frac{5}{6}}{\frac{2}{3}+\frac{1}{4}+1\frac{1}{6}}$. 29. $\frac{3}{14}$ of $\dfrac{4\frac{5}{9}}{6\frac{1}{6}}\div\dfrac{11\frac{5}{7}}{6\frac{3}{11}}$. 30. $\frac{7}{15}\div\dfrac{\frac{3}{4}\text{ of }2\frac{1}{3}}{\frac{5}{6}\text{ of }7\frac{1}{2}}$.

31. $\dfrac{3\frac{1}{3}-\frac{5}{9}+\frac{1}{5}}{2\frac{2}{3}+\frac{13}{45}}+\dfrac{1\frac{1}{5}}{11\frac{1}{5}-2\frac{1}{3}}$. 32. $\dfrac{2\frac{2}{3}+4\frac{4}{7}-3\frac{3}{5}}{\frac{1}{3}+\frac{1}{5}+\frac{1}{7}}\div\left(3-\dfrac{1}{3\frac{13}{19}}\right)$.

(*For Examples in Continued Fractions see page* 409.)

Fractional Equations.

132. The principles of fractions will now be applied to solving certain types of equations of special use in Arithmetic.

EXAMPLE 1. *Find the value of x which satisfies*

(i) $\dfrac{x}{7}=1\frac{3}{5}$. (ii) $\dfrac{4}{3}=\dfrac{5x}{6}$. (iii) $\dfrac{14}{3x}=1\frac{2}{5}$.

Our object is to detach the unknown quantity x from the fraction in which it occurs. This we may do by Axioms 3 and 4 of Art. 62.

(i) In $\frac{x}{7}=1\frac{3}{5}$, we multiply both sides by 7;

this gives $$\frac{x}{7}\times 7=1\tfrac{3}{5}\times 7;$$

or $$x=7\tfrac{21}{5}=11\tfrac{1}{5}.$$

(ii) In $\frac{4}{3}=\frac{5x}{6}$, we divide both sides by 5;

this gives $$\frac{4}{15}=\frac{x}{6}.$$

If now we multiply both sides by 6,

we have $\frac{4\times 6}{15}=x$; or $x=\frac{8}{5}=1\frac{3}{5}$.

These steps may easily be taken together and performed mentally.

(iii) In $\frac{14}{3x}=1\frac{2}{5}$, we multiply both sides by $3x$;

this gives $$14=\tfrac{7}{5}\times 3x.$$

Then in one step divide both sides by 7×3, and multiply by 5;

and we have $$x=\tfrac{10}{3}=3\tfrac{1}{3}.$$

EXAMPLE 2. *Solve the equation* $\frac{3x-4}{5}+\frac{5x}{2}=1\frac{4}{15}$.

First *clear the equation of fractions.* To do this, multiply *both sides* by 30, namely the L.C.D. Then we have

$$\left(\frac{3x-4}{5}\times 30\right)+\left(\frac{5x}{2}\times 30\right)=\tfrac{19}{15}\times 30;$$

or $$(3x-4)\times 6+5x\times 15=19\times 2;$$

that is, $$18x-24+75x=38;$$

hence $$93x=62;$$

$$\therefore\ x=\tfrac{62}{93}=\tfrac{2}{3}.$$

EXAMPLES V. t.

(*Examples* 1–6 *should be taken orally.*)

Read off the values of x which satisfy the following statements:

1. (i) $5x=20$; (ii) $5x=45$; (iii) $5x=4$; (iv) $5x=12$.
2. (i) $35=7x$; (ii) $7x=5$; (iii) $9=7x$; (iv) $7x=34$.
3. (i) $\frac{x}{6}=3$; (ii) $\frac{x}{3}=8$; (iii) $4=\frac{x}{5}$; (iv) $7=\frac{x}{11}$.
4. (i) $\frac{x}{3}=\frac{1}{2}$; (ii) $\frac{x}{4}=\frac{1}{3}$; (iii) $\frac{x}{4}=\frac{1}{16}$; (iv) $\frac{1}{20}=\frac{x}{5}$.
5. (i) $3x=\frac{3}{4}$; (ii) $3x=\frac{1}{4}$; (iii) $\frac{2}{5}=3x$; (iv) $3x=1\frac{1}{2}$.
6. (i) $5x=1\frac{2}{3}$; (ii) $7x=1\frac{3}{4}$; (iii) $7x=2\frac{4}{5}$; (iv) $3\frac{1}{5}=4x$.

Find the value of x in each of the following equations :

7. (i) $\frac{x}{14}=\frac{3}{7}$; (ii) $\frac{4}{5}=\frac{2x}{3}$; (iii) $\frac{3x}{5}=1$; (iv) $3=\frac{7x}{4}$.

8. (i) $\frac{6}{7}=\frac{3x}{2}$; (ii) $\frac{2x}{15}=\frac{8}{45}$; (iii) $\frac{5x}{3}=2\frac{1}{2}$; (iv) $\frac{7}{15}=\frac{8x}{3}$.

9. (i) $\frac{5}{x}=2$; (ii) $\frac{3}{x}=\frac{2}{5}$; (iii) $\frac{7}{x}=3\frac{1}{2}$; (iv) $4\frac{1}{2}=\frac{3}{x}$.

10. (i) $\frac{5}{2x}=\frac{10}{3}$; (ii) $\frac{4}{3x}=\frac{16}{27}$; (iii) $\frac{4}{7}=\frac{8}{21x}$; (iv) $\frac{5}{27}=\frac{1}{3x}$.

Solve the following equations :

11. $\frac{x}{2}+\frac{x}{3}=5.$

12. $\frac{x}{3}-\frac{x}{4}=1.$

13. $x=\frac{x}{4}+6.$

14. $x-5=\frac{3}{4}x.$

15. $2-\frac{x}{5}=4\frac{2}{5}-x.$

16. $\frac{1}{2}(7-x)=2x-4.$

17. $\frac{2x+3}{5}=3.$

18. $\frac{x+1}{3}+\frac{x-1}{2}=4.$

19. $\frac{3x-11}{4}=\frac{x}{2}-1.$

20. $\frac{1}{3}(8x-1)+2x=2.$

21. What number increased by one-third of itself gives 20 ?

22. What number falls short of 20 by one-fourth of itself ?

23. Find the number one-third of which exceeds its fifth part by 2.

24. From the condition $\frac{2}{3}$ of $\frac{6}{7}$ of $x=1\frac{3}{5}$, find x.

25. Find x from the condition $x\times\frac{3}{7}=2\frac{4}{5}$ of $\frac{5}{6}$.

26. Divide 50 into two parts such that one may be $\frac{3}{7}$ of the other.

27. The sum of two numbers is 28 ; their difference is 6. Find them.

28. A and B have 20 shillings between them. If A gives one-third of his share to B, then B will have as much as A had to start with. Find their original shares.

29. If $\frac{m}{n}=\frac{p}{x}$, find x when $m=7$, $n=2$, $p=21$.

30. If $a=6$, $b=4$, $c=7$, find the value of x when

(i) $\frac{3x}{2}=\frac{abc}{14}$; (ii) $\frac{ab^2x}{8}=c$; (iii) $\frac{x}{c}=a+b+c.$

CHAPTER VI.

FRACTIONS CONTINUED. COMPOUND QUANTITIES.

133. **Fractions of Concrete Quantities.** The method of finding the value of a fraction of a concrete quantity will be seen from the following examples.

EXAMPLE 1. *Find the value of* (i) $\frac{3}{16}$ of £7; (ii) $2\frac{3}{28}$ of 5 tons.

(i)

$$16\left\{\begin{array}{r|r} & \text{£.}\quad s.\quad d. \\ 4 & 7\ .\ \ 0\ .\ 0 \\ \hline 4 & 1\ .\ 15\ .\ 0 \\ \hline & 8\ .\ 9 \\ & 3 \\ \hline & 1\ .\ \ 6\ .\ 3 \end{array}\right.$$

Here the unit is £7, and by the definition of a fraction we have to divide this into 16 equal parts and take 3 of them.

Or we may proceed as follows:

$$\tfrac{3}{16} \text{ of } £7 = £7 \times \tfrac{3}{16} = £\tfrac{21}{16} = £\mathbf{1}\tfrac{5}{16},$$

$$£\tfrac{5}{16} = \frac{5 \times \overset{5}{\cancel{20}}}{\underset{4}{\cancel{16}}}\,s. = \tfrac{25}{4}\,s. = \mathbf{6}\tfrac{1}{4}\,s.$$

Thus $\frac{3}{16}$ of £7 = £1. 6*s*. 3*d*.

(ii) Adopting the second method, we have

$$2\tfrac{3}{28} \text{ of } 5 \text{ tons} = 5 \text{ tons} \times 2\tfrac{3}{28} = \mathbf{10} \text{ tons} + 5 \text{ tons} \times \tfrac{3}{28}.$$

$$5 \text{ tons} \times \tfrac{3}{28} = 5 \times \overset{5}{\cancel{20}} \times \frac{3}{\underset{7}{\cancel{28}}} \text{ cwt.} = \tfrac{75}{7} \text{ cwt.} = \mathbf{10}\tfrac{5}{7} \text{ cwt.}$$

$$\tfrac{5}{7} \text{ cwt.} = \tfrac{5 \times 4}{7} \text{ qrs.} = \tfrac{20}{7} \text{ qrs.} = \mathbf{2}\tfrac{6}{7} \text{ qrs.}$$

$$\tfrac{6}{7} \text{ qr.} = \frac{6 \times \overset{4}{\cancel{28}}}{\cancel{7}} \text{ lbs.} = \mathbf{24} \text{ lbs.}$$

Thus $\frac{3}{28}$ of 5 tons = 10 tons 10 cwt. 2 qrs. 24 lbs.

EXAMPLE 2. *Find the value of* $3\frac{5}{8}$ of £3. 13*s*. 7*d*.

$$3\tfrac{5}{8}\text{ of }\pounds3.\ 13s.\ 7d. = \pounds3.\ 13s.\ 7d. \times 3\tfrac{5}{8}.$$

Hence we have to multiply £3. 13*s*. 7*d*. by 3 and by $\frac{5}{8}$, and add the results.

First method.

$$\begin{array}{r|rrll}
 & \pounds. & s. & d. & \\
8 & 3\ . & 13\ . & 7 & \\
\cline{2-4}
 & & 9\ . & 2\frac{3}{8} & =\frac{1}{8}\text{ of the multiplicand} \\
 & & & 5 & \\
\cline{2-4}
 & 2\ . & 5\ . & 11\frac{7}{8} & =\frac{5}{8}\text{ of the multiplicand} \\
 & 11\ . & 0\ . & 9 & =\text{the multiplicand}\times 3 \\
\cline{2-4}
 & 13\ . & 6\ . & 8\frac{7}{8} & =3\frac{5}{8}\text{ of the multiplicand.} \\
\cline{2-4}
\end{array}$$

Second Method. When the proper fraction in the multiplier can be split up into the sum of two or more fractions with *unit numerators* the following arrangement is the most convenient.

Here $\frac{5}{8}=\frac{4+1}{8}=\frac{4}{8}+\frac{1}{8}=\frac{1}{2}+\frac{1}{8}$.

$$\begin{array}{r|rrll}
 & \pounds. & s. & d. & \\
 & 3\ . & 13\ . & 7 & \\
 & & & 3 & \\
\cline{2-4}
 & 11\ . & 0\ . & 9 & \\
\frac{1}{2} & 1\ . & 16\ . & 9\frac{1}{2} & =\frac{1}{2}\text{ of the multiplicand} \\
\frac{1}{8} & & 9\ . & 2\frac{3}{8} & =\frac{1}{8}\text{ of the multiplicand} \\
\cline{2-4}
 & 13\ . & 6\ . & 8\frac{7}{8} & \\
\cline{2-4}
\end{array}$$

Since $\frac{1}{8}=\frac{1}{4}$ of $\frac{1}{2}$, the simplest way of taking $\frac{1}{8}$ of the multiplicand is to take $\frac{1}{4}$ of the result in the previous line.

EXAMPLE 3. *Divide* 4 tons 16 cwt. 3 qrs. *by* $\frac{8}{47}$.

$$4\text{ tons }16\text{ cwt. }3\text{ qrs.}\div\tfrac{8}{47}=4\text{ tons }16\text{ cwt. }3\text{ qrs.}\times\tfrac{47}{8}.$$

Now $\frac{47}{8}=5\frac{7}{8}=6-\frac{1}{8}$; hence we may multiply 4 tons 16 cwt. 3 qrs. by 6 and subtract $\frac{1}{8}$ of 4 tons 16 cwt. 3 qrs. from the result.

$$\begin{array}{r|rrll}
 & \text{tons} & \text{cwt.} & \text{qrs.} & \\
 & 4\ . & 16\ . & 3 & \\
 & & & 6 & \\
\cline{2-4}
 & 29\ . & 0\ . & 2 & \\
\text{subtract }\frac{1}{8} & & 12\ . & 0\frac{3}{8} & =\frac{1}{8}\text{ of 4 tons 16 cwt. 3 qrs.} \\
\cline{2-4}
 & 28\ . & 8\ . & 1\frac{5}{8} & \\
\cline{2-4}
\end{array}$$

NOTE. When the divisor is a mixed number it must first be written in the form of an improper fraction.

EXAMPLE 4. *Find the value of*

$\frac{3}{11}$ of 16*s*. 6*d*. + $1\frac{4}{5}$ of 7*s*. 6*d*. + $\frac{5}{7}$ of 1 guinea.

Here it will be best to bring all the sums of money to the same denomination; the most convenient unit is sixpence.

The required value $= \left(\frac{3}{\cancel{11}} \times \overset{3}{\cancel{33}} + \frac{9}{5} \times \overset{3}{\cancel{15}} + \frac{5}{\cancel{7}} \times \overset{6}{\cancel{42}}\right)$ sixpences

$= (9 + 27 + 30)$ sixpences

$= 66$ sixpences $=$ £1. 13*s*.

EXAMPLES VI. a.

(*Examples* 1–4 *may be taken orally.*)

Express in shillings and pence the following fractions of £1:

1. £$\frac{1}{3}$; £$\frac{1}{6}$; £$\frac{1}{8}$; £$\frac{1}{12}$; £$\frac{1}{16}$; £$\frac{3}{8}$; £$\frac{5}{8}$; £$\frac{7}{8}$.

[These values are important, and should be committed to memory.]

2. $\frac{1}{4}$ of 2*s*. 6*d*.; $\frac{1}{2}$ of 7*s*. 6*d*.; $\frac{1}{3}$ of 5*s*.; $\frac{2}{3}$ of £1; $\frac{5}{6}$ of £1.

Find the value of

3. $\frac{1}{9}$ of 1 yd.; $\frac{5}{18}$ of 1 yd.; $\frac{1}{12}$ of 1 ton 4 cwt.; $\frac{3}{5}$ of 1 ton.

4. $\frac{1}{5}$ of 2 qts. 1 pt.; $\frac{2}{7}$ of 3 qts. 1 pt.; $\frac{1}{14}$ of 1 gal. 3 qts.

Find the value of

5. $\frac{5}{9}$ of 2*s*. 3*d*. **6.** $\frac{6}{11}$ of 18*s*. 4*d*. **7.** $\frac{7}{12}$ of £2. 10*s*.

8. $\frac{3}{28}$ of £4. 18*s*. **9.** $\frac{12}{25}$ of £6. 5*s*. **10.** $\frac{7}{16}$ of £4. 1*s*. 4*d*.

11. $\frac{2}{15}$ of £4. **12.** $3\frac{4}{35}$ of 6 tons. **13.** $\frac{5}{18}$ of 5 guineas.

14. $1\frac{7}{36}$ of 8 yds. **15.** $\frac{11}{96}$ of £12. **16.** $2\frac{5}{24}$ of 18 cwt.

17. $5\frac{3}{4}$ of £11. 7*s*. 8*d*. **18.** $2\frac{5}{8}$ of £3. 7*s*. 2*d*.

19. 1 ton 7 cwt. 3 qrs. × $5\frac{3}{8}$. **20.** £4. 3*s*. 8*d*. ÷ $\frac{16}{23}$.

21. $3\frac{7}{8}$ of £9. 6*s*. 4*d*. **22.** 1 ac. 3 r. 8 p. ÷ $1\frac{4}{11}$.

23. $5\frac{7}{12}$ of £24. 11*s*. 0*d*. **24.** 2 cwt. 3 qrs. 8 lbs. ÷ $1\frac{4}{5}$.

Find the value of

25. $\frac{3}{7}$ of 3*s*. 6*d*. + $\frac{7}{22}$ of 5*s*. 6*d*. **26.** $\frac{2}{9}$ of 3 guineas + $\frac{4}{11}$ of 16*s*. 6*d*.

27. $1\frac{2}{5}$ of 7 ft. 6 in. − $\frac{5}{8}$ of 5 yds. 1 ft.

28. $\frac{3}{16}$ of 1 mi. − $\frac{5}{11}$ of 1 fur. **29.** $3\frac{1}{18}$ of 5*s*. 3*d*. + $2\frac{1}{22}$ of 4*s*. 7*d*.

30. $1\frac{5}{11}$ of 3 fur. + $\frac{3}{17}$ of 11 yds. 1 ft.

31. $\frac{5}{14}$ of 1 cwt. + $\frac{3}{13}$ of 1 qr. 24 lbs. + $2\frac{2}{3}$ of 12 lbs.

32. $4\frac{1}{8}$ of $\frac{4}{11}$ of £1. 6*s*. 8*d*. + $\frac{16}{21}$ of £2. 12*s*. 6*d*. + $\frac{5}{18}$ of £1. 16*s*.

33. $1\frac{2}{5}$ of $\frac{1}{14}$ of 10*s*. − $\frac{3}{5}$ of $1\frac{1}{4}$ of 1 florin + $\frac{3}{11}$ of £1. 18*s*. 6*d*.

134. **To express one concrete quantity as the fraction of another of the same kind.** If we have to find how many times one concrete quantity contains another of the same kind we divide the first by the second, and by the principles of quotition (Art. 17) the result is an abstract number. Similarly in the present case we divide the first quantity by the second, after expressing them in terms of the same unit; the result is an abstract fraction.

EXAMPLE 1. *Express* 2*s*. 1*d*. *as a fraction of* 6*s*. 8*d*.

Express both sums in pence, then

$$\text{the required fraction} = \tfrac{25}{80} = \tfrac{5}{16}.$$

EXAMPLE 2. *Express*

2 tons 5 cwt. *as a fraction of* 4 tons 14 cwt. 2 qrs.

The most convenient common unit is the quarter.

$$\text{The required fraction} = \frac{\text{2 tons 5 cwt.}}{\text{4 tons 14 cwt. 2 qrs.}} = \frac{\text{180 qrs.}}{\text{378 qrs.}}$$
$$= \tfrac{180}{378} = \tfrac{20}{42} = \tfrac{10}{21}.$$

EXAMPLE 3. *What fraction is* £2. 12*s*. 6*d*. *of* £3. 3*s*. 4*d*. ?

$$\text{The required fraction} = \frac{\text{£2. 12}s.\text{ 6}d.}{\text{£3. 3}s.\text{ 4}d.} = \frac{\text{£}2\frac{5}{8}}{\text{£}3\frac{1}{6}}$$
$$= 2\tfrac{5}{8} \div 3\tfrac{1}{6} = \tfrac{21}{8} \times \tfrac{6}{19} = \tfrac{63}{76}.$$

NOTE. There will often be a saving of labour if the highest possible denomination can be readily chosen as a common unit.

135. **Ratio** is the relation which one quantity bears to another *of the same kind*, the comparison being made by considering what multiple, part, or parts the first is of the second. Thus the ratio of one quantity to another is measured by *the fraction which the first is of the second.*

Thus, from the preceding examples,

$$\text{the ratio of 2}s.\text{ 1}d.\text{ to 6}s.\text{ 8}d. = \frac{2s.\ 1d.}{6s.\ 8d.} = \frac{5}{16};$$

$$\text{the ratio of £2. 12}s.\text{ 6}d.\text{ to £3. 3}s.\text{ 4}d. = \frac{\text{£2. 12}s.\text{ 6}d.}{\text{£3. 3}s.\text{ 4}d.} = \frac{63}{76}.$$

136. The ratio of two quantities A and B, of the same kind, is usually written A : B. Thus $\frac{A}{B}$ and A : B have the same meaning. Since a ratio expresses the *number* of times (whole or fractional) that one quantity contains another, *every ratio is an abstract number*, either whole or fractional.

Thus from Art. 135 (where for convenience we worked in pence), the ratio of 2*s*. 1*d*. to 6*s*. 8*d*. $= \frac{5}{16}$ (not $\frac{5}{16}$ *of a penny*).

EXAMPLE. *Find the ratio of* 3 qrs. 7 lbs. *to* 2 cwt. 1 qr. 8 lbs.

We may conveniently express each quantity in quarters;

$$\text{then the required ratio} = \frac{3 \text{ qrs. } 7 \text{ lbs.}}{2 \text{ cwt. } 1 \text{ qr. } 8 \text{ lbs.}} = \frac{3\frac{1}{4} \text{ qrs.}}{9\frac{2}{7} \text{ qrs.}}$$

$$= \frac{13}{4} \div \frac{65}{7} = \frac{\cancel{13}}{4} \times \frac{7}{\underset{5}{\cancel{65}}} = \frac{7}{20}.$$

Thus the ratio is 7 : 20.

137. Four quantities are said to be **directly proportional** when the ratio of the first to the second is equal to that of the third to the fourth.

Thus a, b, c, d are in direct proportion if

$$\frac{a}{b} = \frac{c}{d}, \text{ or } a : b = c : d.$$

The word 'direct' is usually omitted.

EXAMPLE. The quantities 3*s*. 6*d*., 10*s*., 2 yds. 1 ft., 6 yds. 2 ft. are proportional, for

$$\text{the ratio of } 3s.\ 6d. \text{ to } 10s. = \frac{3\frac{1}{2}}{10} = \frac{7}{20},$$

$$\text{and the ratio of 2 yds. 1 ft. to 6 yds. 2 ft.} = \frac{2\frac{1}{3}}{6\frac{2}{3}} = \frac{7}{20}.$$

Ratio and Proportion will be more fully discussed in a later chapter.

EXAMPLES VI. b.

(*Examples* 1–5 *may be taken orally.*)

Express as fractions of £1:

1. 4*s*.; 2*s*. 6*d*.; 7*s*. 6*d*.; 12*s*. 6*d*.; 17*s*. 6*d*.
2. 3*s*. 4*d*.; 6*s*. 8*d*.; 13*s*. 4*d*.; 1*s*. 3*d*.; 1*s*. 8*d*.
3. 8*s*. 9*d*.; 3*s*. 6*d*.; 5*s*. 8*d*.; 11*s*. 3*d*.; 18*s*. 4*d*.

Express as fractions of 1 ton:

4. 4 cwt.; 5 cwt.; 7 cwt.; 1 cwt. 1 qr.; 3 cwt. 2 qrs.

Express as fractions of 1 yard:

5. 2 ft.; 1 ft. 6 in.; 6 in.; 10 in.; 2 ft. 3 in.

Express in lowest terms:

6. 7*s*. 4*d*. as a fraction of £2.
7. £3. 15*s*. .... £11. 5*s*.

Express in lowest terms:

8. 5*s*. 6*d*. as a fraction of £3.
9. 15*s*. 9*d*. .... £4. 4*s*.
10. 13 lbs. .... 3 qrs. 7 lbs.
11. 5 tons 12 cwt. .... 7 tons.
12. 7*s*. 10½*d*. .... £7.
13. £2. 16*s*. .... 5 guineas.
14. 6¾*d*. .... 18*s*. 9*d*.
15. 297 yds. .... 1 mile.
16. 8½*d*. .... 1*s*. 0¾*d*.
17. 5½ tons .... 9 tons 18 cwt.

18. $\dfrac{2s.\ 11d.}{5s.\ 3d.}$ 19. $\dfrac{£3.\ 6s.}{£7.\ 3s.}$ 20. $\dfrac{7 \text{ cwt.}}{3 \text{ tons } 10 \text{ cwt.}}$ 21. $\dfrac{242 \text{ yds.}}{1 \text{ mi. } 3 \text{ fur.}}$

22. What fraction of £1. 15*s*. is £1. 6*s*. 3*d*.?

23. Express 3 tons 12 cwt. 2 qrs. as a fraction of 2 tons.

24. Reduce 4 m. 76 cm. to the fraction of 5 m. 44 cm.

25. What fraction is 2 Km. 508 m. of 3 Km. 36 m.?

26. Express 35 sq. yds. 8 sq. ft. as a fraction of 68 sq. yds.

27. Reduce $\frac{3}{5}$ of £1. 17*s*. 6*d*. to the fraction of $\frac{4}{9}$ of £6. 15*s*.

Express as fractions in their lowest terms the following ratios:

28. £1. 8*s*. to £6. 6*s*. 29. £12. 10*s*. to £18. 15*s*.

30. $15\frac{5}{8}$ oz. to 1 lb. 14 oz. 31. 1 mi. 484 yds. to 2 mi.

32. Shew that 4*d*., 5¾*d*., 5 yds. 1 ft., 7 yds. 2 ft. are proportionals.

33. If x lbs., 1 qr., 5*s*. 3*d*., £3. 13*s*. 6*d*. are in proportion, find x.

34. *A* has a yearly salary of £200 and spends $\frac{3}{5}$ of it; *B* is paid at the rate of £12 a month and spends $\frac{5}{8}$ of what he earns. What is the ratio of their savings at the end of the year?

35. If $\frac{7}{15}$ of a guinea be taken from $\frac{1}{12}$ of $\frac{5}{9}$ of £15, what fraction of £3. 9*s*. will remain?

138. **Ratios as Percentages.** Since a ratio can always be expressed as a fraction, when two or more ratios have to be compared the equivalent fractions must be reduced to a common denominator (Art. 111). In all such cases the common denominator most convenient as a general standard of comparison is **100**.

A fraction expressed with 100 as its denominator is called a **percentage**, and the numerator of the fraction is called the **rate per cent.**

Suppose, for example, that a merchant at the end of a year's business has gained £250 with a capital of £5000. His gain is $\frac{250}{5000}$, or $\frac{5}{100}$ of his capital, and he is said to have gained 5 *per cent.* Here 5 is called the *rate per cent.*, and the fraction $\frac{5}{100}$ is called a *percentage.*

The abbreviations 5 p.c., or 5 % are used for the words "5 per cent."

EXAMPLE 1. *Express* (i) the ratio 7 : 15 as a percentage;
(ii) $62\frac{1}{2}$ per cent. as a fraction in its lowest terms.

(i) Here we have to find a fraction equal to $\frac{7}{15}$, having 100 as its denominator, and the required *rate* as its numerator.

Let x = the rate per cent.;

then $$\frac{x}{100}=\frac{7}{15}, \text{ whence } x=46\tfrac{2}{3};$$

thus the *rate per cent.* is $46\frac{2}{3}$, and 7 : 15 is equivalent to a *percentage* $\frac{46\frac{2}{3}}{100}$.

(ii) $62\frac{1}{2}$ per cent. $=\frac{62\frac{1}{2}}{100}=\frac{125}{200}=\frac{5}{8}$.

EXAMPLE 2. *There are two mixtures of wine and water; $\frac{5}{8}$ of the first and $\frac{7}{15}$ of the second is wine. Which is the stronger mixture, and by how much per cent.?*

By Ex. 1, $\frac{5}{8}$ is equivalent to $62\frac{1}{2}$ %;
and $\frac{7}{15}$ .... $46\frac{2}{3}$ %.

Thus the first is the stronger.

Also $62\frac{1}{2}-46\frac{2}{3}=15\frac{5}{6}$; hence the first mixture contains $15\frac{5}{6}$ *per cent.* more wine than the second.

NOTE. Another way of stating this result is as follows: if we consider 100 equal *parts* (gallons, pints, or litres) of each mixture, there are $62\frac{1}{2}$ parts of wine in the first, and $46\frac{2}{3}$ parts of wine in the second.

EXAMPLE 3. *A Division of 7650 men receives reinforcements that bring its strength to 8262; by how much per cent. has it been increased?*

On an original strength of 7650 the gain is 612; thus a percentage is required equal to the fraction $\frac{612}{7650}$.

Let x = the rate per cent.;

then $$\frac{x}{100}=\frac{612}{7650}; \text{ whence } x=\frac{612}{7650}\times 100=8.$$

Thus the increase is 8 %.

EXAMPLE 4. *In a school 56 p.c. of the boys learn Science, and the remaining boys are 220 in number. Find the number of boys in the school.*

If x be the number required, 44 % do *not* learn Science;

$$\therefore \tfrac{44}{100}x = 220; \text{ whence } x = 500.$$

EXAMPLES VI. c.

(*Examples 1–4 should be taken orally.*)

1. Give percentages equivalent to the following fractions:

(i) $\frac{1}{20}$; (ii) $\frac{3}{40}$; (iii) $\frac{2}{5}$; (iv) $\frac{1}{8}$; (v) $\frac{27}{200}$.

2. Give, in lowest terms, the fractions equivalent to

(i) 20 p.c.; (ii) 25 p.c.; (iii) $2\frac{1}{2}$ p.c.; (iv) $3\frac{1}{8}$ p.c.; (v) $16\frac{2}{3}$ p.c.

3. How much per cent. is

(i) 11 of 25? (ii) 7 of 20? (iii) 5*s.* of £1? (iv) 17*s.* 6*d.* of £1?

4. Give the values of

(i) 5 % of £40; (ii) 4 % of £50; (iii) 3 % of £50; (iv) $2\frac{1}{2}$ % of £80.

5. How much per cent. is

(i) £3. 2*s.* 6*d.* of £12. 10*s.*? (ii) 4 cwt. 3 qrs. of 1 ton 8 cwt. 2 qrs.?

6. How much is $37\frac{1}{2}$ p.c. of £106. 13*s.* 4*d.*?

7. In an orchard which contains 350 trees, 6 p.c. are cherry trees: how many trees of other kinds are there?

8. A tradesman's profits are $13\frac{1}{2}$ p.c. of his capital: if his capital is £500, what profit does he make?

9. When the income tax is 1*s.* in the £, how much per cent. does this represent of a man's income?

10. How much per cent. does a man save out of an income of £750 of which he spends £510?

11. The population of a town increased from 23,000 to 24,380; what was the rate per cent. of increase?

12. How many inhabitants are there in a town if 35 per cent. of its population amounts to 1603 persons?

13. Find the sum of which £142 is 4 per cent.?

14. My rates and taxes amount to £81, this being $7\frac{1}{2}$ p.c. of my whole income. Find my whole income?

15. An agent employed to sell a house charged £93. 15*s.* as commission, this being at the rate of $6\frac{1}{4}$ p.c. What price did he obtain for the house?

16. An expeditionary force, having lost $6\frac{2}{3}$ p.c. of its numbers by casualties and disease, has 13,132 men left fit for service. Find its original strength.

17. The expenses of a hospital are £4837. 10*s.*; find the income if there is a deficit amounting to $7\frac{1}{2}$ p.c. of the income.

18. At the end of each year a firm adds to its capital a sum equal to $6\frac{1}{4}$ p.c. of its capital at the beginning of the year. If the capital now stands at £8177, what was it a year ago?

19. In a school 45 p.c. are English, 20 p.c. are Scotch, 13 p.c. are Irish, and there are 44 Welsh boys; how many boys are there altogether?

20. A tradesman in selling off at a reduction receives £1. 7*s.* 6*d.* for an article marked £1. 11*s.* 3*d.*: how much per cent. is taken off the marked price?

Aliquot Parts.

139. Any part which is contained in a quantity an exact (or integral) number of times is said to be an **aliquot part** of that quantity, and may be expressed as a fraction of it with *unit numerator.*

Thus 6*s.* 8*d.* is an aliquot part of £1, being $\frac{1}{3}$ of £1.
2*s.* 6*d.* .... 17*s.* 6*d.*, ... $\frac{1}{7}$ of 17*s.* 6*d.*
14 lbs. .... 1 cwt., ... $\frac{1}{8}$ of 1 cwt.

EXAMPLES VI. d. (*Oral.*)

Read off as aliquot parts of £1:

1. 2*s.* **2.** 5*s.* **3.** 2*s.* 6*d.* **4.** 6*s.* 8*d.*
5. 3*s.* 4*d.* **6.** 1*s.* 8*d.* **7.** 1*s.* 3*d.* **8.** 1*s.* 4*d.*

[The above results should be carefully noted and remembered.]

Express as aliquot parts of 5*s.*:

9. 1*s.* 3*d.* **10.** 1*s.* 8*d.* **11.** 10*d.* **12.** $7\frac{1}{2}$*d.*

Express as aliquot parts of 7*s.* 6*d.*:

13. 2*s.* 6*d.* **14.** 3*s.* 9*d.* **15.** 10*d.* **16.** 6*d.*

Read off as aliquot parts of 2*s.* 6*d.*:

17. $7\frac{1}{2}$*d.* **18.** 10*d.* **19.** 1*s.* 3*d.* **20.** $2\frac{1}{2}$*d.*

Read off as aliquot parts of 1 cwt.:

21. 7 lbs. **22.** 2 qrs. **23.** 8 lbs. **24.** 14 lbs.

Multiplication by Aliquot Parts. Practice.

140. Compound multiplication may often be conveniently performed by means of aliquot parts. The method, known as **Practice**, will be illustrated by examples.

EXAMPLE 1. *Find the cost of* 89 *things at* £5. 6*s*. 8*d*. *each.*

Since 6*s*. 8*d*. = £$\frac{1}{3}$; ∴ the cost of each thing = £5 + £$\frac{1}{3}$.

The work of multiplication may be arranged thus:

	£.	s.	d.	
	89	0	0	= cost of 89 at £**1** each.
			5	
	445	0	0	= cost of 89 at £5 each.
6*s*. 8*d*. = $\frac{1}{3}$ of £1	29	13	4	= .... at 6*s*. 8*d*. each.
	474	13	4	= cost of 89 at £5. 6*s*. 8*d*. each.

It will be seen that the cost at 6*s*. 8*d*. each is found by dividing £89 (that is, the cost at £**1** each) by 3.

EXAMPLE 2. *Find the cost of* 139 *things at* £5. 17*s*. 6*d*. *each.*

Here the cost = £6 *less* 2*s*. 6*d*., or £6 − £$\frac{1}{8}$.

Hence, arranging the work as before, we have:

	£.	s.	d.	
	139	0	0	= cost of 139 at £**1** each.
			6	
	834	0	0	= cost of 139 at £6 each.
less 2*s*. 6*d*., or $\frac{1}{8}$ of £1	17	7	6	= .... 2*s*. 6*d*. each.
By subtraction	816	12	6	= cost of 139 at £5. 17*s*. 6*d*. each.

EXAMPLE 3. *Find the cost of* 75 *things at* £3. 13*s*. 9*d*. *each.*

Since £3. 13*s*. 9*d*. = £3 + 10*s*. + 2*s*. 6*d*. + 1*s*. 3*d*., the value of 75 things at the given price is equal to the sum of their values at £3, 10*s*., 2*s*. 6*d*., and 1*s*. 3*d*. Practice furnishes a convenient arrangement for finding these separate values and their sum.

	£.	s.	d.	
	75	0	0	= cost of 75 at £1 each.
			3	
	225	0	0	= cost of 75 at £3 each.
10*s*. = $\frac{1}{2}$ of £1	37	10	0	= .... 10*s*. each.
2*s*. 6*d*. = $\frac{1}{4}$ of 10*s*.	9	7	6	= .... 2*s*. 6*d*. each.
1*s*. 3*d*. = $\frac{1}{2}$ of 2*s*. 6*d*.	4	13	9	= .... 1*s*. 3*d*. each.
	276	11	3	= cost of 75 at £3. 13*s*. 9*d*. each.

It will be seen that the process consists in separating the given price into parts each of which (after the first) is an aliquot part of one of the preceding parts. Then to find the cost of 75 things at £3 each we multiply the £75 (which is the cost of 75 things at **£1** each) by 3. Next to find the cost at 10*s.* we divide the cost at **£1** by 2. To find the cost at 2*s.* 6*d.* we divide the cost at **10***s.* by 4. Lastly, the cost at 1*s.* 3*d.* is ½ of the cost at **2***s.* **6***d.* By adding these separate results we obtain the cost at £3. 13*s.* 9*d.*

141. The repeated steps of division shewn in the last example will often give rise to inconvenient fractions of a penny in the successive lines of work. Such cases are best dealt with *decimally*, as explained in Chapter XI. Our object here is merely to afford exercise in the manipulation of aliquot parts, and we therefore confine ourselves to examples in which the above-named difficulty does not occur.

EXAMPLES VI. e.

Read off the cost of the following numbers of articles:

1. 800 at 2*s.* 6*d.*
2. 360 at 6*s.* 8*d.*
3. 252 at 3*s.* 4*d.*
4. 640 at 1*s.* 3*d.*
5. 450 at 1*s.* 4*d.*
6. 288 at 1*s.* 8*d.*

Find the cost of the following numbers of articles, adding or subtracting only *one* aliquot part in each case:

7. 4340 at £2. 5*s.*
8. 3791 at £1. 6*s.* 8*d.*
9. 985 at £3. 2*s.* 6*d.*
10. 488 at £4. 1*s.* 3*d.*
11. 480 at £1. 15*s.*
12. 408 at £2. 17*s.* 6*d.*
13. 112 at £7. 3*s.* 4*d.*
14. 132 at 13*s.* 4*d.*

Using only two aliquot parts, find the value of

15. 425 at 12*s.*
16. 960 at 2*s.* 3*d.*
17. 324 at £1. 12*s.* 6*d.*
18. 357 at £2. 3*s.* 9*d.*
19. 464 at £4. 6*s.* 3*d.*
20. 425 at £3. 11*s.* 8*d.*

Find the cost of the following numbers of articles:

21. 714 at £1. 3*s.* 9*d.*
22. 723 at £3. 7*s.* 11*d.*
23. 339 at £1. 9*s.* 6*d.*
24. 355 at £3. 16*s.* 8*d.*
25. 632 at 12*s.* 8½*d.*
26. 6113 at 8*s.* 1½*d.*
27. 427 at £6. 8*s.* 9*d.*
28. 288 at £3. 12*s.* 10*d.*
29. 6184 at £5. 7*s.* 3*d.*
30. 1260 at £1. 8*s.* 11½*d.*

MISCELLANEOUS EXAMPLES II.

1. Shew that there are twice as many pence in £3. 6*s*. 4*d*. as there are pounds in 3 cwt. 2 qrs. 6 lbs.

2. How many lengths of rope, each 3 m. 54 cm. in length, can be cut from a coil containing 53 m. 10 cm.?

3. What is the smallest sum of money which will exactly contain each of the following sums:

1*s*. 3*d*., 1*s*. 4*d*., 2*s*. 2*d*., 1*s*. 8*d*., 3*s*. 6*d*.?

4. Reduce to lowest terms (i) $\frac{105}{280}$; (ii) $\frac{231}{429}$; (iii) $\frac{696}{1624}$.

5. A book case 2 m. 45 cm. in length stands against a wall 5 m. 75 cm. long. If it is placed in the middle, how much of the wall is left on each side?

6. A grocer buys a hundredweight of coffee; after mixing chicory with it he sells the whole at 1*s*. 6*d*. a pound for £9. 15*s*. How much chicory did he add?

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7. Which is the greater $\frac{128}{15}$ or $8\frac{2}{5}$, and by how much?

8. Find the sum of $2\frac{1}{4}$, $\frac{7}{9}$, $\frac{13}{18}$, $\frac{5}{6}$, and the difference between $6\frac{15}{77}$ and $9\frac{44}{105}$.

9. If 3 kilometres are as much under 2 miles as 5 kilometres are over 3 miles, what is the length of a kilometre?

10. At a bankrupt's valuation his bullocks were put at £25 a head. They afterwards sold, one half at £24. 9*s*. a head, and one half at £26. 6*s*. a head, and thus realised £15 more than was expected. How many bullocks had he?

11. I divide a number by 150, and multiply the result by 6, and obtain 36084. Find the number *by division* in one line.

12. Explain why (i) $\frac{27}{20}=1\frac{7}{20}$; (ii) $\frac{24}{5}=4\frac{4}{5}$; in each case illustrating graphically by a line drawn on squared paper.

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13. Simplify (i) $\frac{26\times17\times35}{45\times39\times51}$; (ii) $5\frac{13}{38}\div12\frac{4}{19}$.

14. Find the least number of weeks in which an exact number of half-guineas can be earned, the wages per week being 16*s*. 4*d*.

15. A train is travelling at the rate of 35 miles an hour. How many feet does it travel in a second?

16. A man has a certain sum to pay to three tradesmen *A*, *B*, and *C*. He pays three-fifths of what he had to *A*, two-fifths of the remainder to *B*, and the rest, which amounts to £75, to *C*. How much money does he pay in all?

17. Find the value of £14. 6*s.* $10\frac{1}{2}$*d.* × 1746.

18. Express the fractions $\frac{5}{19}$, $\frac{10}{31}$, $\frac{2}{11}$ with the same *numerator*, and then arrange them in ascending order of magnitude.

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19. If in $5\frac{5}{12}$ hours a trains travels $230\frac{3}{4}$ miles, what is its rate per hour?

20. A train is running at 54 miles an hour: if the telegraph posts are 66 yards apart, what is the greatest number, and what is the least number, of posts a traveller could count in a minute? Explain the two cases.

21. What is the average height of 3 boys whose respective heights are 1 m. 72 cm., 1 m. 61 cm., 1 m. 47 cm.?

22. The H.C.F. of two numbers is 65, and L.C.M. is 9100. If one of the numbers is 260, what is the other?

23. From $15\frac{11}{12}$ take the sum of $2\frac{3}{5}$, $3\frac{9}{10}$, $5\frac{5}{12}$.

24. Prove by a diagram, taking a straight line as the unit, that $\frac{2}{5}=\frac{6}{15}$. What general principle does this illustrate?

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25. A carpenter cuts off $\frac{9}{16}$ of a plank, and then $\frac{5}{14}$ of the remainder; what fraction of the whole is left?

26. By separation into prime factors find which of the numbers (i) 2371600, (ii) 66825, (iii) 571536 are perfect squares. Find the H.C.F. and L.C.M. of their square roots.

27. Complete the division sum given below, by finding the numbers omitted in the first and second lines.

$$\begin{array}{r|ll} 5 & \underline{\qquad\qquad} & \\ 9 & \underline{\qquad\qquad} & \text{rem. 2.} \\ & \;\;1260 & \text{rem. 6.} \end{array}$$

28. If 315 francs will buy 50 kilograms of tea, how much would be gained by selling the whole at 6 fr. 42 c. per kilogram?

29. A man in a train notices that he can just count 21 telegraph posts in one minute. If they are known to be 44 yards apart, at what speed is the train travelling?

30. Add together $3\frac{1}{7}$, $\frac{4}{5}$. 7; divide the result by $\frac{3}{5}$, and find how much must be added to the quotient to make 100.

31. What fraction multiplied by $3\frac{1}{2}$ of $\frac{4}{5}$ produces $2\frac{2}{5}$ of $\frac{4}{13}$?

32. Find the three numbers between 2500 and 3000 which are divisible by all the numbers 21, 24, and 28.

33. Assuming 1 kilometre $= \frac{5}{8}$ mile, shew that 100 yards is very nearly equal to 91 metres.

34. Find the size of the largest square slab which could be used in paving a courtyard 72 ft. long and 57 ft. broad.

35. Separate 249984 into prime factors, and shew that it is the continued product of three consecutive numbers.

36. Out of £12 I pay bills amounting to £$2\frac{5}{8}$, £$\frac{9}{40}$, £$1\frac{17}{20}$, £$4\frac{2}{5}$; how much have I left?

---

37. Simplify, as shortly as you can:

(i) $2\frac{7}{8}+5\frac{3}{7}+\frac{13}{14}+\frac{19}{24}$; (ii) $10\frac{1}{10}-11\frac{1}{11}+22\frac{1}{22}-20\frac{1}{20}$.

38. What fractions are represented by x in the following statements?

(i) $x+3\frac{4}{15}=16\frac{3}{40}$; (ii) $\frac{53}{56}-x=\frac{1}{24}$; (iii) $\frac{4}{5}$ of $\frac{3}{7}\times x=\frac{3}{14}$.

39. Given that 1 kilogram is approximately equal to $2\frac{1}{5}$ lbs., express 11 tons in kilograms, and 500 Kg. in cwt. etc.

40. If a crew, rowing at 32 strokes per minute, rows a mile in $8\frac{1}{4}$ min., find the distance travelled each stroke.

41. Three persons having £$1\frac{11}{15}$, £$2\frac{3}{8}$, £$7\frac{1}{32}$, subscribe respectively $\frac{5}{13}$, $\frac{1}{6}$, and $\frac{2}{15}$ of their money. What is their total subscription?

42. Find the least number which leaves the remainder 6 when divided by 15, 35, or 42.

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43. The cost of 4 acres of land is £121; what is the value per square yard?

44. Simplify (i) $2\frac{1}{5}\div 1\frac{4}{5}\times 1\frac{1}{10}$; (ii) $2\frac{1}{5}\div 1\frac{4}{5}$ of $1\frac{1}{10}$.

45. Find the value of 51 things 4 of which cost £33. 6*s.* 8*d.*

46. An 18 gallon cask contains $82\frac{2}{7}$ litres; how many pints are there in a litre?

47. How can you tell, without working the sum, that

$$\tfrac{9}{10}+\tfrac{17}{25}+\tfrac{19}{30}+\tfrac{27}{40} \text{ is } \textit{not} \text{ equal to } 2?$$

Find the difference between the greatest but one and the least but one of the above fractions.

48. Simplify

(i) $(2\frac{1}{5}+1\frac{4}{7})\div(1\frac{1}{21}-\frac{2}{7})$; (ii) $2\frac{1}{5}+1\frac{4}{7}\div1\frac{1}{21}-\frac{2}{7}$.

49. Express the difference between $\frac{1}{12}$ of £1 and $\frac{1}{14}$ of a guinea as a fraction of half-a-crown.

50. If plums weigh 50 lbs. to the bushel and cost £22 per ton, what is the cost of 100 bushels to the nearest shilling?

51. A man swims 3 miles in $5\frac{1}{2}$ hours. He goes 1 ft. 4 in. by each stroke; how many strokes does he take per minute?

52. A truck of coals containing 8 tons is sold at the pit's mouth for 13*s*. a ton, and delivered at a railway station distant 60 miles for £7. 14*s*. What is the cost of carriage per ton per mile?

53. Divide the product of $7\frac{3}{4}$ and $9\frac{1}{31}$ by the difference between 9 and $6\frac{1}{5}$.

54. Simplify

(i) $\frac{1}{3}$ of $(\frac{3}{4}+\frac{5}{8})$; (ii) $\frac{1}{3}$ of $\frac{3}{4}+\frac{5}{8}$; (iii) $(\frac{1}{3}+\frac{3}{4})$ of $\frac{5}{8}$.

55. A man takes 3 hrs. 45 min. in walking to a certain place and riding back. He would have gained $1\frac{1}{4}$ hr. by riding both ways. How long would he take to walk both ways?

56. Simplify (i) $\dfrac{4\frac{1}{3}+2\frac{1}{5}}{3\frac{1}{3}-\frac{1}{15}}$; (ii) $\dfrac{3\frac{3}{4}-\frac{11}{12}}{\frac{7}{15}\times1\frac{7}{8}\text{ of }\frac{8}{21}}$.

57. Find the value of

(i) 1 yd. 2 ft. 3 in. ÷ $\frac{7}{25}$; (ii) $9\frac{7}{8}$ of £8. 7*s*. 4*d*.

58. After spending $\frac{1}{3}$ of my money, and then $\frac{1}{4}$ of what remained, and finally $\frac{1}{5}$ of what still remained, I found I had £10 left. How much had I at first?

59. Standard gold is worth £3. 7*s*. $10\frac{1}{2}$*d*. per ounce. What is the least number of ounces that can be coined into an exact number of sovereigns?

60. Find the difference between $\frac{3}{14}$ of 7 half-crowns and $\frac{5}{48}$ of £3.

61. Express 26 ft. 3 in. as a fraction of 100 yds., and 3 r. 10 p. as a fraction of $2\frac{1}{2}$ acres.

62. Find the number of square yards in a rectangular piece of ground which measures $13\frac{8}{9}$ yds. by $6\frac{3}{10}$ yds. Into how many pieces each containing $3\frac{1}{2}$ sq. ft. can the rectangle be divided?

63. At a concert £11 were taken; 50 persons took gallery tickets; 30 persons paid 2*s.* more, and 20 persons paid 3*s.* more than those who sat in the gallery. What was the price of each kind of ticket?

64. Simplify (i) $\dfrac{2\frac{1}{7} \text{ of } \frac{3}{4} \div \frac{9}{16}}{1\frac{1}{7} \times 2\frac{1}{2}}$; (ii) $\dfrac{1\frac{29}{36} + 4\frac{1}{8} \times 1\frac{7}{11}}{5\frac{1}{9} - 7\frac{7}{8} \div 9\frac{9}{20}}$.

65. A dealer has six kinds of teas, averaging 2*s.* 10*d.* per lb. Four of them cost respectively 1*s.* 11*d.*, 2*s.* 3*d.*, 2*s.* 6*d.*, and 3*s.* 4*d.* per lb. What was the cost per lb. of the 5th and 6th teas, supposing that one cost three-fourths as much as the other?

66. Use the following approximate equivalents:

1 *metre* $=39\frac{3}{8}$ *inches*; 1 *kilometre* $=\frac{5}{8}$ *mile*;
1 *kilogram* $=2\frac{1}{5}$ *lbs.*; 1 *litre* $=1\frac{3}{4}$ *pints*;

to determine roughly

(i) the number of metres in 84 ft.;
(ii) .... hundredweight in 280 Kg.;
(iii) .... litres in a 9-gallon cask;
(iv) .... kilograms in 2 tons 4 cwt.;
(v) the price of beef per kilogram when it is 10*d.* per lb.;
(vi) the price of a half-pint glass of beer at 7*d.* per litre.

---

67. Express the following ratios as fractions in their lowest terms:

(i) $\dfrac{1 \text{ ac. } 3 \text{ r. } 35 \text{ p.}}{5 \text{ ac. } 1 \text{ r. } 15 \text{ p.}}$; (ii) $\dfrac{£4.\ 13s.\ 9d.}{£5.\ 16s.\ 8d.}$.

68. Find, with as little work as possible, the value of

(i) $2\frac{7}{12} + 4\frac{1}{6} + \frac{7}{9} + 1\frac{3}{4} + \frac{11}{18}$;

(ii) $\frac{6}{11} - 1\frac{4}{9} - 2\frac{7}{33} + 3\frac{25}{36} - \frac{1}{12}$.

69. If the contents of a pint-tankard will fill two tumblers, and a tea-cup holds $\frac{3}{5}$ as much as a tumbler, how many tea-cups can be exactly filled from a litre? What fraction of a litre is left?

70. A train is travelling at the rate of 40 Km. per hour: how many miles will it go in 12 min.?

71. How many allotments of 3 ac. 2 r. 20 p. each may be taken out of a piece of land $14\frac{1}{2}$ miles long and 7 miles wide?

72. If £2. 11*s.* 4*d.* is paid for 20 kilograms of biscuit, how much should be given for 1 cwt.?

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73. Express $\dfrac{15 \text{ cwt.}}{1 \text{ ton}} - \dfrac{£7.\ 10s.}{£15}$ as a fraction in its lowest terms.

74. A man travelled 520 miles by sea, rail, and coach. The distance by coach was one-third that by rail, the distance by rail one-third that by sea. The charges at sea were 1*d.* per mile, by rail 2*d.*, by coach $4\frac{1}{2}d.$ Find the average cost per mile for the whole journey.

75. An empty cask weighs 11 lbs., and when filled with water its weight is 55 Kg.; find how many gallons the cask will hold. [See Ex. 66.]

76. If a farmer lays two tons of lime on an acre of land, how many grams is that per square metre? Express the result to the nearest integer.

77. Shew that the fraction $\frac{7}{16}$ may be expressed in the form

$$\tfrac{1}{4} + \tfrac{1}{2} \text{ of } \tfrac{1}{4} + \tfrac{1}{2} \text{ of } \left(\tfrac{1}{2} \text{ of } \tfrac{1}{4}\right).$$

Use this result to find the value of

$$£33.\ 2s.\ 8d. \div \left(\tfrac{2}{29} \text{ of } 2\tfrac{2}{3}\right).$$

78. One train travels at an average speed of 50 kilometres an hour, and another at half-a-mile a minute. If the second starts immediately after the first, how long will it be before it is 5 miles behind? [See Ex. 66.]

CHAPTER VII.

THE UNITARY METHOD.

142. IN the present chapter we shall consider questions of which the following are typical examples.

EXAMPLE 1. *If* 12 *yards of cloth cost* 42 *shillings, how much should be paid for* 16 *yards?*

Given that 12 yards cost 42 shillings, it follows that *one* yard will cost *one-twelfth* as much;

that is, 1 yard costs 42 shillings $\times \frac{1}{12}$.

Again 16 yards will cost *sixteen times* as much as 1 yard costs;

$\therefore$ 16 yards cost 42 shillings $\times \frac{16}{12}$.

Hence the required sum $= 42s. \times \frac{4}{3} = 56s.$

EXAMPLE 2. *If* 35 *men can do a piece of work in* 6 *days, in how many days could* 15 *men do it?*

Given that 35 men do the work in 6 days, it follows that *one* man will take *thirty-five times* as long;

that is, 1 man does the work in 6 days $\times$ 35.

Again 15 men will take *one-fifteenth* of the time that 1 man takes;

$\therefore$ 15 men do the work in 6 days $\times \frac{35}{15}$.

Hence the required time $=$ 6 days $\times \frac{7}{3} =$ 14 days.

143. From the above Examples it will be seen that the method depends on first finding

in (1), how much **one** yard of cloth costs;

in (2), how long **one** man takes for the work.

For this reason the process is known as **Reduction to the Unit**, or the **Unitary Method.**

The middle steps in these Examples, namely those in which the reduction to the unit is performed, should be carefully noted and compared. In the first Example, the *smaller* the number of yards the *smaller* the cost; while in the second, the *smaller* the number of men the *greater* the time taken. In every example the pupil must begin by asking himself whether, on reducing to the unit, a smaller or a greater result is to be expected.

The actual arrangement of Examples should be as follows:

EXAMPLE 1. *If I spend* £49. 1*s.* *in* 18 *weeks, how much should I spend in a year at the same rate?*

In 18 weeks I spend £49. 1*s.*

$\therefore$ in 1 week .... £49. 1*s.* $\times \frac{1}{18}$.

$\therefore$ in 52 weeks .... £49. 1*s.* $\times \frac{52}{18}$.

$\therefore$ req$^{\text{d}}$. sum $= £49.\ 1s. \times \frac{26}{9}$

$= £141.\ 14s.$

$$\frac{52}{18} = \frac{26}{9} = 3 - \frac{1}{9}.$$

$$\begin{array}{r|r}
 & £. \quad s. \\
 & 49\ .\ \ 1 \\
 & 3 \\
\hline
 & 147\ .\ \ 3 \\
\text{Less } \frac{1}{9} & 5\ .\ \ 9 \\
\hline
 & 141\ .\ 14
\end{array}$$

NOTES.

(i) In the first line of work it is convenient to arrange the statement so that the quantity of the same kind as the answer comes last.

(ii) In the reduction of the last line it is in some cases best to express the given quantity fractionally, in others to deal with it as a compound quantity by one of the methods explained in Chapter VI. Here we adopt the latter course.

EXAMPLE 2. *A train takes* 3 hrs. 12 min. *to perform a certain journey if it travels at the average speed of* 35 *miles an hour. How long would the same journey take at* 40 *miles an hour?*

At 35 miles an hour the train takes $3\frac{1}{5}$ hours.

$\therefore$ at 1 mile an hour .... $3\frac{1}{5}$ hours $\times$ 35;

$\therefore$ at 40 miles an hour .... $3\frac{1}{5}$ hours $\times \frac{35}{40}$.

Hence req$^{\text{d}}$. time $= (\frac{16}{5} \times \frac{7}{8})$ hours $= 2\frac{4}{5}$ hours

$=$ 2 hrs. 48 min.

EXAMPLES VII. a.

(*Examples* 1–10 *should be done orally.*)

1. If 8 copies of a book cost 24 shillings, what would one copy cost?

2. If 5 men do a piece of work in 12 days, in how many days would 1 man do it?

3. If 8 sheep cost £18, how much would 1 sheep cost?

4. If 10 men dig a ditch in 9 days, how long would 1 man take at the same rate?

5. The stores in a block-house would last 9 men for 9 days; how long would they last 1 man?

6. In 8 weeks my earnings are £40; what are my average earnings for 1 week? What do I earn in 11 weeks?

7. For a certain sum 7 cwt. are carried 30 miles; for the same sum how far should 1 cwt. be carried? How far should 10 cwt. be carried?

8. Travelling 9 miles an hour I make a journey in 10 hours; how long would the same journey take at 1 mile an hour? How long at 15 miles an hour?

9. I pay 12 shillings for 8 lbs. of coffee; how much do I pay (i) for 1 lb.; (ii) for 6 lbs.; (iii) for x lbs.?

10. If x lbs. of tea cost 15 shillings, how much does 1 lb. cost? How much do y lbs. cost?

11. How long will it take me to earn £68 if I can earn £51 in 12 weeks?

12. How many trucks would be wanted to carry 175 tons of coal at the rate of 12 trucks for 100 tons?

13. If 125 men can do a piece of work in 120 days, how many men would be sufficient to do it in 100 days?

14. In how many days would 35 horses eat a supply of corn which lasts 20 horses for 14 days?

15. If half-a-crown will buy 3 lbs. of meat, what will be the cost of 16 lbs.?

16. A certain quantity of stores will last 77 men for 16 weeks. How many men will the same stores last for 22 weeks?

17. By travelling at an average speed of 39 kilometres an hour I take 16 hours for a journey; how many hours should I take at 24 kilometres an hour?

18. What will be the cost of 16 yards of cloth if 30 yards cost £6. 5*s*.?

19. What should be paid for 28 days' work at the rate of £3. 7*s*. 6*d*. for 12 days?

20. A cart delivers 15 tons 12 cwt. of sand in 13 loads, how much could be brought in 5 loads?

21. If I save £61. 13*s*. 4*d*. in 37 weeks, how much should I save in 43 weeks at the same rate?

22. A train running at an average speed of 48 miles an hour performs a journey in 6 hrs. 25 min.; how long would the same journey take if the speed were reduced to 33 miles an hour?

23. (i) If 3 men take 5 hours to load a consignment of coal, how long would 10 men take?

(ii) If x men take p hours for this work, how long would y men take?

24. (i) At the rate of $13\frac{1}{2}$ miles in 3 hours, what distance could be covered in 8 hours?

(ii) What distance could be covered in x hours at the rate of m miles in p hours?

25. How far will a train travel in 17 minutes at the rate of 30 miles an hour? What distance would be travelled in x minutes at this speed?

26. (i) At what speed in miles per hour is a train travelling if it covers 1056 yards in a minute?

(ii) Express a speed of a yards per minute in miles per hour.

27. (i) If a motor travels 90 miles in 1 hour, how many feet does it go in 1 second?

(ii) Express a speed of m miles per hour in yards per minute.

28. (i) If an engine of 60 horse-power will raise a certain load up the shaft of a mine in $\frac{3}{4}$ of a minute, how long would it take an engine of 36 horse-power to do the same work?

(ii) An engine of x horse-power does certain work in a minutes; in how many minutes would an engine of y horse-power do the same work?

29. If 40 tons of provisions would last 840 men for 57 days, for how many days would the same supply last 1330 men?

30. A man makes a journey in 10 hours travelling by coach at 12 miles an hour; how much time would he save if he made the journey by train at 45 miles an hour?

31. If 95 sacks of flour weigh 2 tons 15 cwt., find the weight of 133 sacks.

32. I give £62. 10*s*. 8*d*. for 56 tons of coal; how much should I give for 72 tons at the same rate?

33. What will be the cost of 108 quarters of wheat, if 63 quarters are sold for £95. 5*s*. 9*d*.?

34. If 135 lbs. of coffee cost £8. 8*s*. 9*d*., what should be given for 375 lbs.?

35. A watch loses 2 hrs. 13 min. a week; how much does it lose in 36 hours?

36. A man's expenditure is at the rate of £5. 15*s*. 6*d*. in 35 days; how much will he spend in a year at this rate?

37. If I save money at the rate of £42 a month, I shall have enough to buy a certain house in 4 years 7 months. How soon should I be able to buy this house if I were to increase my rate of saving to £70 a month?

38. Find the cost of 209 copies of a book of which 176 copies can be bought for £46. 18*s*. 8*d*.

39. The rent of 119 ac. 1 r. 16 p. of land is £264; how much land can be rented for £105?

40. A besieged garrison of 4000 men had provisions left for 42 days, when 1760 men broke through the enemy's lines. For how long could the provisions last the remainder of the garrison?

144. EXAMPLE 1. *If* $\frac{6}{11}$ *of a ton of gun-metal is worth* £90, *find the value of* (i) *a whole ton,* (ii) $\frac{13}{15}$ *of one hundred-weight.*

Since $\frac{6}{11}$ of a ton is worth £90,
$\frac{1}{11}$ of a ton .... £$90\times\frac{1}{6}$;
$\therefore$ $\frac{11}{11}$, or *one*, ton .... £$90\times\frac{11}{6}$, viz. £165.

Hence 1 cwt. is worth £$165\times\frac{1}{20}$;
$\therefore$ $\frac{13}{15}$ cwt. .... £$165\times\frac{1}{20}\times\frac{13}{15}$.

Thus the required result = £$\frac{143}{20}$, or £7. 3*s*.

EXAMPLE 2. *Of the annual profits of a firm the senior partner takes* $\frac{3}{8}$, *and the junior partner* $\frac{7}{10}$ *of the remainder. If* £378. 3*s*. *is left to carry forward to next year's account, what are the total profits?*

When $\frac{3}{8}$ of the profits have been taken, there remains $\frac{5}{8}$;
$\therefore$ the junior partner takes $\frac{7}{10}$ of $\frac{5}{8}$ of the total;
so that there is then left $\frac{3}{10}$ of $\frac{5}{8}$, or $\frac{3}{16}$ of the total.

Since $\frac{3}{16}$ of profits amount to £378. 3*s*.,
$\therefore$ $\frac{1}{16}$ of profits .... £378. 3*s*. $\times\frac{1}{3}$;
$\therefore$ $\frac{16}{16}$, or *whole*, profits .... £378. 3*s*. $\times\frac{16}{3}$.

Thus the required sum = £2016. 16*s*.

EXAMPLES VII. b.

1. If $\frac{3}{8}$ of a legacy is worth £93, what is the whole worth?

2. What are the total profits of a business, when $\frac{5}{11}$ of the profits amount to £42. 10*s*.?

3. What is the length of a pole which is shorter by 54 inches when $\frac{6}{7}$ of it has been cut off?

4. After $\frac{17}{20}$ of my journey is done, I have still 6 miles to go. How far have I gone already?

5. If $\frac{4}{5}$ of my income is £308, find $\frac{2}{7}$ of it.

6. What is the value of $\frac{3}{8}$ of an estate, when $\frac{5}{24}$ of it is worth £1065?

7. A creditor who has a claim upon $\frac{3}{7}$ of a bankrupt's property gets £87. 15*s*. What will another creditor get who can claim $\frac{2}{9}$?

8. If $\frac{5}{13}$ of an estate is worth £250, what part of the estate is worth £650?

9. When I have read 240 pages, I shall have got through $\frac{4}{7}$ of a book. What fraction of the book shall I have read when I have got to the end of page 350?

[*In Examples* 10–25 *the following approximate equivalents are to be used*:

1 *metre*	$=39\frac{3}{8}$ *inches*;	1 *kilometre*	$=\frac{5}{8}$ *mile*;
1 *sq. metre*	$=10\frac{3}{4}$ *sq. feet*;	1 *hectare*	$=2\frac{1}{2}$ *acres*;
1 *gram*	$=15\frac{1}{2}$ *grains*;	1 *kilogram*	$=2\frac{1}{5}$ *lbs.*;
1 *litre*	$=1\frac{3}{4}$ *pint* $=61$ *cubic inches.*]		

10. To cut a road costs £240 per mile; find the approximate cost per kilometre.

11. If 1 kilometre of telegraph wire costs £31. 5*s*., find approximately the cost per mile?

12. Find the cost per lb. of tea which is sold for 3*s*. 8*d*. per kilogram.

13. If sugar costs 11*s*. 8*d*. per cwt., quote the corresponding price per lb., and per kilogram.

14. What should be the price per litre of ale which costs 3*s*. 4*d*. a gallon?

15. How much should be given for a pint of claret which is sold at 6*s.* 5*d.* per litre?

16. If cloth is sold at 2*s.* 11*d.* per metre, what is the price per yard?

17. Ribbon costs 8*d.* a yard; at what rate is this per metre?

18. If two glasses go to the pint, how much should be paid for a glass of beer that is sold at 7*d.* per litre?

19. Which is the greater speed 96 kilometres an hour or a mile a minute?

20. The distance from Paddington to Bristol is 119 miles; if the run is made in 2 hrs. 20 min., express the average speed in kilometres per hour.

21. The rent of land is £2. 5*s.* per acre; find roughly the rent per hectare.

22. A plot of 15 hectares is sold for £4650; find roughly the price per acre.

23. How many metres of silk could be bought for £3. 10*s.* if the price per yard is 5*s.* 4*d.*?

24. Calico which cost $8\frac{3}{4}$*d.* per metre is sold at $8\frac{1}{2}$*d.* a yard. What gain is made on selling 48 yards?

25. Spirit bought at 5*s.* 4*d.* per quart is sold at the same price per litre. What is the total gain on selling 1900 litres?

26. Find the acreage of a farm which consists of pasture and corn-land, if the pasture amounts to $\frac{7}{13}$ of the whole, and there are 216 acres of corn-land.

27. A battalion, after $\frac{5}{16}$ of its whole strength had been disabled, had 748 men left fit for duty. In a week's time 816 men were reported fit for duty. By what fraction of its full strength was the battalion still short?

28. After travelling $\frac{7}{8}$ of a journey by rail, and $\frac{1}{16}$ of it by coach, I find I have still 8 miles to walk. What was the length of the whole journey?

29. Three partners own respectively $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$ of a coasting vessel, while the rest belongs to the master; what is the latter's share? If the master's share is worth £937. 6*s.*, what is the whole vessel worth?

30. A man pays $\frac{1}{10}$ of his income in rates and taxes, and $\frac{1}{12}$ in insurances; he has then left £492. 13*s.* 1*d.* What is his income?

CHAPTER VIII.

DECIMALS.

145. Numeration. The numbers 10, 100, 1000, 10,000, ..., are known as **powers of ten.** 10 is itself the *first* power; 100, or 10^2, is the *second* power; 1000, or 10^3, is the *third* power, and so on.

Similarly $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, ..., are called **inverse** powers of ten, and may be written $\frac{1}{10}$, $\frac{1}{10^2}$, $\frac{1}{10^3}$, ...

146. We already know that the figures of any whole number express that number in *descending powers of ten* and in units. For instance

5893 means 5 *thousands* + 8 *hundreds* + 9 *tens* + 3 *units*,

$$\text{or } 5\times10^3 \quad +8\times10^2 \quad +9\times10+3.$$

If we now extend this system fractionally *beyond the units' figure*, we may express *tenths*, *hundredths*, *thousandths*, and so on. This extended notation is shewn in the following Table.

....	ten-thousands	thousands	hundreds	tens	**units**	tenths	hundredths	thousandths	ten-thousandths	....
		5	8	9	**3**	4	6	7		

The above figures represent

5 *thousands* + 8 *hundreds* + 9 *tens* + 3 *units* together with

4 *tenths* + 6 *hundredths* + 7 *thousandths.*

To mark the position of the *units' figure* we place a dot called the **decimal point** immediately after it. Thus the mixed number given above in words is written in figures as follows:

5893·467.

In the same way

2 *hundreds* + 6 *units* + 5 *tenths* + 1 *thousandth*

is written 206·501, the two ciphers being used to indicate the absence of *tens* and *hundredths*, and to fix the position of the other figures.

Similarly 0·8002 indicates 8 *tenths* + 2 *ten-thousandths*, the cipher on the left being used to mark the units' place.

NOTES. (i) When there is no whole number, the cipher before the decimal point is sometimes omitted. Thus 0·8002 and ·8002 have the same meaning, though the first form is usually to be preferred.

(ii) Ciphers appended to the *right* of any decimal do not alter its value. Thus in 3·5200, the final ciphers merely indicate absence of *thousandths* and *ten-thousandths*, so that 3·5200 and 3·52 have the same value.

147. A fraction (or whole number followed by a fraction) thus expressed in *tenths*, *hundredths*, *thousandths*, etc., is called a **decimal fraction** or simply a **decimal.**

In reading off a decimal fraction the digits are named in order, without specifying *tenths*, *hundredths*, etc. Thus 57·4067 is read as follows: "fifty-seven, *decimal* (or *point*) four, nought, six, seven."

EXAMPLES VIII. a. (*Oral.*)

1. Read off as decimals the figures given in the following Table:

	ten-thousands	thousands	hundreds	tens	**units**	tenths	hundredths	thousandths	ten-thousandths
(i)					3	4			
(ii)				2	6	7			
(iii)					8	9	1		7
(iv)		1	4	5		3			
(v)					7		8	0	0
(vi)			4		1			7	0

Read off as decimals:

2. (i) 7 units + 5 tenths; (ii) 6 tenths + 2 hundredths.

3. 4 units + 6 tenths + 8 hundredths.

4. 39 + 1 tenth + 2 hundredths.

5. (i) 50 + 5 hundredths; (ii) 500 + 5 thousandths.

6. (i) 3 tenths + 4 hundredths; (ii) 30 + 4 ten-thousandths.

7. (i) 9 thousandths; (ii) 8 millionths.

State in words (naming *tenths*, *hundredths*, *thousandths*, etc.) the equivalents of:

8. 4·7.
9. 3·68.
10. 51·96.
11. 7·02.
12. 10·08.
13. 0·451.
14. 0·07.
15. 5·005.
16. 0·01.
17. 3·196.
18. 31·96.
19. 319·6.
20. 20·8.
21. 2·08.
22. 0·208.
23. 0·0208.
24. 23·23.
25. 0·023.

26. Read off as decimals: $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, $\frac{1}{10000}$.

27. State (naming tenths, hundredths, etc.) the meanings of

(i) ·72, (ii) ·0072, (iii) ·7200.

What effect have the ciphers in the second decimal?

What effect have the ciphers in the third decimal?

28. On a rough measurement I make the length of a line to be 8·6 inches. The true length is 8·63 inches. State in words my error.

29. Which is the greatest of these decimals: ·0202, ·202, ·2002? Which is the least?

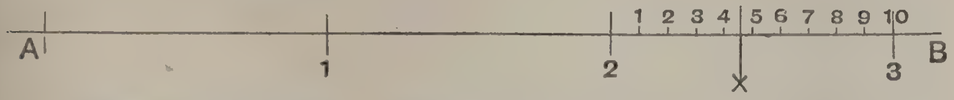
148. Approximate Decimals. It must specially be noticed that the number of digits in a decimal fraction gives no indication of its *value*. Thus 0·736 denotes a smaller quantity than 0·8;

$$\begin{aligned}\text{for } 0{\cdot}736 &= \frac{7}{10}+\frac{3}{100}+\frac{6}{1000}=\frac{736}{1000},\\ \text{while } 0{\cdot}8 &= \frac{8}{10} \qquad\qquad\qquad\quad =\frac{800}{1000}.\end{aligned}$$

And however many decimal figures we might append to 0·736..., the result would still be less than 0·8.

149. The beginner should always consider the digits of a decimal *one by one* in order from left to right, and he should try to realize the *rapidly diminishing importance of successive digits*.

For example, suppose it is required to mark off from the line **AB** a part equal to 2·46 inches.



Now $2·46 = 2 + \frac{4}{10} + \frac{6}{100}$. Hence measuring from **A**, we first take 2 inches; then along the next inch, which is divided into 10 equal parts, we take 4 *tenths*; so that we have now taken 2·4 inches. To get the remaining 6 hundredths, we should have to sub-divide the next tenth into ten equal parts, and of these take 6. We shall then have obtained a line 2·46 inches in length.

Now suppose we wanted a line 2·468 inches long. In order to get the additional 8 *thousandths*, we should have to sub-divide the next *one-hundredth* of an inch into *ten* parts, and take 8 of them. But it is clear from the diagram that this would require a sub-division far more minute than is possible with ordinary instruments.

The best we can do is to count 8 *thousandths* as roughly 1 *hundredth*, and accordingly to estimate a length of 2·**47** inches as an approximate substitute for 2·468 inches. This of course involves an error of **2** thousandths; whereas if we had simply omitted the third decimal figure, our error would have been **8** thousandths.

Thus 2·47 is the nearest equivalent of 2·468 that can be given with *two* decimal figures; and it is therefore said to give the value **correct to the nearest hundredth**, or **correct to the 2nd decimal place.**

If, in the above illustration, the 3rd decimal figure had been less than 5, the nearest equivalent with two decimal figures would clearly have been got by merely omitting the 3rd figure.

Thus we have

$$\left.\begin{array}{l} 2·464 = 2·\mathbf{46} \\ 2·468 = 2·\mathbf{47} \end{array}\right\} \begin{array}{l}\text{correct to the 2nd decimal figure,} \\ \text{or nearest } \textit{hundredth}.\end{array}$$

$$\left.\begin{array}{l} 0·32125 = 0·\mathbf{321} \\ 0·32172 = 0·\mathbf{322} \end{array}\right\} \begin{array}{l}\text{correct to the 3rd decimal place,} \\ \text{or nearest } \textit{thousandth}.\end{array}$$

EXAMPLES VIII. b. (*Oral.*)

1. I am required to measure off a line 2·147 inches long: what errors do I make in taking (i) 2·14 inches, (ii) 2·15 inches as near equivalents?

What is the equivalent of 2·147 *correct to the 2nd decimal figure?*

What figures represent 2·147 *correct to the nearest hundredth?*

2. Instead of 3·128 inches I take 3·13 inches: what error is involved? Is the approximation correct to the nearest hundredth?

3. Correct 4·786 to the nearest hundredth. What error is accepted in doing so?

4. Supposing that you can measure *inches*, *tenths*, *hundredths*, *and no further*, what figures would you take to represent the following lengths?

(i) 3·742 inches. (ii) 3·746 inches. (iii) 2·087 inches.
(iv) 4·063 inches. (v) 0·8847 inch. (vi) 6·12784 inches.

5. As an approximation to 3·685, which is nearer, 3·68 or 3·69?

6. As an approximation to 3·6857, correct to the nearest hundredth, which is the closer, 3·68 or 3·69?

7. In the decimal £0·111, find in pence or the fraction of a penny, the value of each decimal figure separately.

8. How will the following decimals be most nearly represented with *three* decimal figures?

(i) 0·4768. (ii) 3·4909. (iii) 0·07923. (iv) 0·6799.

9. Read off the following decimals correct to the nearest *thousandth.*

(i) 4·6872. (ii) 2·6877. (iii) 0·90884. (iv) 0·9099.

10. Read off the value of 3·618293 correct to *five*, *four*, *three*, *two*, *one* decimal places; and also correct to the *nearest unit.*

150. Place-Value illustrated by the Metric Table of Length. We have seen that the position of any digit in a decimal relatively to the *units' figure* fixes its **place-value**; that is, it determines whether the digit represents *tens*, *hundreds*, *thousands*, etc., or *tenths*, *hundredths*, *thousandths*, etc. For instance, the 3rd digit to the *left* of the units' figure indicates *thousands*, while the 3rd digit to the *right* indicates *thousandths.* Thus the place-value of each digit throughout a decimal is *ten-times* the place-value of the next digit on the right, and consequently *one-tenth* that of the next digit on the left.

Now in the Metric Table of Length the successive denominations are connected in exactly the same way. Hence, taking 1 metre as unit, we may immediately express a series of metric denominations as the decimal of 1 metre; and, conversely, read off a decimal of 1 metre as a compound quantity.

1000	100	10	**1**	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
Km.	Hm.	Dm.	**metre**	dm.	cm.	mm.
	4		6		5	
7		3		8		

Examples. From the above Table, we have at once

(i) 4 Hm. 6 m. 5 cm. = 406·05 metres.
(ii) 7030·8 metres = 7 Km. 3 Dm. 8 dm.

Obs. **1 metre = 39·370113 ... inches = $39\frac{3}{8}$ inches** (nearly).
1 inch = 2·5400 ... centimetres.

EXAMPLES VIII. c.

(*Most of the following examples should be taken orally.*)

Read off as decimals of a metre :

1. 8 m. 5 dm. **2.** 8 m. 5 cm. **3.** 7 m. 4 dm. 3 cm.
4. 7 Dm. 4 m. 3 dm. **5.** 7 dm. 4 cm. 3 mm. **6.** 7 m. 4 cm. 3 mm.

7. Remembering that 8 m. 1 dm. 2 cm. = 8·12 metres,
is it true that 8 yds. 1 ft. 2 in. = 8·12 yards?
Give your reason.

Read off (as compound quantities) in a series of metric denominations :

8. 9·6 m. **9.** 90·6 m. **10.** 0·96 m. **11.** 9·06 m.
12. 7·52 m. **13.** 0·752 m. **14.** 70·502 m. **15.** 7005·2 m.

Read off as decimals :

16. 10 + 2 hundredths + 7 thousandths.

17. 7 tenths + 1 thousandth + 4 ten-thousandths.

18. 21 + 3 thousandths. **19.** 5 + 7 ten-thousandths.

20. $4+\frac{3}{10}+\frac{7}{100}+\frac{9}{1000}$. **21.** $51+\frac{7}{10}+\frac{8}{10^2}$.

22. $\frac{1}{10}+\frac{1}{1000}$. **23.** $\frac{3}{10000}$.

24. $\frac{7}{10^3}$. **25.** 4 hundred-thousandths.

State in words the equivalents of

26. 0·3003. **27.** 0·0072. **28.** 0·0702. **29.** 0·09002.

State in words, and also in inverse powers of 10, the equivalents of

30. ·01. **31.** ·0001. **32.** ·000001.

33. Which is greater, one farthing or ·001 of a sovereign? Why?

34. Read off as compound quantities in metric denominations

(i) 4·036 Km. (ii) 0·409 Km. (iii) 7·007 Km. (iv) 8·8 Km.

35. Using only *two decimal* figures, give the nearest equivalents of

(i) 4·612. (ii) 0·089. (iii) 5·6832. (iv) 0·1781.

36. State as decimals *correct to the nearest thousandth* the value of the following:

(i) 0·0717. (ii) 3·62812. (iii) 0·8008. (iv) 4·50691.

(v) $4+\frac{7}{10}+\frac{3}{100}+\frac{6}{1000}+\frac{8}{10000}$. (vi) $7+\frac{1}{10^2}+\frac{4}{10^3}+\frac{9}{10^4}$.

37. To how many places must the decimal of a *kilometre* be correct to give a result *true to the nearest metre?* Illustrate by an example.

38. Read off as compound quantities *correct to the nearest metre*:

(i) 3·0069 Km. (ii) 9·0483 Km. (iii) 0·10892 Km.

39. To how many places must the decimal of a *metre* be correct to give a result *true to the nearest millimetre?*

40. Read off as compound quantities *correct to the nearest millimetre*:

(i) 0·0431 metre. (ii) 4·0619 metres. (iii) 0·00683 metre.

151. Multiplication and Division by Powers of Ten. It will now be understood that if each digit of a decimal is moved one place to the *left* (the decimal point remaining fixed), the place value of each digit is *increased* ten-fold.

Hence, *to multiply a decimal* by **10**, **100**, **1000**, ..., that is to say, by 10^1, 10^2, 10^3, ..., *move each* **digit 1** *place*, **2** *places*, **3** *places*, ..., *to the* **left**; or, what serves the same purpose, *move the* **decimal point** *so many places to the* **right.**

EXAMPLE 1.

26·324 × 100
= 2632·4.

EXAMPLE 2.

0·00268 × 1000
= 2·68.

EXAMPLE 3. $8·71 \times 10^4 = 87100$.

152. Similarly, if each digit of a decimal is moved one place to the *right*, the place-value of each digit is *diminished* ten-fold.

Hence *to divide a decimal by* **10**, **100**, **1000**, ... (that is to say by 10^1, 10^2, 10^3, ...), *move each* **digit 1** *place*, **2** *places*, **3** *places*, ..., *to the* **right**; or, *move the* **decimal point** so many places to the **left.**

EXAMPLE 1.

916·4 ÷ 100
= 9·164.

EXAMPLE 2.

4·62 ÷ 1000
= 0·00462.

EXAMPLE 3. $5038 \div 10^3 = 5·038$.

153. Standard Form. A decimal (such as **4**·803) which has a whole number consisting of a *single digit*, is said to be in **standard form.**

Now 480·3 = **4**·803 × 10^2;
and 0·04803 = **4**·803 ÷ 10^2.

Thus any decimal can be presented in standard form by using some power of 10 as a multiplier or divisor.

154. As before, the above principles may be illustrated from the Metric Table of length.

Example 1. 4·0816 kilometres = 4081·6 metres
= 40·816 Hm.
= 408160 cm.

Example 2. 8 Km. 37 m. 6 dm., or
8 Km. 0 Hm. 3 Dm. 7 m. 6 dm. = 8·0376 kilometres
= 8037600 millimetres.

Example 3. 70·9004 hectometres = 7 Km. 9 Dm. 4 cm.

From these instances we see that in metric measurements all processes of *Reduction* can be performed without numerical calculation, by merely considering the place-value of the different denominations.

155. We add some further illustrations to shew the advantage of Tables in which successive denominations are connected by *tens* or *powers of ten.*

(i) Express 51 fr. 31 c. in francs only.

Since 100 c. = 1 fr., to bring centimes to francs divide by 100.

Now 31 ÷ 100 = ·31.
∴ 51 fr. 31 c. = 51·31 fr.

(ii) Similarly 36 fr. 7 c. = 36·07 fr.

(iii) Again, since 100 cents = 1 dollar;
8 dollars 35 cents = 8·35 dollars.

(iv) Conversely, 82·31 fr. = 82 fr. 31 c.
And 40·09 dollars = 40 dollars 9 cents.

(v) Since 100 links = 1 chain;
∴ 14 chains 8 links = 14·08 chains;
and 7·32 chains = 7 chains 32 links.

EXAMPLES VIII. d.

(*Most of the following examples should be taken orally.*)

Multiply (reading off the results as decimals)

1. 2·467 by 10, by 100, by 1000.

2. 3·062 by 100, by 1000, by 10,000.

3. 41·32 by 10, by 10^2, by 10^3.

4. 0·0091 by 100, by 10^3, by ten-thousand.

5. 0·083 by 10^4, by 100, by one million.

State the number of

6. *Centimetres* in (i) 70·31 metres, (ii) 1·05 metres, (iii) 8·2 metres.

7. *Metres* in (i) 4·708 Km., (ii) 0·312 Km., (iii) 0·87 Km.

8. *Millimetres* in (i) 3·024 metres, (ii) 0·7 metre, (iii) 0·08 metre.

9. *Centimetres* in (i) 1 Km., (ii) 0·061 Km., (iii) 0·0607 Km.

10. State in words the equivalent of 70·34; and explain why

$$70{\cdot}34 \div 10 = 7{\cdot}034.$$

Divide (reading off the results as decimals)

11. 409·32 by 10, by 100, by 1000.

12. 803·2 by 100, by 10^3, by ten-thousand.

13. 0·7 by 10, by 1000, by 10^4.

14. 6234 by 100, by 1000, by ten-thousand.

15. 90 by 1000, by 10^4, by one million.

Express

16. In *metres* (i) 483 cm., (ii) 79 cm., (iii) 8 cm.

17. In *kilometres* (i) 3081 m., (ii) 149 m., (iii) 85 m.

18. In *metres* (i) 4105 mm., (ii) 9300 mm., (iii) 800 mm.

19. In *kilometres* (i) 504384 cm., (ii) 73009 cm., (iii) 301 cm.

20. Multiply $2+\frac{3}{10}+\frac{4}{100}+\frac{5}{1000}$ by 100, reading off the product as a decimal.

21. Divide 36401 by 10000, stating the quotient as a decimal.

22. By what power of ten must 0·0324 be multiplied to give the result 3·24?

23. By what power of ten must I divide 4090·2 to get 4·0902 as the quotient?

Express in francs:

24. 3 fr. 47 c. **25.** 50 fr. 40 c. **26.** 18 fr. 30 c.
27. 18 fr. 3 c. **28.** 4817 centimes. **29.** 7020 centimes.

Read off in francs and centimes:

30. 21·73 francs. **31.** 8·06 francs.
32. 9·04 francs. **33.** 9·4 francs.

34. Bearing in mind that

3 fr. 17 c. = 3·17 francs,

may we conclude that £3. 17*s.* = £3·17? Give your reason.

Remembering that 1 florin = 2 shillings = £$\frac{1}{10}$ = £·1, express as decimals of a pound:

35. £4. 2*s.* **36.** £5. 6*s.* **37.** £8. 18*s.* **38.** £17. 14*s.*

Express as decimals of one metre:

39. 31 m. 8 cm. **40.** 4707 cm. **41.** 4·708 Km.
42. 6 dm. 5 cm. **43.** 5 Dm. 9 dm. **44.** 0·063 Km.

Read off as decimals of a dollar:

45. 3 dollars 14 cents. **46.** 26 dollars 4 cents.
47. 18$. 30¢. **48.** 9$. 80¢.

49. How many cents are there in (i) 9·07 dollars, (ii) 9·7 dollars?

Express as decimals of one kilometre:

50. 5 Km. 4 Dm. 7 m. **51.** 9 Km. 3 Hm. 2 m.
52. 416 metres. **53.** 6 m. 26 cm.

Read off in kilometres and metres the equivalents of:

54. 4·976 Km. **55.** 13·027 Km.
56. 8 Km. 4 Hm. 3 Dm. **57.** 5 Hm. 28 m.

58. Multiply 7 m. 2 dm. 3 mm. by 10; and express the result in successive metric denominations, and also as the decimal of a kilometre.

59. Taking an average pace to be 76·2 cm., find in metres the equivalent of 1000 paces.

60. If £1 can be exchanged for 25 fr. 18 c., find in francs the equivalent of £100,000.

61. The circumference of a bicycle wheel measures 239 cm.; how many metres will it travel in making ten thousand revolutions?

62. A surveyor's chain of 22 yards is nearly equal to 20 m. 12 cm. Express in kilometres and metres the length of 100 chains.

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63. By what must I multiply (i) 0·5481 to give 5·481?
(ii) 0·00893 to give 8·93?
(iii) 0·000071 to give 7·1?

64. By what must I divide (i) 702·6 to give 7·026?
(ii) 8004·71 to give 8·00471?
(iii) 35·021 to give 3·5021?

65. Express the following decimals by their equivalents in *standard form*:
(i) 381·6. (ii) 4068·7. (iii) 0·867. (iv) 0·00425.

66. Read off the following *correct to the nearest unit*:
(i) 706·8. (ii) 70·683. (iii) 8909·7. (iv) 999·9.

67. Read off the value of the following in metres, *correct to the nearest metre*:
(i) 3·6858 Km. (ii) 0·84927 Km. (iii) 0·9098 Km.

68. Read off the following in francs and centimes, *correct to the nearest centime*:
(i) 83·924 francs. (ii) 17·0791 francs. (iii) 37·698 francs.

69. Express in dollars and cents, *correct to the nearest cent*:
(i) 46·837 dollars. (ii) 101·0961 dollars. (iii) 99·999 dollars.

Decimal Numeration illustrated from Metric Square Measure, Cubic Measure, Capacity, Weight.

156. Square Measure.

Since

1 Dm. = 10 metres, ∴ 1 *square* Dm. = 10^2, or 100, sq. metres;

Similarly

1 dm. = $\frac{1}{10}$ metre, ∴ 1 *square* dm. = $\frac{1}{10^2}$, or $\frac{1}{100}$, sq. metre;

and so on.

Thus since each denomination of length is 10 times the next lower denomination of length, therefore *each denomination of* **area** (expressed as the square of a length) *is* **100 times** *the next lower denomination of area.* This is shewn in the following Table:

$(1000)^2$	$(100)^2$	$(10)^2$	**1**	$\frac{1}{(10)^2}$	$\frac{1}{(100)^2}$	$\frac{1}{(1000)^2}$
sq. Km.	sq. Hm.	sq. Dm.	**sq. metre**	sq. dm.	sq. cm.	sq. mm.

Thus 4364 sq. metres = 43·64 sq. decametres
= 436400 sq. decimetres.

NOTE. The symbols *Km.*^{2}, *m.*^{2}, *cm.*^{2} are sometimes used for *square kilometre, square metre, square centimetre.*

157. For land measurement the unit of area is the **Are**, viz. 1 *square decametre.*

Hence 1 **are** = $(10)^2$, or 100, square metres.

1 **hectare** (viz. 100 ares) = 10000 square metres
= $(100)^2$ ”

So that the hectare (Ha.) is *one square hectometre.*

Obs. **1 square metre = 1550·0 square inches.**
1 square inch = 6·4516 sq. centimetres.
1 hectare = 2·4771 acres (or $2\frac{1}{2}$ acres nearly).

EXAMPLES VIII. e.

(*Most of the following Examples should be taken orally.*)

Express in terms of *square metres*:

1. 482·1 sq. decametres.
2. 6020 sq. decimetres.
3. 0·0046 sq. kilometre.
4. 8701 sq. centimetres.
5. 14·68 sq. hectometres.
6. 4 Dm.^{2} 5 m.^{2} 3 dm.^{2}

Express in terms of hectares:

7. 416 ares.
8. 50026 sq. metres.
9. 8 Ha. 14 ar.
10. 14 Ha. 8 ar.
11. 6 Ha. 6 ar. 6 sq. m.
12. 7 ar. 7 sq. m.
13. How many hectares are there in a square kilometre?
14. How many square decimetres in an are?

158. Cubic Measure.

Since

1 Dm. = 10 metres ; ∴ 1 *cubic* Dm. = $(10)^3$, or 1000, cubic metres.

Similarly

1 dm. = $\frac{1}{10}$ metre ; ∴ 1 *cubic* dm. = $\frac{1}{(10)^3}$, or $\frac{1}{1000}$, cubic metre,

and so on.

Thus since each denomination of length is 10 times the next lower denomination of length, it follows that *each denomination of volume* (expressed as the cube of a length) *is* **1000 times** *the next lower denomination of volume.*

For instance, 3648 *cubic decimetres* = 3·648 *cubic metres*

= 3648000 cubic centimetres.

NOTE. The cubic metre is sometimes called a **stere.**

A *cubic metre*, *cubic decimetre*, and *cubic centimetre* may be denoted by the symbols *m.*³, *dm.*³, *cm.*³.

159. Capacity. The capacity of **one cubic decimetre** is called a **litre.**

One **litre** being taken as the unit, the prefixes *Kilo-*, *Hecto-*, *Deca-*, *deci-*, *centi-*, *milli-*, are attached with their usual meaning.

Thus 1 hectolitre = 100 litres ; 1 decilitre $= \frac{1}{10}$ litre ;

1 kilolitre = 1000 litres ; 1 centilitre $= \frac{1}{100}$ litre.

Since 1 cubic metre = $(10)^3$, or 1000, *cubic decimetres*,

∴ 1 cubic metre contains 1000 litres, viz. *one kilolitre.*

Obs. **1 litre contains 61·024... cubic inches**

or **1·759... pints.**

160. Weight. The weight of **one cubic centimetre of water** (at a temperature of 4° C.) is called a **gram.**

Obs. **1 gram = 15·432... grains** (or $15\frac{1}{2}$ grains, roughly).

1 kilogram (or 1000 grams) = **2·2046... lbs.**

Since 1 litre contains 1 cubic decimetre,

viz. 1000 cubic centimetres ;

∴ **1 litre of water** weighs **1000 grams,**

viz. **1 kilogram.**

Since 1 cubic metre = 1000 cubic decimetres, or litres,

∴ 1 cubic metre of water weighs 1000 kilograms (or 1 tonne).

EXAMPLE 1. *How many litres are there in a tank containing* 5·86 *cubic metres?*

5·86 cu. metres = 5860 cu. decimetres = 5860 litres.

EXAMPLE 2. *How many kilograms of water are there in a vessel containing* 8270 *cubic centimetres?*

8270 cu. cm. = 8·27 cu. dm. = 8·27 litres;
and since each litre of water weighs 1 Kg.,
∴ the required weight = 8·27 Kg.

EXAMPLE 3. *What is the weight of a litre of mercury; if mercury weighs* 13·5 *times its bulk of water?*

1 litre of water weighs 1 Kg.
∴ 1 litre of mercury weighs 13·5 Kg.

EXAMPLES VIII. f. (*Oral.*)

Express in *cubic decimetres*:

1. 3·421 cubic metres.
2. 0·08 cubic metre.
3. 4090 cubic centimetres.
4. 0·009 cubic decametre.

Give the number of *litres* in vessels containing

5. 250 cubic decimetres.
6. 6·97 cubic metres.
7. 0·046 cubic metre.
8. 16200 cubic centimetres.
9. 0·075 kilolitre.
10. One million cubic millimetres.

Express in *kilograms* the *weight of water* in vessels containing

11. 1000 litres.
12. 150 cubic decimetres.
13. 4·75 cubic metres.
14. 72500 cubic centimetres.
15. 0·025 kilolitre.
16. $2\frac{1}{2}$ millions of cubic mm.

State in kilograms the weight of the following:

17. 10 cubic decimetres of copper; copper being 8·8 times as heavy as water.

18. 1 cubic metre of elm; elm weighing 0·72 of its bulk of water.

19. 10 litres of sulphuric acid, this acid weighing 1·84 of its bulk of water.

20. 1 litre of alcohol, the weight of alcohol being 92 per cent. of the weight of the same bulk of water.

21. What weight of water could be held in a tank whose dimensions are as follows:

Length = 5 dm., breadth = 4 dm., depth = 3 dm.?

Addition and Subtraction.

161. Since the place-value of every digit throughout a decimal is ten-times the place-value of the next digit on the right, the four fundamental processes, viz. *addition*, *subtraction*, *multiplication*, and *division* may be performed on decimals in exactly the same way, *as regards numerical work*, as if the figures of the decimals represented whole numbers.

162. In Addition and Subtraction the decimals are to be placed so that the units' figures are in one column, and consequently the decimal points in one vertical line. By this means the *tenths* in the given decimals form one column, the *hundredths* another, and so on. And in the result the decimal point will be under those of the given decimals.

Example 1. *Add together* 14·149, 2·897, *and* 0·768.

tenths hundredths thousandths

$$\begin{array}{r} 14{\cdot}149 \\ 2{\cdot}897 \\ 0{\cdot}768 \\ \hline 17{\cdot}814 \end{array}$$

Explanation. On adding the right-hand (in this case the thousandths') column the result is 24 *thousandths*, viz. 2 *hundredths* + 4 *thousandths*: therefore we set down **4** in the *thousandths'* column, and carry 2 to the *hundredths'* column. Adding the latter, we have 21 *hundredths*, viz. 2 *tenths* + 1 *hundredth*: therefore we set down **1** hundredth, and carry 2 to the *tenths'* column. Adding the *tenths*, we have 18 tenths, viz. 1 *unit* + 8 *tenths*: therefore we set down **8** tenths, and carry 1 to the units' column. *Thus the numerical work is exactly the same as that of adding integers.*

Example 2. *Simplify*

30·48 + 3 + 206·321 + 0·0006.

$$\begin{array}{r@{}l} 30 & {\cdot}48 \\ 3 & {\cdot} \\ 206 & {\cdot}321 \\ 0 & {\cdot}0006 \\ \hline 239 & {\cdot}8016 \end{array}$$

Example 3. *Find the sum of*

4·23 metres, 350 centimetres, and 11 m. 27 cm.

$$\begin{array}{r@{}ll} 4 & {\cdot}23 & \text{metres} \\ 3 & {\cdot}50 & ,, \\ 11 & {\cdot}27 & ,, \\ \hline 19 & {\cdot}00, & \text{or 19 metres.} \end{array}$$

Example 4. *From* 1·21 *take* 0·3708.

$$\begin{array}{r} 1{\cdot}21\mathit{00} \\ 0{\cdot}3708 \\ \hline 0{\cdot}8392 \end{array}$$

Here the vacant places in the upper decimal are represented by ciphers. The subtraction is performed just as if the decimals represented whole numbers, the reason for the process being analogous to that given for Addition in Example 1.

EXAMPLES VIII. g.

Add together

1. 4·3, 3·4, 1·7. **2.** 7·2, 5, 3·8. **3.** 6·07, 7·1, 4·33.
4. 8, 0·8, 8·08. **5.** 0·31, 4·007, 2·683. **6.** 13·002, 5·378, 2·12.

Find the value of

7. 36 + 0·72 + 3·3. **8.** 7·1 + 2·81 + 0·09.
9. 0·63 + 9·072 + 300. **10.** 14·4 + 7 + 0·23 + 8·37.

11. Add together 27·6 cm., 62 cm., and 123·4 cm.; and give the sum in metres.

12. Find the sum of 0·76 Km., 5 Km., and 1·64 Km. (Answer in metres.)

13. What is the total distance round a four-sided plot of ground, the sides measuring 1·5 Km., 1·21 Km., 0·86 Km., and 730 metres? (Answer in kilometres.)

14. I spend 13 fr. 90 c. at my grocer's, 5 fr. 5 c. at the butcher's, buy 45 centimes' worth of apples, also a book costing 10 francs. What is my total expenditure? And how much have I left out of 30 francs?

Subtract

15. 4·9 from 7·2. **16.** 2·4 from 3·81. **17.** 0·56 from 1·85.
18. 0·89 from 1·7. **19.** 0·03 from 3. **20.** 0·007 from 5·2.

Find the difference between

21. 0·75 and 1. **22.** 0·25 and 0·031. **23.** 0·7 and 7·0.
24. 2 and 1·006.

25. The sides of a triangle measure 4·86 in., 5·52 in., and 3·61 in.; by how much does the sum of the sides fall short of 14 inches?

26. A fishing rod is in three pieces whose lengths are 1·33 metres, 1·06 metres, and 0·9 metre; what is its total length, allowing a loss of 9 centimetres for the fitting together of the joints?

27. I cycle to a Railway Station distant 15·26 Km.; travel 73·5 Km. by train, then walk 740 metres to my house: by how much does my total journey fall short of 90 kilometres?

28. What error is made in taking 6 kilometres as equivalent to the sum of 4·206 Km., 0·078 Km., and 1·717 Km.? What fraction is the error of the total length?

29. From 100 yards of wire I cut off lengths of 14·2 yards, 17·4 yards, and 3·5 yards. If I call the remainder 65 yards, what decimal of a yard do I neglect? Shew that the error is more than 3 inches but less than 4 inches.

30. If I take 16·3 in. to represent the perimeter (*i.e.* the sum of the sides) of a triangle, whose sides are 4·64 in., 5·02 in., and 6·7 in., what is my error? Would 16·4 in. be a nearer estimate?

Simplify the following, affixing, when necessary, a negative sign to the result:

31. $5+0{\cdot}07-1{\cdot}63$.
32. $0{\cdot}005+2{\cdot}495-1{\cdot}5$.
33. $1{\cdot}93-2{\cdot}4$.
34. $0{\cdot}06-0{\cdot}6$.
35. $1{\cdot}23+3{\cdot}47+2{\cdot}68-7{\cdot}38$.
36. $5{\cdot}99-(3{\cdot}64+2{\cdot}36)$.
37. $7-0{\cdot}7-0{\cdot}007-0{\cdot}0007$.
38. $6-(3{\cdot}46+7{\cdot}6)-4{\cdot}94$.

39. What decimal must be added to the sum of 5·7, 0·08, and 14·219 to make the total 20?

40. If
$$4{\cdot}04+11{\cdot}4+0{\cdot}291+x=16{\cdot}821,$$
what decimal is represented by x?

41. What decimal must be subtracted from 3 to give the difference 1·693?

42. What decimals, positive or negative, are represented by x in the following statements?

(i) $5{\cdot}68-x=2{\cdot}871$.
(ii) $7{\cdot}84-x=10$.
(iii) $13{\cdot}02-6{\cdot}9-x=5{\cdot}6+18$.

43. Add together 3·92, 14, 0·876, 5·61, 0·0003, and 1·4037. Give the complete result, also the sum *correct to the nearest unit.*

Multiplication of Decimals.

163. *Multiplication by a* **single figure** (*integral or decimal*).

Example. *Multiply* 4·397 (i) *by* 8, (ii) *by* 80, (iii) *by* 800; *also* (iv) *by* 0·8, *and* (v) *by* 0·08.

$$\text{(i)}\ \begin{array}{r} 4{\cdot}397 \\ 8 \\ \hline 35{\cdot}176 \end{array} \qquad \text{(ii)}\ \begin{array}{r} 4{\cdot}397 \\ 80 \\ \hline 351{\cdot}76 \end{array} \qquad \text{(iii)}\ \begin{array}{r} 4{\cdot}397 \\ 800 \\ \hline 3517{\cdot}6 \end{array}$$

In (i) we proceed as if multiplying a whole number, setting down the *thousandths*' figure in the product under the *thousandths*' figure in the multiplicand, the *hundredths*' figure under the *hundredths*' figure, and so on. Hence the decimal point in the product falls under that of the multiplicand. The reason for the process is the same as that given for Addition (Art. 161, Ex. 1).

In (ii) we multiply by 8 as in (i), and the result by 10; the last step being performed by moving every digit in the product *one* place to the *left*. The two steps are taken together.

In (iii), to multiply by 800, we multiply by 8 and place each digit of the product *two* places to the *left*, taking the two steps together.

(iv) $$\begin{array}{r} 4{\cdot}397 \\ 0{\cdot}8 \\ \hline 3{\cdot}5176 \\ \hline \end{array}$$

(v) $$\begin{array}{r} 4{\cdot}397 \\ 0{\cdot}08 \\ \hline {\cdot}34176 \\ \hline \end{array}$$

In (iv), to multiply by ·8, or 8 *tenths*, we multiply by 8, and *divide the result by* 10; the last step being performed by moving each digit of the product *one* place to the *right* (Art. 151). The two steps are taken together.

In (v), to multiply by ·08, or 8 *hundredths*, we multiply by 8, and set down each digit *two* places to the right.

NOTE. It is convenient to notice that if the *units' figure* of the multiplier is placed underneath the last figure of the multiplicand, then in all the above cases the *first figure to be set down in the product falls immediately below the multiplying figure that produced it.*

164. When we multiply ·02 by ·4, we multiply 2 by 4 *and set down the result one place to the right of the multiplied figure*;

that is $$\cdot 02 \times \cdot 4 = \cdot 008.$$

Hence a **2nd** decimal figure multiplied by a **1st** decimal figure gives rise to a **3rd**, or $(2+1)^{\text{th}}$, decimal figure.

Similarly $$\cdot 002 \times \cdot 04 = \cdot 00008\,;$$

thus a **3rd** decimal figure multiplied by a **2nd** decimal figure gives rise to a **5th**, or $(3+2)^{\text{th}}$ decimal figure.

Generally, *a decimal figure in the* $\mathbf{m}^{\text{th}}$ *place multiplied by a decimal figure in the* $\mathbf{n}^{\text{th}}$ *place gives a decimal figure in the* $(\mathbf{m}+\mathbf{n})^{\text{th}}$ *place of the product.*

NOTE. From this it may be seen that the number of decimal figures in a product is always equal to the sum of the numbers of decimal figures in the factors.

EXAMPLES VIII. h.

(*Examples* 1–18 *may be taken orally.*)

Multiply

1. 2·5 by 3. **2.** 4·2 by 4. **3.** 0·7 by 6.

4. 0·9 by 7. **5.** 3·5 by 8. **6.** 0·04 by 9.

7. *Explain the reason* for each step in the multiplication shewn in the margin. (See explanation of Art. 163, Ex. (i).)

$$\begin{array}{r} 2{\cdot}63 \\ 7 \\ \hline 18{\cdot}41 \end{array}$$

8. Each volume of a book is 1·7 inches thick; what is the total thickness of 3 volumes?

9. If 1 kilogram = 2·2 lbs., what is the equivalent of 7 kilograms?

10. Each side of a square field measures 0·15 Km.; what is the distance all round it (in metres)?

11. How many pints are there in 5 litres, if 1 litre contains 1·76 pints?

12. When 3·816 is multiplied by 7, the product is 26·712. Read off the products when the same decimal is multiplied (i) by 70, (ii) by 700, (iii) by 0·7.

13. Given that 27·08 × 9 = 243·72, read off the products when 27·08 is multiplied by (i) 900, (ii) by 0·9, (iii) by 0·009.

Multiply

14. 1·07 (i) by 5, (ii) by 50, (iii) by 0·5.

15. 0·41 (i) by 8, (ii) by 800, (iii) by 0·08.

16. 2·15 (i) by 0·6, (ii) by 6000, (iii) by 0·006.

17. If a figure in the 4th decimal place is multiplied by a figure in the 2nd decimal place, in what place of the product should the resulting figure be set down?

18. Multiply 0·0003 by 0·02; and give the result as a decimal, and also in words.

Multiply

19. 23·684 (i) by 700, (ii) by 0·07, (iii) by 7 million.

20. 0·0375 (i) by eight-thousand, (ii) by 0·08, (iii) by $\frac{8}{1000}$.

21. 4·06 (i) by 9×10^2, (ii) by $\frac{9}{10^2}$, (iii) by $\frac{9}{10^4}$.

22. I walk 115 metres a minute; how many kilometres do I walk in an hour?

23. Express 0·56 of a right angle in degrees.

24. How many yards are there in 5000 metres, supposing that 1 metre = 3·2808 feet?

25. A manufacturer makes silk for 3 fr. 45 c. a metre, and sells it for 5 fr. 30 c. What does he gain on an order for 800 metres?

26. Find the total length of 9 rails, each measuring 13 m. 75 cm.

165. Multiplication by any decimal number.

EXAMPLE. *Multiply* 46·527 *by* 29·83.

Since the multiplier 29·83 = 20 + 9 + ·8 + ·03, we have to multiply separately by 20, by 9, by ·8, and by ·03, and finally to add the partial products.

The work may be arranged as follows:

$$\begin{array}{l} 46{\cdot}527 \\ 29{\cdot}83 \\ \hline 930{\cdot}54 \\ 418{\cdot}743 \\ 37{\cdot}2216 \\ 1{\cdot}39581 \\ \hline 1387{\cdot}90041 \end{array}$$

Begin the multiplication with the digit of the highest order, namely, that on the extreme left; and proceed with the multiplying digits in succession from left to right, each partial product being formed according to the principles explained in the last section.

NOTES. (i) If we arrange the work so that the *units' figure of the multiplier* stands under the *last figure of the multiplicand*, we see that in each partial product *the first figure set down is placed under the multiplying figure that produced it.*

(ii) In order to detect any large mistake (such as the misplacement of the decimal point), it is very desirable *before multiplication* to make a rough estimate of the required product, so that we may know beforehand what sort of result to expect.

Here, for instance, the multiplicand is a little over 46, and the multiplier a little under 30; so that the product should be somewhere about 1380, a rough result confirming the position of the decimal point in our actual product 1387·90041.

166. Multiplication. Alternative Arrangement.

For reasons which will appear later (in connection with *contracted* work), the following arrangement of multiplication by a decimal is recommended.

EXAMPLE 1. *Multiply* 46·527 *by* 29·83.

Instead of the actual *multiplier* take the corresponding decimal of *standard form* (Art. 153); that is, move the decimal point (to the right or left, as may be necessary) so as to give *one integral digit.*

Thus instead of the multiplier 29·83, take **2**·983.

Accordingly the decimal point in the *multiplicand* must be moved the same number of places *in the opposite direction.*

Thus instead of the multiplicand 46·527, take 465·27.

In this case we are *dividing* one factor by 10, and *multiplying* the other factor by 10, so that the product is left unchanged.

The work then stands thus:

$$\begin{array}{r}
465{\cdot}27\\
2{\cdot}983\\
\hline
930{\cdot}54\\
418{\cdot}743\\
37{\cdot}2216\\
1{\cdot}39581\\
\hline
1387{\cdot}90041
\end{array}$$

$$46{\cdot}527 \times 29{\cdot}83$$
$$= 465{\cdot}27 \times \mathbf{2}{\cdot}983.$$

On comparing the work with that given on the last page, it will be seen that the partial products (and therefore also the final result) are all unaltered by this change.

Example 2. *Find the product of* 0·02007 *and* 3568·9.

Here it is advantageous to take 0·02007 as the multiplier, as it contains only two *effective* digits.

Rough Estimate. The multiplier is rather more than ·02, *i.e.* $\frac{2}{100}$, or $\frac{1}{50}$; ∴ the product should be about $\frac{1}{50}$ of 3569, that is, about 71.

$$\begin{array}{r}
35{\cdot}689\\
2{\cdot}007\\
\hline
71{\cdot}378\\
{\cdot}249823\\
\hline
71{\cdot}627823
\end{array}$$

Move the decimal point two places *forward* in the multiplier, and consequently two places *back* in the multiplicand; that is, *multiply* and *divide* respectively by 100. Hence

$$3568{\cdot}9 \times 0{\cdot}02007$$
$$= 35{\cdot}689 \times \mathbf{2}{\cdot}007.$$

EXAMPLES VIII. k.

Multiply

1. 2·4 by 2·1. **2.** 5·3 by 3·2. **3.** 6·5 by 1·4.
4. 0·35 by 3·4. **5.** 0·06 by 5·2. **6.** 32 by 3·1.

7. Multiply 86·54 (i) by 2, (ii) by ·3, (iii) by ·07.
Hence shew how to multiply 86·54 by 2·37.

Find the products of

8. 42·21 and ·25. **9.** 1·56 and ·75. **10.** 13 and ·029.
11. 1·32 and 0·019. **12.** 0·023 and 84. **13.** 400 and 3·012.
14. 46·5 and 0·072. **15.** 37·5 and 0·88. **16.** 42·01 and 20·08.

17. Given that 1 inch = 2·54 cm., express 1 yard in centimetres; also in metres.

18. Find the cost of 6·4 metres of cloth at 7 fr. 50 c. a metre.

19. How many pence are there in £0·875?

Find the value of

20. 10·3 × 0·243. **21.** 8·016 × 125. **22.** 84 × 0·023.
23. 0·0087 × 11·9. **24.** 640 × 0·001275. **25.** 31·05 × 24·02.

26. Find the value of 16 × 2·5; hence write down the values of 1·6 × 0·25 and 160 × 0·025.

Find the area of the rectangles in which:

27. Length = 6·4 in., breadth = 4·5 in. [Ans. in sq. inches.]

28. Length = 9·75 m., breadth = 8·4 m. [Ans. in sq. metres.]

29. Length = 1·45 m., breadth = 80 cm. [Ans. in sq. metres.]

30. Which has the greater area: a square on a side of 10 inches, or a rectangle measuring 10·1 in. by 9·9 in.? State in words the difference of area.

31. A rectangular piece of ground measures 2 Km. 375 m. long by 1 Km. 800 m. wide. Express these dimensions in kilometres, and then find the area.

32. Find the total steam pressure on a square plate, each side of which measures 4·6 inches, at the rate of 15 lbs. per square inch.

33. Write down at sight the continued products

(i) ·7 × 7 × ·01, (ii) ·07 × ·07 × 1000, (iii) ·7 × ·07 × ·001.

34. Find the value of

(i) 100 × 1·6 × ·0125, (ii) 2·5 × 6·4 × ·16, (iii) 12·7 × ·15 × 40.

35. Assuming that the circumference of a circle is given (very nearly) by the formula

circumference = *diameter* × 3·1416;

find (i) The circumference of a bicycle wheel, 28 inches in diameter. (In your answer discard all fractions of an inch beyond *tenths.*)

(ii) The length of a circular track whose diameter is 450 feet. (In your answer give merely the nearest whole number of feet.)

(iii) The length of a steel wire, which, evenly laid, goes 40 times round a winding-drum 15 feet in diameter. (In your answer give only the nearest whole number of feet.)

Find in *grams* the weight of the following:

36. 36 cubic cm. of silver; silver weighing 10·5 times its bulk of water. (See Art. 160.)

37. 0·28 cubic dm. of mercury; mercury being 13·5 times as heavy as water.

38. 3·2 cubic cm. of gold; gold being 19·25 times as heavy as water.

Find in *kilograms* the weight of

39. 5 cubic metres of earth, weighing 1·15 times its bulk of water.

40. 375 cubic metres of coal, which weighs 1·28 times as much as water.

Find the value of

41. $0{\cdot}0875 \times 50 \times 0{\cdot}8$.

42. $1{\cdot}25 \times 3{\cdot}2 \times 2{\cdot}375$.

43. $(0{\cdot}2)^2 \times (0{\cdot}5)^3 \times 200$.

44. $1{\cdot}625 \times (0{\cdot}4)^2 \times 0{\cdot}5$.

Division of Decimals.

167. *Division by a* **single figure** (*integral or decimal*).

Example 1. *Divide* 45·71 *by* 7.

$$\begin{array}{r|l} 7 & 45{\cdot}71 \\ \hline & 6{\cdot}53 \end{array}$$

Divide throughout just as if the figures of the decimal represented a whole number, placing the decimal point in the quotient under that in the dividend.

Explanation.

(i) 45 divided by 7 gives **6** *units*, and 3 units over.

(ii) 3 *units* + 7 *tenths*, viz. 37 *tenths*, divided by 7 gives **5** *tenths*, and 2 tenths over.

(iii) 2 *tenths* + 1 *hundredth*, viz. 21 *hundredths*, divided by 7 gives **3** *hundredths* without remainder.

Example 2. *Divide* 0·0072 *by* 9.

$$\begin{array}{r|l} 9 & 0{\cdot}0072 \\ \hline & 0{\cdot}0008 \end{array}$$

Here in the quotient there can be no *units*, no *tenths*, and no *hundredths*. Beginning the division therefore at the *thousandths*, we say: "9 into 7 *thousandths* gives **0** *thousandths*; 9 into 72 *ten-thousandths* gives **8** *ten-thousandths*."

Example 3. *Divide* 0·38 *by* 8.

$$\begin{array}{r|l} 8 & 0{\cdot}3800 \\ \hline & 0{\cdot}0475 \end{array}$$

Here on dividing 38 *hundredths* by 8, we have **4** *hundredths*, and 6 *hundredths* over.

If now we add ciphers to the right of the dividend (thereby not altering its value), we are able to carry on the division until it terminates.

Example 4. *Divide* 4·72 *by* 9.

$$\begin{array}{r|l} 9 & 4{\cdot}72000\ldots \\ \hline & 0{\cdot}52444\ldots \end{array}$$

In this case the process of division can never terminate. For, following out the work as in the last example, we say

(i) 9 into 47 gives **5**, and remainder 2;

(ii) 9 into 22 gives **2**, and remainder 4;

(iii) 9 into *40* gives **4**, and *remainder 4*.

From this point if the process is continued, the same *quotient figure* and the same *remainder* must be continually repeated.

Note. Results of this kind will be explained later on. Meantime the beginner may be told that no difficulty will result from this indefinite extension of the quotient. For owing to the *very rapid decrease in the place-value of successive decimal figures*, figures beyond the first few places denote quantities relatively so small, that in most practical calculations they may be omitted without materially affecting the result.

Example 5. *Divide* 4·3778 (i) *by* 0·007, (ii) *by* 700.

Here we must multiply or divide (as may be necessary) both divisor and dividend by the same power of 10, namely that power which will give the *divisor* a *single integral figure.* The work will then follow the method already explained, and the quotient will be unaltered by the change.

$$\text{(i)}\quad \frac{4{\cdot}3778}{0{\cdot}007}=\frac{4377{\cdot}8}{\mathbf{7}}=625{\cdot}4.$$

$$\text{(ii)}\quad \frac{4{\cdot}3778}{700}=\frac{0{\cdot}043778}{\mathbf{7}}=0{\cdot}006254.$$

168. If the divisor is a whole number readily expressed in factors, we may divide by each factor in turn, using short division.

Example. *Divide* 25·645 *by* 315 *correct to four decimal figures.*

$$315\left\{\begin{array}{l|l} 9 & 25{\cdot}645 \\ \hline 7 & 2{\cdot}84944\ldots \\ \hline 5 & 0{\cdot}40706\ldots \\ \hline & \underline{\underline{0{\cdot}08141\ldots}} \end{array}\right.$$

Here the divisor $=9\times7\times5$; and the result true to the 4th decimal figure (or the nearest ten-thousandth) is 0·0814.

EXAMPLES VIII. 1.

(*Examples* 1–20 *may be taken orally.*)

Divide

1. 6·9 by 3. **2.** 5·2 by 4. **3.** 9·1 by 7.

4. 5·6 by 8. **5.** 4·3 by 6. **6.** 3·69 by 3.

Complete the following statements :

7. $5{\cdot}6\div4=$. **8.** $29{\cdot}4\div7=$. **9.** $\frac{33{\cdot}6}{8}=$

10. $\frac{0{\cdot}81}{2}=$. **11.** $0{\cdot}81\div9=$. **12.** $\frac{0{\cdot}4}{8}=$

13. $\frac{1}{4}$ of $5{\cdot}28=$. **14.** $7{\cdot}2\times\frac{1}{8}=$. **15.** $0{\cdot}27\div5=$

16. *Explain the reason* for each step in the division shewn in the margin.

$$\begin{array}{r|l} 7 & 0{\cdot}245 \\ \hline & 0{\cdot}035 \end{array}$$

17. A line 3·5 inches long is divided into 5 equal parts: how many tenths of an inch are there in each part?

18. If 9 yards of material cost 6·30 fr., what is the cost of 1 yard?

19. How far do I go in 15 minutes, if I walk 6·4 Km. an hour?

20. If £4·8 is to be divided equally among 8 people, how many tenths of £1 would each get? What would each share be worth in shillings?

Divide

21. 11·22 by 6.
22. 1·841 by 7.
23. 0·342 by 9.
24. 0·0504 by 8.
25. 0·38 by 5.
26. 1·71 by 4.
27. 218·4 by 80.
28. 47·7 by 90.
29. 190·8 by 600.
30. 3·75 by 0·5.
31. 2·66 by 0·7.
32. 0·585 by 0·09.

33. Divide 3·3 by 8; hence find the values of $33 \div 80$, and $0·33 \div 0·8$.

Find the value of

34. $\frac{1}{4}$ of 2·7.
35. $\frac{23·7}{50}$.
36. $7·38 \div 0·06$.
37. $\frac{0·0413}{0·07}$.
38. $5·3 \div 0·02$.
39. $\frac{1802·7}{900}$.
40. $0·01 \div 0·002$.
41. $\frac{0·003}{0·05}$.
42. $\frac{1}{70}$ of 54·6.
43. $5·13 \times \frac{1}{900}$.
44. $121 \div 4$.
45. $\frac{2196}{400}$.
46. $59 \div 8$.
47. $1 \div 8$.
48. $\frac{7}{40}$.
49. $0·29 \times \frac{1}{0·008}$.

50. A vessel steams 193·95 Km. in a 9 hours' trial trip: what is her average speed per hour?

51. A profit of 8368 dols. 80 cents has to be divided among 80 share-holders: what does each receive?

52. If 400 revolutions of the driving wheel carry a bicycle 1001 yards, find the circumference of the wheel in inches. Give your answer to the nearest tenth of an inch, and note the error you thus incur.

53. The total length of 500 steel rails is 6·045 Km.; what is the length of each rail in metres?

54. In a geometry lesson five boys independently measure the lengths of a straight line. Their records are 7·05″, 6·98″, 7·00″, 6·99″, and 7·03″. Find the average of these measurements.

55. A pile of metal discs is 27·3″ in height: if each disc is 0·7″ thick, how many are there in the pile?

Divide, giving the quotient *correct to the* 4^{th} *decimal place,*

56. 4·7 by 6. **57.** 8·6 by 9. **58.** 17 by 7.
59. 1 by 9. **60.** 4 by 7. **61.** 18 by 11.

Divide, using the method of factors,

62. 155·4 by 42. **63.** 45·92 by 56. **64.** 15·12 by 168.
65. 9·046 by 135, *correct to the* 3^{rd} *decimal figure.*
66. 349·968 by 385, *correct to the nearest thousandth.*

Obtain the quotients of the following *to the nearest ten-thousandth* :

67. 6·13 ÷ 15. **68.** 23 ÷ 36. **69.** 0·65 ÷ 56.

169. Division by any whole number or decimal. From what has gone before it is clear that when the divisor is any whole number or decimal, the *figures* of the quotient will be found by long division just as if both divisor and dividend were whole numbers. All that remains is to fix the position of the decimal point in the quotient.

Example 1. *Divide* 3·3562 *by* 0·097.

Instead of the actual *divisor* take the corresponding decimal of *standard form* ; that is, move the decimal point (to the right or left as may be necessary) so as to give *one integral digit.*

To compensate for this change, move the decimal point in the dividend the same number of places *in the same direction.*

Thus $$\frac{3{\cdot}3562}{0{\cdot}097} = \frac{335{\cdot}62}{\mathbf{9}{\cdot}7},$$

for we have multiplied both denominator and numerator by 100, the quotient being therefore unchanged.

The new divisor 9·7 lies between 9 and 10, and the new dividend is a little over 335; so that the quotient will evidently have *two integral figures.*

$$\begin{array}{r l}
 & \underline{\;34{\cdot}6\;} = \textit{Quotient.} \\
9{\cdot}7\,) & 335{\cdot}62 \\
 & \underline{291} \\
 & \;\;446 \\
 & \;\;\underline{388} \\
 & \;\;\;\;582 \\
 & \;\;\;\;\underline{582}
\end{array}$$

It is convenient to write the quotient over the dividend, first placing the decimal point of the former over that of the latter. Now divide as if both divisor and dividend were whole numbers, so placing the figures of the quotient as to give *two* integral digits.

Thus $$3{\cdot}3562 \div 0{\cdot}097 = 335{\cdot}62 \div 9{\cdot}7 = 34{\cdot}6.$$

EXAMPLE 2. *Divide* 15·8055 *by* 257.

Here $$\frac{15\cdot8055}{257}=\frac{0\cdot158055}{\mathbf{2}\cdot57}$$

(dividing denominator and numerator by 100).

And since the new divisor lies between 2 and 3, we see by trial that the first figure in the quotient will be in the hundredths' place.

FULL WORK.

$$\begin{array}{rl}
 & \overline{0\cdot0615}=\textit{Quotient.}\\
2\cdot57\,) & 0\cdot158055\\
 & \underline{\;1542\;}\\
 & \;\;385\\
 & \;\;\underline{257}\\
 & \;\;1285\\
 & \;\;\underline{1285}
\end{array}$$

ABRIDGED BY ITALIAN METHOD.

$$\begin{array}{rl}
 & \overline{0\cdot0615}=\textit{Quotient.}\\
2\cdot57\,) & \underline{0\cdot158055}\\
 & \;\;\underline{385}\\
 & \;\;1285
\end{array}$$

$$\therefore\ 15\cdot8055\div257=0\cdot0615.$$

NOTE. If the divisor is a *whole number*, the work may be done as follows, without first preparing the divisor and dividend.

$$\begin{array}{rl}
 & \overline{0\cdot0615}=\textit{Quotient.}\\
257\,) & 15\cdot8055\\
 & \underline{15\;42}\\
 & \;\;385\\
 & \;\;\underline{257}\\
 & \;\;1285\\
 & \;\;\underline{1285}
\end{array}$$

Here since 257 into 15 *units* will not go, we set down **0** as the units' figure of the quotient and place the decimal point above that of the dividend. Next, 257 into 158 *tenths* gives no tenths, therefore we set down **0** in the tenths' place of the quotient. Then 257 into 1580 *hundredths* gives **6** hundredths; and from this point the work follows the ordinary course.

EXAMPLES VIII. m.

Divide

1. 1·95 by 1·5. **2.** 1·98 by 1·8. **3.** 5·52 by 2·4.

4. 73·1 by 43. **5.** 59·2 by 37. **6.** 375·2 by 67.

7. 0·943 by 0·41. **8.** 3·23 by 0·19. **9.** 3·285 by 7·3.

10. 25·48 by 91. **11.** 8·547 by 0·077. **12.** 15·812 by 23·6.

Find the value of

13. 40·6 ÷ 290. **14.** 0·01426 ÷ 0·031. **15.** 2075 ÷ 8·3.

16. $\frac{2\cdot044}{56}$. **17.** $\frac{4\cdot4226}{0\cdot63}$. **18.** $\frac{0\cdot3}{64}$.

19. Divide 30·26 by 89; then *write down* the values of 302·6÷8·9, and $\frac{3·026}{890}$.

20. Divide 18·53 by 10·9; then *write down* the values of $\frac{1·853}{109}$ and $\frac{185·3}{1·09}$.

21. The product of two quantities is 10·22; one factor is 280; what is the other? Without further numerical work write down the quotients of 1·022÷2·8, and 1022÷0·28.

22. Find the value of x, where (i) $x=\frac{20·387}{703}$. (ii) $x\times 4·7=33·37$. (iii) $8·2\times x=319·8$. (iv) $x\times 2·8=4·8\times 3·5$.

23. How many persons could receive 5 fr. 65 c. from a fund of 1210 francs, and how much would be left over?

24. How many steel rails, each 12·2 metres in length, would be wanted to lay 18·300 Km. of double line?

25. If 1 yard=91·44 cm., how many centimetres are there in 1 inch?

26. Find the value of 1 metre in terms of feet, supposing that 1700 metres=5576 feet.

27. What is the width of a sheet of paper whose length is 12·5″ and area 92·5 sq. in.?

28. The area of a rectangular courtyard is 407·79 sq. metres, and its length is 23·64 metres, find its breadth.

Simplify the following:

29. $\frac{7·2\times 8·4}{5·6}$. **30.** $\frac{5·6\times 14·4}{0·63}$. **31.** $\frac{0·085.}{0·34\times 1·25}$.

Divide, giving the quotient *correct to the 4th decimal place*:

32. 0·05 by 6. **33.** 1440·1 by 18. **34.** 2·71 by 13.

Divide, giving the quotient *correct to the nearest ten-thousandth*:

35. 0·08571 by 25·603. **36.** 4·6513 by 596·8.

37. 28·294 by 21·37. **38.** 2·6576 by 72·81.

39. From the following equations find the value of x, retaining only 3 decimal figures in the result:

(i) $x\times 79=500$. (ii) $23\times x=11·87$. (iii) $x\times 2·735=7·29$.

40. Assuming that in a circle

$$\textit{circumference} = \textit{diameter} \times 3{\cdot}1416 \text{ (nearly)};$$

find (i) the diameter of a circle whose circumference is known to be 49·6 inches. [Give the result true to the nearest *tenth* of an inch.]

(ii) the diameter of a wheel which makes 500 revolutions in running a mile. [Give the result true to the nearest hundredth of an inch.]

41. Equal lengths of 0·42 inch are marked off along a rod 19 inches long. How many such lengths would there be, and what would be the remainder?

42. Find the remainder when 37·8241 is divided

(i) by 9, (ii) by 12, (iii) by 293, (iv) by 90·7.

Decimals and Fractions.

170. *To convert a decimal into a common fraction.* The method will appear from the following examples.

Example. *Find fractions equivalent to*

(i) 0·237, (ii) 14·63, (iii) 0·0029.

(i) $$0{\cdot}237 = \frac{2}{10} + \frac{3}{100} + \frac{7}{1000} = \frac{200+30+7}{1000}$$

$$= \frac{237}{1000}, \text{ or 237 } \textit{thousandths}.$$

Or thus: $$0{\cdot}237 = 237 \div 10^3 = \frac{237}{1000}.$$

(ii) $$14{\cdot}63 = 1463 \div 10^2 = \frac{1463}{100}.$$

(iii) $$0{\cdot}0029 = 29 \div 10^4 = \frac{29}{10000}.$$

Thus in each case *the numerator is the number formed by the figures of the given decimal, and the denominator is that power of ten which corresponds to the number of decimal figures.*

The following equivalents should be verified and remembered.

$${\cdot}5 = \tfrac{1}{2}; \quad {\cdot}25 = \tfrac{1}{4}; \quad {\cdot}75 = \tfrac{3}{4};$$

$${\cdot}125 = \tfrac{1}{8}; \quad {\cdot}375 = \tfrac{3}{8}; \quad {\cdot}625 = \tfrac{5}{8}; \quad {\cdot}875 = \tfrac{7}{8}.$$

171. *To convert a fraction into a decimal.* Here we have to express the given fraction in *tenths*, *hundredths*, *thousandths*, etc.

Now a fraction may be regarded as the *quotient* obtained by dividing the numerator by the denominator. Thus we have merely *to divide the numerator by the denominator and to express the quotient decimally.*

EXAMPLE. *Convert* (i) $\frac{317}{10000}$, (ii) $\frac{742}{875}$, *into decimals.*

(i) $\frac{317}{10000} = 317 \div 10^4 = {\cdot}0317.$

(ii) $\frac{742}{875} = \frac{106 \times 7}{125 \times 7} = \frac{106}{125}$

$= 106 \div 125$

$= 0{\cdot}848.$

$$\begin{array}{r|l} 5 & 106{\cdot}0 \\ \hline 5 & 21{\cdot}2 \\ \hline 5 & 4{\cdot}24 \\ \hline & \underline{0{\cdot}848} \end{array}$$

The above method applies to all fractions, but the division will not usually terminate.

EXAMPLE. *Find decimals equivalent to* (i) $\frac{5}{6}$, (ii) $\frac{49}{396}$.

(i) $\frac{5}{6} = 5 \div 6 = 0{\cdot}8333\ldots.$

$$\begin{array}{r|l} 6 & 5{\cdot}0000\ldots \\ \hline & \underline{0{\cdot}8333\ldots} \end{array}$$

Here, after the first step of division, the remainder is **2**. Now 6 into 20 gives 3, and again the remainder is **2**. Thus in the quotient the figure 3 will be continually repeated, and the process will never terminate.

(ii) $\frac{49}{396} = 49 \div 396$

$= 0{\cdot}123737\ldots.$

$$\begin{array}{rl} & \overline{0{\cdot}1237\ldots} = \textit{Quotient.} \\ 396\,) & 49{\cdot}00000\ldots \\ & 940 \\ & \mathbf{1480} \\ & 2920 \\ & \mathbf{1480} \end{array}$$

Here, after the second step of division, the remainder is **148**, and the next dividend is 1480. The next two figures of the *quotient* are 37, when again the remainder becomes **148**, and the next dividend 1480. Therefore in the quotient the figures 37 are repeated, and so on without limit.

172. Decimals in which one or more figures are continually repeated in the same order are called **recurring**, **repeating**, or **circulating** decimals; and the figures thus repeated form the **recurring period**. The recurring period is shewn by placing a dot over its first and last figures.

Thus $0{\cdot}8333\ldots$ is written $0{\cdot}8\dot{3}$;

and $0{\cdot}123737\ldots$ is written $0{\cdot}12\dot{3}\dot{7}$.

Observe that $0{\cdot}8\dot{3} = 0{\cdot}833$ *correct to the nearest thousandth* [Art. 149]

and $0{\cdot}12\dot{3}\dot{7} = 0{\cdot}124$ " " "

173. In converting a fraction into a decimal we divide the numerator by the denominator, *affixing ciphers to the numerator as may be required.* Therefore all *twos* and *fives* contained as *factors* in the denominator will eventually divide out, leaving no remainder; *and if there is no other factor, the quotient will terminate.*

If however the denominator contains any factors other than 2 and 5 (the fraction *being in its lowest terms*), there will always be a remainder, and the resulting decimal will not terminate.

NOTE. A fraction whose denominator has no other factors than *twos* and *fives* may be decimalized thus:

$$\text{(i)}\quad \frac{106}{125}=\frac{106}{5^3}=\frac{106\times 8}{5^3\times 2^3}=\frac{848}{10^3}=0{\cdot}848.$$

$$\text{(ii)}\quad \frac{7}{16}=\frac{7}{2^4}=\frac{7\times 625}{2^4\times 5^4}=\frac{4375}{10^4}=0{\cdot}4375.$$

174. The notation of fractions may often be usefully combined with that of decimals.

For instance, just as $38\frac{5}{9}$ denotes 3 *tens* + $8\frac{5}{9}$ *units* (or $38\frac{5}{9}$ units);

So $\cdot 38\frac{5}{9}$ may be taken to denote 3 *tenths* + $8\frac{5}{9}$ *hundredths* (or $38\frac{5}{9}$ hundredths).

By this means the exact decimal value of a fraction may be represented at any stage.

EXAMPLE 1. *Express* $\frac{7}{13}$ *in tenths, hundredths, thousandths, and a fraction of one thousand.*

$$\begin{array}{r|l} 13 & \underline{7{\cdot}000} \\ & 0{\cdot}538,\ \text{rem}^{\text{r}}\ 6\ \textit{thousandths.} \end{array}$$

Here, after three steps of division, the remainder is 6 *thousandths*;

and 6 *thousandths* ÷ 13 = $\frac{6}{13}$ of *one-thousandth.*

Hence the quotient is completely represented by $0{\cdot}538\frac{6}{13}$.

NOTE. Here $\frac{6}{13}$ affixed to the 3^{rd} or *thousandths'* figure is the equivalent of all the figures of the decimal after the 3^{rd}. This fraction, being less than $\frac{1}{2}$, shews that the equivalent of $\frac{7}{13}$ is $0{\cdot}\mathbf{538}$ *true to the nearest thousandth.*

This notation sometimes saves labour in converting a decimal into a fraction.

EXAMPLE 2. *Convert* $0{\cdot}09375$ *into a fraction.*

Since $\cdot 375=\frac{3}{8}$,

$$\therefore\quad \cdot 09375=\cdot 09\tfrac{3}{8}=\frac{9\frac{3}{8}}{100}=\frac{75}{800}=\frac{3}{32}.$$

EXAMPLES VIII. n.

Express the following decimals as fractions in their lowest terms.

1. 0·65. **2.** 0·85. **3.** 0·36. **4.** 0·44.
5. 5·38. **6.** 0·0125. **7.** 7·275. **8.** 0·1875.

Express as decimals

9. $\frac{23}{100}$. **10.** $3\frac{9}{1000}$. **11.** $\frac{827}{100}$. **12.** $\frac{41}{100000}$.
13. 13 thousandths. **14.** 52 hundred-thousandths. **15.** 73 millionths.

Convert the following fractions into decimals :

16. $\frac{7}{40}$. **17.** $3\frac{7}{8}$. **18.** $9\frac{5}{16}$. **19.** $1\frac{22}{25}$. **20.** $\frac{1}{640}$.

21. State (without actually performing the work) which of the following fractions give rise to terminating and which to non-terminating decimals. Give your reason in each case.

(i) $\frac{13}{25}$. (ii) $\frac{17}{14}$. (iii) $\frac{1}{64}$. (iv) $\frac{3}{128}$. (v) $\frac{5}{48}$.
(vi) $\frac{87}{375}$. (vii) $\frac{41}{275}$. (viii) $\frac{77}{275}$. (ix) $\frac{63}{490}$. (x) $\frac{107}{250}$.

Express as decimals *correct to the third decimal figure*

22. $\frac{5}{48}$. **23.** $2\frac{1}{13}$. **24.** $3\frac{4}{35}$. **25.** $\frac{11}{19}$. **26.** $7\frac{14}{23}$.

27. Represent the decimal equivalent of $\frac{6}{17}$ as nearly as possible with (i) *one* figure, (ii) *two* figures, (iii) *three* figures, (iv) *four* figures.

Express the following fractions in *tenths*, *hundredths*, *thousandths*, and *the fraction of a thousandth*. Hence give the decimal values *correct to the nearest thousandth*.

28. $\frac{5}{7}$. **29.** $\frac{13}{17}$. **30.** $\frac{11}{23}$. **31.** $\frac{261}{16000}$. **32.** $\frac{33}{1400}$.

Find the equivalents of the following fractions (i) as *recurring decimals*; (ii) as *three-figure decimals with fractional remainders*; (iii) as nearly as possible with *three decimal figures*.

33. $\frac{1}{3}$, $\frac{2}{3}$. **34.** $\frac{1}{9}$, $\frac{4}{9}$, $\frac{7}{9}$. **35.** $\frac{1}{11}$, $\frac{3}{11}$, $\frac{8}{11}$. **36.** $\frac{1}{7}$, $\frac{3}{7}$, $\frac{5}{7}$.
37. $\frac{13}{90}$. **38.** $\frac{13}{44}$. **39.** $4\frac{2}{13}$. **40.** $2\frac{11}{37}$. **41.** $5\frac{4}{27}$.

Write down the following in their complete decimal form :

42. $2{\cdot}13\frac{3}{4}$. **43.** $0{\cdot}32\frac{5}{8}$. **44.** $3{\cdot}19\frac{7}{8}$. **45.** $5{\cdot}32\frac{5}{11}$.

Convert into fractions by the method of Art. 174, Ex. 2 :

46. 0·4375. **47.** 0·1875. **48.** 0·03125. **49.** 0·071875.

50. Convert into decimals at sight :

(i) $\frac{3}{5^2}$. (ii) $\frac{5}{2^3}$. (iii) $\frac{7}{5^3 \times 2^2}$. (iv) $\frac{11}{5^2 \times 2^3}$.

CHAPTER IX.

DECIMAL REDUCTION OF COMPOUND QUANTITIES.

DECIMALIZATION OF MONEY.

175. WE have already seen [Art. 150] that if the Tables are constructed on the *decimal* (or centesimal) scale, all processes of Reduction from any one denomination to any other, either higher or lower, can be performed at sight without numerical work. For example :

(i) 8 m. 5 cm. 7 mm. = 8·057 metres.
(ii) 4 fr. 37 c. = 4·37 francs.

Conversely,

(iii) 0·03809 kilometres = 38 m. 0 dm. 9 cm.
(iv) 18·06 dollars = 18 dols. 6 cents.

In all other Tables these processes must be worked by methods of successive division or multiplication.

EXAMPLE 1. *Reduce* £3. 5*s*. $4\frac{1}{2}$*d*. *to pounds, expressing the result decimally.*

12 | **4·5** pence
20 | **5**·375 shillings
3·26875 pounds

Set down $4\frac{1}{2}$*d*. as **4·5** *pence*, and divide by 12 ; this gives $4\frac{1}{2}$*d*. as the decimal of a *shilling*.

Bring down the **5** shillings, and divide by 20 ; this gives 5*s*. $4\frac{1}{2}$*d*. as the decimal of a *pound*. Finally bring down the **£3**.

Hence £3. 5*s*. $4\frac{1}{2}$*d*. = £3·26875
or 3·26875 of £1.

EXAMPLE 2. *Express* £5. 13*s*. $4\frac{1}{4}$*d*. *in* pounds, (i) *correct to the* fifth *decimal figure ;* (ii) *correct to the nearest* thousandth *of a pound.*

12 | **4**·25 pence
20 | **13**·354166... shillings
5·667708... pounds

The work proceeds as before, but neither division terminates.

Thus £5. 13*s*. $4\frac{1}{4}$*d*. = £5·6677**1**, correct to the 5th decimal figure
= £5·**668**, correct to the nearest thousandth.

EXAMPLE 3. *Reduce* 5 ac. 2 r. 34 p. (i) *to the decimal of* 1 *acre;* (ii) *to the decimal of* $8\frac{1}{4}$ *acres.*

(i)
$$\begin{array}{r|ll} 40 & \mathbf{34{\cdot}00} & \text{poles} \\ 4 & \mathbf{2{\cdot}85} & \text{roods} \\ \hline & \underline{\mathbf{5}{\cdot}7125} & \text{acres} \end{array}$$

(ii)
$$\begin{array}{rr|l} & & 5{\cdot}7125 \div 8\tfrac{1}{4} \\ & & \quad 4 \\ \hline 33\Big\{ & 11 & 22{\cdot}85 \\ & 3 & 2{\cdot}07727\ldots \\ \hline & & \underline{0{\cdot}69242\ldots} \end{array}$$

Hence

(i) 5 ac. 2 r. 34 p. $= 5{\cdot}7125$ of 1 acre.

(ii) req^d decimal of $8\frac{1}{4}$ acres $= \dfrac{5{\cdot}7125}{8\frac{1}{4}} = 0{\cdot}6924$, correct to the nearest ten-thousandth.

EXAMPLES IX. a.

Express as decimals of 1 shilling:

1. 6*d*. **2.** 3*d*. **3.** 9*d*. **4.** $4\frac{1}{2}$*d*.

5. $10\frac{1}{2}$*d*. **6.** $8\frac{1}{4}$*d*. **7.** $6\frac{3}{4}$*d*. **8.** 4*d*.

Express as decimals of £1:

9. 5*s*. **10.** 15*s*. **11.** 2*s*. 6*d*. **12.** 4*s*. 6*d*.

13. 12*s*. 6*d*. **14.** 7*s*. 6*d*. **15.** 11*s*. 6*d*. **16.** 1*s*. 3*d*.

17. £5. 6*s*. 6*d*. **18.** £7. 8*s*. 6*d*. **19.** £8. 14*s*. 6*d*.

Reduce to *pounds*, expressing the result decimally:

20. £4. 11*s*. 6*d*. **21.** £3. 15*s*. 6*d*. **22.** £1. 6*s*. 3*d*.

23. 8*s*. 9*d*. **24.** £8. 17*s*. 9*d*. **25.** £5. 8*s*. $1\frac{1}{2}$*d*.

Express as decimals of £1, correct to the nearest *thousandth*:

26. £11. 13*s*. $10\frac{1}{2}$*d*. **27.** 16*s*. $7\frac{1}{2}$*d*. **28.** £8. 15*s*. 4*d*.

Express as decimals of £1, *correct to the* 4th *decimal figure*:

29. £2. 10*s*. $8\frac{1}{4}$*d*. **30.** £5. 7*s*. $5\frac{1}{2}$*d*. **31.** 3*s*. $4\frac{1}{4}$*d*.

32. Express 5 tons 13 cwt. 3 qrs. as the decimal of 1 ton.

33. Reduce 4 ac. 1 r. 36 p. to acres and the decimal of an acre.

34. What decimal is 5 yds. 1 ft. 6 in. of 1 mile?

35. Express 2 tons 3 cwt. 7 lbs. as the decimal of 5 tons.

36. What decimal is 19*s*. $9\frac{3}{4}$*d* of 25 shillings?

176. *Conversely*, Reduction from a higher to a lower denomination requires successive *multiplication.*

Example 1. *Express* £8·715 *in* £., *s.*, *d.*, (i) exactly; (ii) *correct t the* nearest penny.

8·715 pounds
20
14·30 shillings
·12
3·6 pence

Here we reserve the **£8** for the answer, an multiply the *decimal only* by 20, to bring it t *shillings.* Next we reserve the 14 shillings fo the answer, and multiply the *decimal only* b 12 to bring it to *pence.*

Thus, £8·715 = £8. 14*s.* 3·6*d.*, or £8. 14*s.* 4*d.*, to the nearest penny.

Example 2. *Find in compound quantities the value of*

(i) 1·788 *of* 5 yards; (ii) 0·28125 *of* 7 tons.

(i)
1·788
5
8·940 yards
3
2·82 feet
12
9·84 inches

∴ 1·788 of 5 yards
= 8 yds. 2 ft. 9·84 in. (exactly),
or 8 yds. 2 ft. 10 in., *to the nearest inch.*

(ii)
0·28125
7
1·96875 tons
20
19·375 cwt.
4
1·5 qr.

∴ 0·28125 of 7 tons
= 1 ton 19 cwt. $1\frac{1}{2}$ qr.

177. We reserve for Chapter XI. discussion of the following processes:

(i) *To find the value of a decimal of a* **compound** *quantity; e.g.* to find 3·27 of £5. 4*s.* 7*d.*

(ii) *To express one compound quantity as the decimal of another* **compound** *quantity; e.g.* to express 2 tons 7 cwt. as the decimal of 7 tons 3 cwt. 2 qrs.

178. Approximate results. It is important to notice that £·001 (or £$\frac{1}{1000}$) is a little less than one farthing (or £$\frac{1}{960}$). Hence any decimal of £1 corrected to the nearest *thousandth* (or 3rd decimal place) must yield a result *within one farthing of the true value.*

For example: £3·40682... = £3·407 nearly; the error being less than £·001, and therefore less than 1 farthing. Hence to reduce £3·40682... to £. *s. d.* and *the nearest farthing*, it is sufficient to work with £3·407 (by the method of Art. 176, Ex. 1).

179. Again, in order to reduce a decimal of 1 ton to pounds, we multiply by 2240; and this number being less than 10000 or 10^4, we see that if a decimal of 1 ton is correct to the 4th decimal figure, it will yield a result *true to the nearest pound.*

$$\left.\begin{array}{lll}\text{Thus} & 4{\cdot}0316824 & \text{of 1 ton}\\ \text{and} & 4{\cdot}031\mathbf{7} & \text{of 1 ton}\end{array}\right\}\ \begin{array}{l}\text{give results which differ by less}\\ \text{than 1 lb.}\end{array}$$

EXAMPLES IX. b.

(*Examples* 1–12 *may be taken orally.*)

Find the number of shillings in

1. £0·3. **2.** £0·7. **3.** £0·9. **4.** £0·05.
5. £0·55. **6.** £0·65. **7.** £0·80. **8.** £0·95.

Find the number of pence in

9. 0·75*s.* **10.** 0·125*s.* **11.** 0·625*s.* **12.** 0·875*s.*

Reduce to £. *s.* *d.*, giving the exact results:

13. £3·225. **14.** £5·425. **15.** 9·875 of £1.
16. £2·0875. **17.** £11·5625. **18.** 0·40625 of £1.

Find, *correct to the nearest penny*, the value of:

19. £4·63. **20.** £5·72. **21.** 14·823 of £1.

Find, *correct to the nearest farthing*, the value of:

22. £9·89. **23.** £0·634. **24.** 11·047 of £1.

Find in £. *s.* *d.* the value of the following, giving the exact results:

25. 0·28125 of £2. **26.** 2·10625 of £3.
27. 3·11875 of £5. **28.** 0·53125 of £10.

Find the value of

29. 7·304 of £9, *correct to the nearest penny.*

30. 0·78 of £11, *correct to the nearest farthing.*

Reduce

31. £3. 15*s.* $4\frac{1}{2}$*d.* to the decimal of £4, *correct to the 4th figure.*
32. £7. 9*s.* 4*d.* to the decimal of £8, ...
33. £15. 17*s.* 8*d.* to the decimal of £40, ...
34. £11. 7*s.* $9\frac{3}{4}$*d.* to the decimal of £125, ...

Find in compound quantities the value of

35. 1·8875 tons. **36.** 6·475 acres. **37.** 0·895 of a day.

38. 2·03125 miles (in miles and yds.). **39.** 0·0425 of 10 tons.

40. 0·9375 of 7 tons. **41.** 9·9375 of 3 guineas.

42. What decimal of a guinea (correct to the 4th place) is 12*s.* 9*d.*?

43. What decimal of $2\frac{1}{2}$ acres is 3 r. 10 p.?

44. Express 9 wks. 5 days 6 hrs. as the decimal of 5 years.

Discarding unnecessary decimal figures, find, *correct to the nearest farthing*, the value of

45. £4·324814. **46.** £3·07179. **47.** £0·70996.

48. £0·099872. **49.** £7·$\dot{7}$. **50.** £0·7$\dot{5}\dot{3}$.

51. To how many places must a decimal of 1 *metre* be correct to give a result *true to the nearest millimetre*?

52. To how many places must a decimal of 1 *kilometre* be correct to give a result true (i) *to the nearest metre*, (ii) *to the nearest centimetre*?

53. Prove that a decimal of 1 *mile* correct to the 4th place will give a result *true to the nearest foot.*

54. To how many places must a decimal of 1 *acre* be correct in order to give a result *true to the nearest square pole*?

Discarding unnecessary decimal figures, find the value of

55. 4·23784 of 1 metre, in millimetres, *to the nearest mm.*

56. 7·09978 of 1 kilometre, in metres, *to the nearest metre.*

57. 0·017492 of 1 ton, in pounds, *to the nearest pound.*

58. 0·218971 of 1 mile in yards, *to the nearest yard.*

59. 4·72082 acres in square poles *to the nearest unit.*

60. 0·1$\dot{7}\dot{2}$ of 1 mile in yards and feet, *to the nearest foot.*

180. Decimalization of money at sight. The words *to decimalize* in this section will mean *to express decimally in terms of* £1.

The *one-thousandth* part of £1 (or £·001) will be called a **mil.**

Observe that £1 = 960 farthings = 1000 mils;

hence **24** farthings = **25** mils.

181. *To decimalize any number of* **shillings** (less than 20).

Since 1 shilling = £$\frac{1}{20}$ = £·05 ;

∴ 13 shillings = £·05 × 13 = £·65 ;

and 18 shillings = £·05 × 18 = £·90.

Thus to decimalize shillings, *we multiply the number of shillings by* **5**, *and prefix the decimal point.*

182. *To decimalize any number of* **farthings** (less than 48).

Since 24 farthings = 25 mils = £·025 ;

∴ **1** farthing = £·**001**$\frac{1}{24}$;

so that 9 farthings = £·009$\frac{9}{24}$.

Similarly 21 farthings = £·021$\frac{21}{24}$;

and 38 farthings = £·038$\frac{38}{24}$ = £·03**9**$\frac{14}{24}$.

In each case the decimal equivalent may be carried to any number of figures by decimalizing the terminal fractions.

EXAMPLE 1. *Decimalize* 12*s.* 4$\frac{1}{2}$*d.*

Here 12*s.* = £·60

4$\frac{1}{2}$*d.* = 18*f.* = £·018$\frac{18}{24}$;

∴ 12*s.* 4$\frac{1}{2}$*d.* = £·618$\frac{3}{4}$ = £·61875.

This process should be performed *mentally* at one step.

NOTE. By observing whether the terminal fraction is greater or less than $\frac{1}{2}$, the decimal may be at once written down correct to the *third* figure.

For example, £·618$\frac{3}{4}$ = £·619, correct to the 3rd decimal place.

EXAMPLE 2. *Decimalize* 8*s.* 9$\frac{1}{2}$*d.*, *correct to* five *figures.*

Here 8*s.* = £·40

9$\frac{1}{2}$*d.* = 38*f.* = £·038$\frac{38}{24}$ = £·039$\frac{7}{12}$;

∴ 8*s.* 9$\frac{1}{2}$*d.* = £·439$\frac{7}{12}$ = £·439583....

NOTE. We have seen that 1*s.* = £·05 ;

hence 6*d.* = £·025.

Or otherwise : 6*d.* = 24*f.* = 25 mils = £·025.

It is useful to remember this equivalent.

EXAMPLES IX. c. (*Oral.*)

State as decimals of £1

1. 5*s.* **2.** 8*s.* **3.** 17*s.* **4.** 12*s.*
5. 11*s.* 6*d.* **6.** 9*s.* 6*d.* **7.** 16*s.* 6*d.* **8.** 1*s.* 6*d.*

Read off as decimals of £1 as far as thousandths and fractions of a thousandth :

9. $1\frac{1}{4}$*d.* **10.** $2\frac{1}{2}$*d.* **11.** $3\frac{3}{4}$*d.* **12.** $5\frac{1}{4}$*d.*
13. $7\frac{1}{2}$*d.* **14.** $8\frac{1}{4}$*d.* **15.** 10*d.* **16.** $11\frac{1}{2}$*d.*

Decimalize at sight (i) as far as thousandths and fractions of a thousandth ; (ii) correct to the nearest thousandth :

17. £2. 7*s.* 1*d.* **18.** £3. 11*s.* 4*d.* **19.** £7. 14*s.* 5*d.*
20. £4. 8*s.* 7*d.* **21.** £5. 18*s.* $6\frac{1}{2}$*d.* **22.** £18. 0*s.* $9\frac{3}{4}$*d.*

Read off the complete decimal equivalents of

23. £2. 11*s.* 3*d.* **24.** £5. 2*s.* 3*d.* **25.** 14*s.* $1\frac{1}{2}$*d.*
26. £16. 1*s.* $7\frac{1}{2}$*d.* **27.** 3*s.* $10\frac{1}{2}$*d.* **28.** £22. 4*s.* $4\frac{1}{2}$*d.*
29. 9*s.* $2\frac{1}{4}$*d.* **30.** £11. 4*s.* $5\frac{1}{4}$*d.* **31.** 17*s.* $3\frac{3}{4}$*d.*

Decimalize at sight, correct to five decimal places :

32. 4*s.* 1*d.* **33.** £3. 14*s.* 2*d.* **34.** 9*s.* $0\frac{1}{2}$*d.*
35. £1. 3*s.* $3\frac{1}{2}$*d.* **36.** 7*s.* $5\frac{1}{4}$*d.* **37.** 11*s.* $2\frac{1}{2}$*d.*
38. £8. 10*s.* $4\frac{1}{4}$*d.* **39.** 18*s.* $10\frac{1}{4}$*d.* **40.** £3. 14*s.* $2\frac{3}{4}$*d.*
41. 16*s.* $6\frac{1}{4}$*d.* **42.** £2. 14*s.* 8*d.* **43.** 12*s.* $11\frac{1}{4}$*d.*

183. Converse Process. *To read off a decimal of* £1 *as shillings, pence, and farthings.*

We have seen that any decimal of a pound corrected to the 3rd decimal place gives a result within a *mil*, and therefore within a *farthing*, of the true value. [See Art. 178.] Hence for practical purposes we need retain no decimal figure beyond the third, this figure being duly *corrected.*

184. In a decimal of £1 the first two decimal figures taken together denote *hundredths.* Now 1*s.* = **5** *hundredths* of £1 ; therefore the number of *shillings* is found *by dividing the first two decimal figures by* 5.

Thus £·75 $= (\frac{1}{5}$ of 75) shillings = 15*s.*
£·8 $= (\frac{1}{5}$ of **8**0) shillings = 16*s.*
£·371 = 7 shillings + a remainder.

The last example may be written

$$\pounds{\cdot}371 = \pounds{\cdot}35 + \pounds{\cdot}021$$
$$= 7s. + 21 \text{ mils}.$$

If then we treat each mil as roughly equivalent to 1 farthing, we get 21 farthings, or $5\frac{1}{4}d$. This result is evidently a little *too great.*

185. In counting mils as farthings the error increases with the number of mils so treated, at the rate of 1 in every 25 mils or roughly $\frac{1}{2}$ in every twelve mils. Hence a result true to the nearest farthing may be obtained thus:

Count as farthings any number of mils ranging
from **1** to **12**, without correction (the error being less than $\frac{1}{2}f.$);
from **13** to **36**, subtract 1 (the error being between $\frac{1}{2}f.$ and $1\frac{1}{2}f.$);
from **37** to **48**, subtract 2 (the error being between $1\frac{1}{2}f.$ and $2f.$).

Thus £·009 = 9 farthings (no correction) $= 2\frac{1}{4}d.$;
£·032 = 32 farthings (less 1) $= 7\frac{3}{4}d.$;
£·044 = 44 farthings (less 2) $= 10\frac{1}{2}d.$

Example 1. *Evaluate £0·372 to the nearest farthing.*

The mental process is performed thus:

5 into 37 gives 7 (set down 7*s.*); remainder 22 mils.
4 into **21** (22 less 1) gives $5\frac{1}{4}$.
∴ £0·372 = 7*s.* $5\frac{1}{4}d.$, to the nearest farthing.

This process is known as the **Five-and-Four** Rule.

Example 2. *Reduce £6·39683 to £. s. d. and the nearest farthing.*

Here £6·39683 = £6·397 (correct to the 3rd decimal figure).
Now **5** into 39 gives 7, remainder 47 mils.
4 into **45** (47 less 2) gives $11\frac{1}{4}$.
∴ £6·39683 = £6. 7*s.* $11\frac{1}{4}d.$

Example 3. *Find to the nearest penny the value of* $\pounds 4{\cdot}\dot{2}\dot{9}$.

Here $\pounds 4{\cdot}\dot{2}\dot{9}$ = £4·2929... = £4·29**3**, correct to 3rd place,
= £4. 5*s.* 10*d.*, to the nearest penny.

EXAMPLES IX. d.

Read off in £. *s.* *d.* correct to the nearest farthing:

1. £0·475. **2.** £0·825. **3.** £0·307.

4. £0·559. **5.** 0·661 of £1. **6.** £0·460.

7. £0·089. **8.** 0·192 of £1. **9.** 0·497 of £1.

10. £3·098. **11.** £11·111. **12.** £7·509.

13. £4·3. **14.** £4·03. **15.** £4·003.

Evaluate at sight to the nearest penny, by first correcting to the 3rd decimal place:

16. £9·42791. **17.** £0·0299. **18.** £4·82529.

19. £4·63219. **20.** £7·4168. **21.** $\pounds 0{\cdot}91\dot{9}$.

22. $\pounds 2{\cdot}36\dot{1}\dot{8}$. **23.** $0{\cdot}3\dot{7}\dot{1}$ of £1. **24.** $\pounds 2{\cdot}0\dot{9}$.

25. Prove that $\frac{1}{4}d.$ is equivalent to $\pounds{\cdot}001 \times (1 + \frac{1}{24})$.

26. Express the difference between one *farthing* and one *mil* as the decimal of £1 correct to six decimal places.

27. Explain each step in the following work:

$$\begin{aligned} 17s.\ 11\tfrac{1}{2}d. &= 8 \text{ florins} + 3 \text{ sixpences} + 22 \text{ farthings} \\ &= \pounds{\cdot}8 \qquad + \pounds{\cdot}075 + \pounds({\cdot}022 + {\cdot}001 \times \tfrac{22}{24}) \\ &= \pounds{\cdot}898 \text{ to the nearest thousandth.} \end{aligned}$$

28. Find the value of 2·624 of £5 as the decimal of £1; and read off the result in £. *s.* *d.*

29. Express as a decimal $\frac{7}{100}$ of £463; and read off the result in £. *s.* *d.* to the nearest penny.

30. Find 4 per cent. of £632 to the nearest penny.

CHAPTER X.

DECIMAL APPROXIMATIONS. LIMITS OF ERROR. SIGNIFICANT FIGURES. CONTRACTED METHODS.

186. WE have already seen [Art. 149] how to modify a decimal so as to represent its value as correctly as possible with *one*, *two*, *three*, ... decimal figures; in other words to give an approximation true to the nearest *tenth*, *hundredth*, *thousandth*, ... The method may be re-stated thus.

Consider the decimals ·1462... and ·1468..., which both lie between ·14**6** and ·14**7**.

Now ·1462..., being *below* ·146**5** (or $\cdot146\frac{1}{2}$), is

nearer to ·14**6** than to ·14**7**;

while ·1468..., being *above* ·146**5** (or $\cdot146\frac{1}{2}$), is

nearer to ·14**7** than to ·14**6**.

Thus if ·1462... and ·1468... are to be represented as nearly as possible with *three* decimal figures only,

for ·1462... we must take ·14**6**;

for ·1468... we must take ·14**7**:

the error, in excess or defect, being in each case less than **·0005**, or $\mathbf{\cdot000\frac{1}{2}}$, or **$\frac{1}{2}$ of one thousandth.** The decimals thus modified are said to be **correct to the 3rd decimal place,** or **true to the nearest thousandth.**

187. *Conversely*, if the value of a decimal is given *correct to a specified decimal place*, all we know is that its real value lies between two definite limits.

For example, ·14**7** (correct to the 3rd place) may represent *any* decimal whatever, greater than $\cdot146\frac{1}{2}$ and less than $\cdot147\frac{1}{2}$; that is, any decimal between ·1465 and ·1475. In other words, the limits between which the unmodified decimal must lie may be expressed by ·147 ± ·0005. A decimal such as ·147 true to the 3rd place, is sometimes written ·147 ±.

NOTE. The phrases "true *within* one-thousandth" and "true *to the nearest* thousandth" are not identical, the latter being the closer approximation. In the first case the error is less than ± ·001; in the second less than $\pm\cdot000\frac{1}{2}$.

188. Significant Figures. The meaning of the term *significant figures* is best explained by examples.

(i) The approximate distance of the earth from the sun may be given as 93,000,000 miles, meaning 93 **millions** *correct to the nearest* **million.**

Here the unit of measurement may be considered as *one million miles*, and the distance is stated to be 93 such units, *correct to the nearest unit.* In this case the figures **93**, which give the *number* of units, are said to be **significant** ; while the six ciphers, which point out the *magnitude* of the unit, are said to be **non-significant.**

(ii) Again, suppose it is said that the distance from Southampton to Cape Town is 6100 miles.

If this statement is to be understood *in round numbers* as 61 *hundreds*, correct to the nearest *hundred*, then *one hundred miles* is the assumed unit of measurement ; so that the figures **61** are significant, and the two ciphers non-significant.

If however it is implied that a definite course has been measured or calculated to the nearest *mile*, all the figures **6100** become significant.

(iii) It is given that 1 millimetre = 0·039 inch, correct to the third decimal place.

This implies that 1 millimetre = 39 *thousandths* of an inch correct to the nearest *thousandth.* So that here the assumed unit is *one-thousandth of an inch* ; the significant figures are **39** ; and the ciphers at the beginning are non-significant.

(iv) Distinguish between the following approximate equivalents :

1 inch = 25·4 millimetres ;
1 inch = 25·400 millimetres.

The first statement merely gives the equivalent of 1 inch correct to the nearest *tenth* of a millimetre ; the second statement implies definite calculation correct to the nearest *thousandth.* Thus

1 inch = 254 *tenths* of 1 mm., to the nearest *tenth* ;
1 inch = 25400 *thousandths* of 1 mm., to the nearest *thousandth.*

So that in the second approximation the two ciphers on the right are significant.

189. To sum up : **Significant figures** are those which in any approximate result or measurement tell the *number* of units, correct to the nearest such unit. Thus ciphers at the *end* of a whole number or decimal may or may not be significant according to the degree of accuracy implied.

Ciphers are **non-significant** when they merely point out the *magnitude* of the assumed unit, *by fixing the* **place-value** *of the significant figures.* Thus ciphers at the beginning of a decimal are necessarily non-significant.

190. Relative Errors. In all approximate work it matters less whether the actual (or *absolute*) error is large or small, than whether it amounts to *a large or small fraction* of the quantity we are estimating. Suppose, for instance, in judging a distance of 40 yards we are wrong by 4 yards, while in judging a distance of 100 yards we are wrong by 5 yards. The absolute error in the latter case is greater than in the former, but it is really of less importance. For in the second case the error is only at the rate of *one in twenty*, or $\frac{1}{20}$ of the whole distance; while in the first case it is at the rate of *one in ten*, or $\frac{1}{10}$ of the distance to be judged. Errors thus measured as fractions of the true value are called **relative errors.**

Thus $$\textit{relative error} = \frac{\textit{absolute error}}{\textit{true value}}.$$

NOTE. In applying this principle, if (as is usually the case) the *true* value cannot be found, we must use the nearest ascertainable value for the denominator.

191. *Approximate results represented by the* **same significant figures** *are subject to the* **same relative errors.**

For instance:

(i) 24800 denotes **248** *hundreds* to the nearest *hundred.*

(ii) 2·48 .... **248** *hundredths* .... *hundredth.*

(iii) ·000248 .... **248** *millionths* .... *millionth.*

Here the *absolute* errors (in excess or defect) are respectively less than

(i) $\frac{1}{2}$ *hundred,* (ii) $\frac{1}{2}$ *hundredth,* (iii) $\frac{1}{2}$ *millionth.*

So that the *relative* errors are respectively less than

(i) $\frac{\frac{1}{2}\textit{ hundred}}{248\textit{ hundreds}}$, (ii) $\frac{\frac{1}{2}\textit{ hundredth}}{248\textit{ hundredths}}$, (iii) $\frac{\frac{1}{2}\textit{ millionth}}{248\textit{ millionths}}$.

Each of these evidently $=\frac{0·5}{248}$.

192. Percentage Errors. It is often convenient to express *relative errors* as *percentages*, thus exhibiting the absolute error as a percentage of the whole quantity to be represented.

EXAMPLE. *In expressing a length of* 81·472 Km. *as nearly as possible with* three *significant digits, find* (i) *the absolute error,* (ii) *the relative error,* (iii) *the percentage error.*

81·472 Km., corrected to three significant digits = 81·5 Km.

(i) *Absolute error* = (81·5 − 81·472) Km. = 0·028 Km.

$$\text{(ii) } \textit{Relative error} = \frac{0{\cdot}028}{81{\cdot}472} = \frac{28}{81472}, \text{ or } 0{\cdot}00034\ldots.$$

(iii) Let x denote the required rate per cent.

$$\text{Then } \frac{x}{100} = 0{\cdot}00034\ldots;$$

$$\therefore\ x = 0{\cdot}034, \text{ correct to two significant digits.}$$

Thus the *percentage error* = 0·034 % of the whole length.

193. The following method of roughly estimating the accuracy of a result from the number of significant digits is sometimes useful.

Any number consisting of three significant figures is *greater than* 100. Hence a result true to *three significant* digits has a relative error *less than* $\frac{0{\cdot}5}{100}$, and still less than $\frac{1}{100}$.

Thus a result true to *three* significant figures must be true within *one hundredth* of the real value.

Similarly a result true to *four* significant figures must be true within *one-thousandth* of the real value. And so on.

NOTE. It is clear that the actual limit of error may be much narrower than that indicated by this means. The method however shews that the real accuracy of a result is to be judged not so much by the number of *decimal places* to which it is carried, for this merely fixes the *absolute* error; but by the number of *significant figures* known to be correct, for this limits the *relative* error.

EXAMPLES X. a.

(*Examples* 1–37 *should be taken orally.*)

Read off the following numbers correct to the nearest *thousand*; and name the significant figures in your answers.

1. 78912. **2.** 481390. **3.** 369841.

Read off the following decimals correct to the nearest *thousandth*; and name the significant figures in your answers.

4. 2·76428. **5.** 0·012638. **6.** 3·40081.

7. $7{\cdot}4\dot{1}\dot{7}$. **8.** $7{\cdot}4\dot{7}\dot{1}$. **9.** $0{\cdot}\dot{9}0\dot{9}$.

10. 19 ten-thousandths. **11.** 901 millionths. **12.** $9{\cdot}999\frac{5}{8}$.

Read off the following quantities correct to *three significant figures*:

13. 6·2684. **14.** 81·492. **15.** 704·856.

16. 0·68214. **17.** 0·09188. **18.** 0·0040275.

19. $4{\cdot}\dot{7}$. **20.** $8{\cdot}\dot{6}\dot{1}$. **21.** $0{\cdot}0\dot{1}\dot{8}$.

22. 37295. **23.** 70077. **24.** 462130.

Name the significant digits in the following:

25. Liverpool to Quebec, 2630 miles, to the nearest 10 miles.

26. London to Lisbon, 1120 miles, to the nearest mile.

27. Length of Equator, 24,900 miles, to the nearest 100 miles.

28. One cubic inch of water weighs 0·016 Kg., correct to 1 gram.

29. Atmospheric pressure per square inch = weight of 0·760 metre of mercury, correct to 1 millimetre.

30. Illustrate by three instances (one integral and two decimal) how you distinguish *significant* from *non-significant* figures.

The following figures denote quantities correct to the *second decimal place*: between what limits must the actual quantities lie?

31. £3·47. **32.** 41·63 tons. **33.** 3·60 miles. **34.** 0·07 metre.

The following represent quantities *correct to four significant* figures: between what limits must the actual quantities lie?

35. (i) £40·76; (ii) £0·4076; (iii) £4076; (iv) £407600.

36. (i) 9·347 Km.; (ii) 93·47 Hm.; (iii) 9347 metres; (iv) 934700 cm.

37. Sums of money are approximately represented by 74·20 francs, 7420 francs, 7·420 francs, and 74200 francs respectively. Between what limits must the actual sums lie, if the above figures are true (i) to *three* significant digits, (ii) to *four* significant digits?

In expressing the following quantities as nearly as possible with *three* significant digits, find in each case the resulting *absolute* error, the *relative* error, and the *percentage* error.

38. (i) £37·482; (ii) £374·82; (iii) £0·37482.

39. (i) 912·48 miles; (ii) 9·1248 miles; (iii) 0·0091248 of 1 mile.

40. The population of Liverpool in 1905, viz. 723,471.

41. The area of Kent, viz. 989,890 acres.

42. The National Debt in 1903, viz. £770,778,000.

43. The length of a nautical mile, viz. 6087·23 feet.

44. The equivalent of 1 kilogram in grains, viz. 15432·35 grains.

Contracted Addition and Subtraction.

194. In adding or subtracting decimals so as to obtain a result true to a certain number of decimal figures, it is usually enough to retain in each line no more than two decimal figures beyond the number specified.

EXAMPLE. *Find, correct to three decimal places, the sum of* $\frac{11}{12}$, $3\cdot6\dot{2}\dot{4}$, 10·0151923, *and* 832 *millionths.*

$$\begin{array}{rr|l} \frac{11}{12} = & \cdot916 & 66\ldots \\ 3\cdot6\dot{2}\dot{4} = & 3\cdot624 & 24\ldots \\ & 10\cdot015 & 19\ldots \\ 832\text{ millionths} = & \cdot000 & 83\ldots \\ \hline & 14\cdot556 & 92 \end{array}$$

∴ result correct to three places $= \underline{14\cdot557}$.

Here we first draw a vertical line after the third decimal figure to indicate that every figure to the left of this is to be "correct." To secure this, two more figures are retained in each line.

NOTE. No absolute rule can be given to secure the correct carrying figures; the number of extra columns to be retained depending mainly on the number of lines to be added. To add correctly to 3 places, it is generally enough to keep *four* decimal figures, if the 4th figure in each decimal is *corrected* before adding. For the errors so introduced are likely to be some in excess and some in defect, and consequently tend to cancel one another on addition. The sum given above would then stand as in the margin; the sign − being attached when the decimal is *above*, and + when *below* the real value.

$$\begin{array}{r|l} \cdot916 & 7- \\ 3\cdot624 & 2+ \\ 10\cdot015 & 2- \\ \cdot000 & 8+ \\ \hline \underline{14\cdot557} & \end{array}$$

EXAMPLES X. b.

Add together the following, giving results *true to the second decimal figure*:

1. 41·0462, 0·00178, and 21·892.

2. 1·35642, 0·0402, and 4·5019.

3. 5·0819, $1{\cdot}\dot{6}$, and $0{\cdot}0\dot{3}$.

Find the value of the following, giving the results as decimals *true to the nearest thousandth*:

4. 716 millionths $+\frac{17}{2000}+0{\cdot}00008$.

5. $\frac{2}{7}+\frac{5}{6}+3{\cdot}25614$.

6. $0{\cdot}3+\frac{2}{125}+3{\cdot}\dot{6}8\dot{2}+99$ hundred-thousandths.

Express as whole numbers or decimals *correct to three significant digits*:

7. $26{\cdot}0981+306{\cdot}9+0{\cdot}009$.

8. $41{\cdot}682-0{\cdot}1009$.

9. $37{\cdot}261+982{\cdot}8-19{\cdot}9$.

10. $\frac{4}{19}+\frac{6}{17}+\frac{5}{13}$.

Without adding (or writing down) unnecessary figures, find the sum of the following, *correct to the nearest hundred-thousand.*

11. 4,682,531, 806,908, 17,492,714, and 93,602.

12. 971,462, 8,792,031, 92,807, and 17,827,543.

13. The national income for the year 1903-1904 was made up as follows:

Customs and Excise £65,409,431; Property and Income-tax £30,814,016; Estate, House duty, and Land tax £15,659,703; Post and Telegraph Service £28,458,376; Stamps, Fees, Patents, etc. £8,474,371; Crown lands £460,838; Suez Canal Dividends £936,151; Miscellaneous £675,427.

Obtain the total, correct to the nearest £100,000.

14. Find the value of x^2, x^3, and x^4, when $x=0{\cdot}08$.

Hence find, *correct to three decimal places*, the value of

(i) $x+x^2+x^3$; (ii) $x+x^2+x^3+x^4$;

and explain why the results are the same.

15. Express each of the following fractions as a decimal:

$$\frac{1}{5},\ \frac{1}{5^2},\ \frac{1}{5^3},\ \frac{1}{5^4},\ \text{and}\ \frac{1}{5^5};$$

and find their sum *true to the second decimal figure.*

Shew that the exact value of the sum differs from $\frac{1}{4}$ by less than 0·0001.

16. Working in decimals, find, *correct to the nearest* £1, the sum of the following:

£350. 16*s.*, £23·681, £8. 6*s.* 8*d.*, and £$663{\cdot}9\dot{4}$.

17. Find within *one-thousandth of the total distance* the sum of

361·4 Km., 28 Km. 795 m., 0·63 Km., and 1764 metres.

18. Express each of the following quantities as a decimal of £1 *correct to four decimal figures*:

£3. 14*s.* 7*d.*, £$\frac{19}{24}$, and £$12{\cdot}\dot{7}\dot{2}$.

Find their sum; and read off the result *correct to the nearest farthing.*

Explain why correctness to four decimal figures in each line of work is *necessary* and *sufficient* to give the sum to the nearest farthing.

Using decimals, add together the following, and read off the results to the nearest farthing:

19. £2·43, £7·1492, and £$0{\cdot}42\dot{1}$.

20. £5·4862, £3. 12*s.* 6*d.*, and £$\frac{8}{9}$.

Contracted Multiplication of Decimals.

195. The object of this process is to multiply two decimals together so as to get the product true to any specified number of decimal figures without calculating the superfluous figures which would follow if the work were done at full length.

Begin, as recommended in full multiplication, by making *a rough estimate* of the required result, so that any large error, such as might arise from misplacement of the decimal point, may be readily detected.

Next choose as multiplier that decimal which has fewest *effective digits* [see Art. 166, Ex. 2].

Then move the decimal point in the multiplier (if necessary) so as to bring it to *standard form* (*i.e.* to give it *one* integral figure), and make the compensating change in the multiplicand.

The rest of the process will be explained by examples.

EXAMPLE 1. *Multiply* 60·9185 *by* 0·473, *giving the product correct to* two *decimal figures.*

(*Rough Estimate.* The product should be less than 61 × ·5; say, somewhat less than 30.)

Here 60·9185 × 0·473 = 6·09185 × **4**·73.

As the final product is to be true to *two* decimal figures, correctness to *three* decimal figures must be secured in each partial product. On the left we give the work in full, printing in italics those digits which are not required in order to obtain a result true to the 2nd decimal place.

(i) Full Work.

6·09	185
	4·73
24·36	7*40*
4·26	4*295*
·18	*27555*
28·81	4*4505*

(ii) Contracted Work.

6·09	1 ~~85~~
	4·73
24·36	7
4·26	4
·18	3
28·81	4

First step. In (i), on multiplying by 4, the superfluous figures *40* arise from the figures 85 of the multiplicand. Hence in (ii), to get a result true to *two* decimal places, we begin by erasing from the multiplicand all decimal figures *beyond the third* (placing the unit digit of the multiplier under the last figure retained).

We now multiply by the 4, but we carry 3 to the first figure set down, just as we should have done if the fourth decimal figure (8) had not been erased.

Second step. Before multiplying by the 7, *we reject from the multiplicand the last figure remaining on the right* (viz. 1), using it however to supply the carrying figure to the first figure set down.

Third step. Again *reject from the multiplicand the last figure on the right* (viz. 9), and multiply by the 3, carrying as before to the first figure set down. (In this case, since 3 times 9 gives a number nearer 30 than 20, we carry 3 to the first figure.)

Proceeding thus, it is clear that *the first figure set down in each line of work must stand in the 3rd decimal place*, i.e. *immediately to the right of the vertical line*, which has been drawn as before to indicate that all figures to the left of it are to be correct.

We now add the partial products, as explained in Art. 194.

Thus the product = 28·81, correct to two decimal figures.

Note. At each stage care must be taken that the multiplication is begun at the proper figure of the multiplicand. This may be tested by the law explained in Art. 164: viz. that *a figure in the* **m**th *decimal place multiplied by a figure in the* **n**th *decimal place produces a figure in the* (**m** + **n**)th *place.*

For instance in the last example, where each line of work is to have *three* decimal figures:

the 3rd decl fig. in line 1 is due to a **3**rd decl fig. × a *units* fig.

the 3rd decl fig. in line 2 ,, ,, **2**nd decl fig. × a **1**st decl fig.

the 3rd decl fig. in line 3 ,, ,, **1**st decl fig. × a **2**nd decl fig.

Example 2. *Multiply* 0·00926347 *by* 280·435; *the product to be correct to the* nearest thousandth.

(*Rough Estimate.* The product should be a little less than 0·01 of 280, or 2·8.)

Here 0·00926347 × 280·435 = 0·926347 × **2**·80435.

$$\begin{array}{r|l} 0{\cdot}926 & 3\ \cancel{47} \\ & \underline{2{\cdot}80435} \\ \hline 1{\cdot}852 & 7 \\ {\cdot}741 & 0 \\ {\cdot}003 & 7 \\ {\cdot}000 & 3 \\ \hline 2{\cdot}598 & \end{array}$$

Since the product is to be true to *three* decimal figures, retain *four* decimal figures in the new multiplicand.

On reaching the 0 in the multiplier, remember to reject the 6 in the multiplicand; and before multiplying by 4, reject the next figure 2 of the multiplicand.

The process is continued till all the figures in the multiplicand are in turn rejected; but after rejecting the last digit 9, we still multiply it by 3, for this supplies a carrying figure (3) to the fourth decimal column.

Observe that the last figure of the multiplier is not in this case used, since it would contribute nothing to the first four decimal places.

The fourth column is added merely to correct the third. The sum in this case is 17; and this being nearer to 20 than to 10, we carry 2 to the third column.

196. The approximate multiplication of *recurring* decimals introduces no new principle, since the decimals may be extended to as many figures as the conditions of the question require.

For example, if it were required to multiply $0{\cdot}01\dot{2}\dot{8}$ by 236, we should write the product in the form

$$1{\cdot}282828\ldots \times \mathbf{2}{\cdot}36,$$

and then proceed as before.

EXAMPLES X. c.

Find the following products, correct to *two decimal figures*:

1. 5·62 × 3·27.
2. 0·7028 × 4·73.
3. 9·4357 × 5·25.
4. 0·03175 × 4·116.
5. 50·281 × 0·317.
6. 702·84 × 0·0541.
7. 78·95 × 0·0909.
8. 23·76 × 0·35782.
9. 86·43 × 0·028947.

Find, correct to the *nearest unit*, the product of:

10. 46·207 and 8·136.
11. 709·28 and 0·0428.
12. 0·09602 and 86·31.
13. 4004·04 and 0·0606.

Multiply out the following, correct to the *third decimal place*:

14. 0·00561023 × 597·001.
15. $(1{\cdot}41421)^2$.
16. 42·37 × 0·240089.
17. $(1{\cdot}73205)^2$.

Find the value, correct to *three significant digits*, of:

18. 80·6251 × 0·6317.

19. 0·0020476 × 24·06.

20. 41·0208 × 0·7305.

21. 5961·59 × 0·030807.

Multiply out the following, correct *to the nearest thousandth*:

22. $7{\cdot}8\dot{6} \times 0{\cdot}421$.

23. $0{\cdot}5\dot{7}\dot{3} \times 3{\cdot}052$.

24. $0{\cdot}\dot{8}\dot{2} \times 5{\cdot}26\dot{3}$.

25. $0{\cdot}05\dot{6}3\dot{2} \times 32{\cdot}\dot{7}$.

Find the following products within *one thousandth of their true value*:

26. 760·35 × 0·00580079.

27. 3·1416 × 0·31831.

28. Multiply 79364 by 6024, giving the product correct only *to the nearest million.*

[The number of millions in 79364 × 6024

$$= \frac{79364 \times 6024}{1{,}000{,}000} = 79{\cdot}364 \times 6{\cdot}024;$$

a result to be worked out *to the nearest unit.*]

Find approximately the following products:

29. 4085 × 746, correct to the nearest *thousand.*

30. 4528 × 6402, to the nearest *hundred-thousand.*

31. 80460 × 5073, to the nearest *million.*

32. 5807023 × 43·07, to the nearest *million.*

33. Given that 1 metre = 39·3701 inches, find, correct to the *nearest unit*, the number of inches in 7·52 metres.

34. Express $14\frac{3}{4}$ yards in metric measure, having given that 1 inch = 25·400 millimetres. [A result true *to the nearest centimetre* is sufficient.]

35. A train runs at an average rate of $45\frac{1}{4}$ miles an hour. Find in kilometres the distance run in 5 hours; having given 1 mile = 1·6093 Km. [A result true only *to the nearest kilometre* is wanted.]

36. If a pressure expressed in *pounds per square inch* is multiplied by the constant 0·07031, the result expresses the same pressure in *kilograms per square centimetre.* Use this equivalent to express a boiler pressure of 222·6 lbs. per sq. in. in terms of kilograms per sq. cm. [The result is to be true to the nearest hundredth.]

37. The railways of the United Kingdom had 22,435 miles open for traffic in 1904. Estimating their traffic receipts at £4595 per mile, find the total gross earnings for the year to the nearest £100,000.

38. The out-put from the Transvaal mines for the year 1903 was 2,972,897 ounces of fine gold, valued at £4. 4*s.* $11\frac{1}{4}$*d.* per ounce. Find the total value to the nearest £100,000. [Decimalize £4. 4*s.* $11\frac{1}{4}$*d.*, and work by contracted multiplication.]

Contracted Division.

197. Our object is to shorten the ordinary process of Division by discarding all figures which become superfluous when only some specified number of digits is required in the quotient.

Example 1. *Divide* 7944·80625 *by* 320·6402, *as far as the* second decimal figure *of the quotient.*

Begin, as in full division, by bringing the *divisor* to standard form, *i.e.* move the decimal point so as to give *one* integral figure; and make the compensating change in the dividend.

Thus $$\frac{7944{\cdot}80625}{320{\cdot}6402}=\frac{79{\cdot}4480625}{\mathbf{3}{\cdot}206402}.$$

Next determine by inspection how many *integral* figures there will be in the quotient.

Here the dividend is slightly over 79, and the divisor lies between 3 and 4; therefore the quotient will contain *two* integral figures. These, together with the two *decimal* figures, make *four* significant figures to be obtained by division.

We give below the full work, printing in italics all figures not required to obtain the first four digits of the quotient.

Full Work.	Contracted Work.
24·77... = *Quotient*	24·77... = *Quotient.*
3·2064*02*) 79·448 | *06245*	3·2ˎ0ˎ6ˎ4) 79·448~~06245~~
64 128 | *04*	64 128
15 320 | *022*	15 320
12 825 | *608*	12 826
2 494 | *4144*	2 494
2 244 | *4814*	2 244
249 | *93305*	250
224 | *44814*	224
25 | *48491*	26

To perform the contracted work we retain in the divisor *five* figures, that is to say *one more than the number of significant figures required in the quotient*; and in the dividend we keep as many figures as are needed to take the first step of division: in this case also *five.*

We then proceed with the division in the ordinary way, except that, at each stage, instead of bringing down a new figure from the dividend, *we reject a figure from the right of the divisor*; taking care however on multiplying, to use the figure last rejected for the purpose of obtaining a carrying number.

Thus, after rejecting *three* figures of the divisor, we find the first *four* figures of the quotient to be 24·77.

NOTE. Two figures remain in the divisor, so that the work may be carried one step further. Marking off the 2, and dividing as before, we get an extra figure, viz. 8, for the quotient. Now though this cannot be relied upon as the actual fifth figure, it is near enough to enable us to correct the fourth.

Hence the quotient *correct to the 2nd decimal figure* is 24·78.

EXAMPLE 2. *Divide* 2·6289475 *by* 306·5, *giving the first* seven *decimal figures of the quotient.*

Here $$\frac{2\cdot6289475}{306\cdot5}=\frac{0\cdot026289475}{\mathbf{3}\cdot065};$$

and we see at once that in the quotient *two ciphers* will follow the decimal point. Hence *five* figures must be got by division. This requires in the divisor (according to the rule already stated) *six* figures; a difficulty which may be met thus:

(i) by affixing two ciphers, to be rejected successively as already explained; or

(ii) by starting with the original divisor, and postponing the rejection of figures from the divisor until the number of figures still to be found in the quotient is one less than the number of figures in the divisor: that is, in this case we obtain *two* figures by ordinary division and *three* by contraction.

Both methods are given below. In (ii) the work is further shortened by the "Italian" arrangement.

(i)

$$\begin{array}{rrl}
 & 0\cdot0085773\ldots & =\textit{Quotient.}\\
3\cdot0,6,5,0,0\,) & 0\cdot02628947\cancel{5} & \\
 & \underline{2452000} & \\
 & 176947 & \\
 & \underline{153250} & \\
 & 23697 & \\
 & \underline{21455} & \\
 & 2242 & \\
 & \underline{2146} & \\
 & 96 & \\
 & \underline{92} & \\
 & 4 &
\end{array}$$

(ii)

$$\begin{array}{rrl}
 & 0\cdot0085773\ldots & =\textit{Quotient.}\\
3\cdot0,6,5\,) & 0\cdot02628947\cancel{5} & \\
 & \underline{17694} & \\
 & \underline{23697} & \\
 & \underline{2242} & \\
 & \underline{96} & \\
 & 4 &
\end{array}$$

NOTE. Carrying the division one more step, the 8th figure of the quotient appears to be 1; from which we may safely conclude that the real 8th figure is less than 5. Hence the quotient *true to the seventh decimal figure* is 0·0085773.

198. If the dividend or divisor (or both) *recur*, the quotient may be obtained to any specified number of figures by extending the data to as many digits as the conditions of the question require.

EXAMPLES X. d.

Obtain the following quotients true to *three* significant figures in each case.

1. $8{\cdot}6439 \div 1{\cdot}057$.
2. $759{\cdot}56 \div 43{\cdot}01$.
3. $2{\cdot}71835 \div 8{\cdot}2121$.
4. $6{\cdot}52742 \div 218{\cdot}42$.

Find the following quotients correct to the *second* decimal figure :

5. $282{\cdot}10051 \div 29{\cdot}332$.
6. $1{\cdot}576343 \div 0{\cdot}029364$.
7. $11{\cdot}32042 \div 24{\cdot}0604$.
8. $6{\cdot}955714 \div 0{\cdot}08425$.

Find the value of the following, giving results correct to the *third* decimal place :

9. $1481{\cdot}88 \div 435{\cdot}07$.
10. $0{\cdot}043524 \div 0{\cdot}021979$.
11. $\dfrac{1{\cdot}4105}{58{\cdot}664}$.
12. $\frac{1}{8043}$ of $416{\cdot}247$.
13. $17{\cdot}3 \div 13{\cdot}408$.
14. $11 \div 3{\cdot}31662$.
15. $\dfrac{1}{3{\cdot}14159}$.
16. $\dfrac{3}{1{\cdot}73205}$.
17. $8{\cdot}\dot{2}\dot{6} \div 3{\cdot}1762$.
18. $1{\cdot}92\dot{7} \div 3{\cdot}\dot{0}\dot{8}$.

Decimalize the following fractions, using contracted division and giving the results true to three significant digits :

19. $\frac{897}{9135}$.
20. $\frac{73984}{2971}$.
21. $\frac{4381}{308017}$.

From the following equations determine the value of x correct in each case to the nearest unit :

22. $x \times 2{\cdot}03 = 1830{\cdot}26$.
23. $7309x = 8276341$.
24. $0{\cdot}0318973 \times x = 35{\cdot}78364$.

Obtain by contracted division approximate quotients of the following :

25. $346{,}423{,}521 \div 7816$, correct *to the nearest thousand.*
26. $905{,}982{,}715 \div 21638$, correct *to three significant digits.*
27. $65{,}481{,}275 \div 8{,}063{,}742$, correct *to the 3rd decimal figure.*

28. The velocity of sound in air is 1142 feet per second ; and on a still day at sea the discharge of a big gun may be heard 20 miles off. How long does it take the report to travel this distance ? Answer only to the nearest second.

29. The mean circumference of the Earth is 131,385,456 feet. The whole circumference contains 360 × 60 minutes of arc. A knot is the length of one minute of arc. Find the length of 1 knot in feet to the nearest tenth.

30. A company's profits of £56,248 are allotted equally to 12,481 shares. How much is received on each share?

[Obtain a result by contracted division correct to the 3rd decimal place, and read off the result to the nearest penny. Art. 185.]

31. Decimalize £12. 15*s.* 6*d.* If the equivalent of this sum in French money is 322 fr. 31 c., find the equivalent of £1, correct to the nearest centime.

32. From the following statistics at the last census, find the average number of persons per square mile in England and Wales, in Scotland, and in Ireland respectively.

	England and Wales.	Scotland.	Ireland.
Area in sq. mi. -	57,909	29,796	31,759
Population - -	32,527,843	4,472,103	4,458,775

33. One statute mile = 5280 feet, and 1 knot = 6082·66 feet.

Find (i) 1 knot in terms of a mile, to the nearest thousandth.

(ii) 1 mile in terms of a knot, ,, ,, ,,

34. Given that 1 mètre = 39·3701... inches, express

(i) 1 inch in metres, and hence in millimetres, correct to five significant figures.

(ii) 1 chain in metres, to the nearest cm.

(iii) 1 mile in kilometres, to the nearest metre.

35. Prove by contracted division that 1·41421 is approximately the square root of 2.

36. A Canadian Land Company reports the sale of 351,276 acres for $2,429,620. Find the average price per acre to the nearest cent: also the equivalent price in British money, given that $1 = 4*s.* 1½*d.*

37. One gallon of water weighs 10 lbs. One cubic foot of water weighs 62·321 lbs. From these data find the number of cubic inches in a gallon correct to 0·1 cu. in.

38. Find to the nearest penny the British equivalent of 10,000 francs, when the value of £1 in exchange on Paris is quoted at 25 fr. 18½ c.

39. The following statistics refer to the Railways of the United Kingdom for the year 1904:

Miles open for Traffic.	Total Traffic receipts.	Working Expenses.
22,435.	£103,079,191.	£68,561,855.

Find to the nearest £1 the *net* receipts per mile.

40. Find to the nearest penny the liability per head in each of the following cases:

	Population at last census.	National Debt.
United Kingdom -	41,976,827	£759,135,471
France - - - -	38,961,945	£1,172,368,000
German Empire -	56,367,178	£143,799,000

Further Examples of Contracted Work.

199. When two or more contracted operations are to be performed in succession, as in finding the value of

(i) $124{\cdot}9732 \times 81{\cdot}02 \times 0{\cdot}0516$;

(ii) $61{\cdot}924 \times 0{\cdot}07046 \div 401{\cdot}235$;

begin by shifting the points so as to get an equivalent expression in which each *operating* decimal is in *standard form* (that is, has one integral figure). We may thus ascertain to how many decimal places the first stage must be carried in order that the final result may have the required degree of accuracy.

EXAMPLE 1. *Find the continued product of* 124·9732, 81·02, *and* 0·0516, *correct to* two *decimal figures.*

(*Rough Estimate.* $125 \times 80 \times \frac{5}{100}$, or 500.)

The required product $= 12{\cdot}49732 \times \mathbf{8}{\cdot}102 \times \mathbf{5}{\cdot}16$.

12·49	$73\cancel{2}$
	8·102
99·97	86
1·24	97
2	50
101·25	33
	5·16
506·26	65
10·12	53
6·07	52
522·47	

Here the multipliers slightly exceed 8 and 5; hence the first decimal is eventually to be multiplied by a number between 10 and 100.

We therefore keep in the multiplicand *two* decimal figures beyond the number required in the final result; viz. *four* decimal figures in all.

EXAMPLE 2. *Find the value of* $\dfrac{61{\cdot}924 \times 0{\cdot}07046}{401{\cdot}235}$, *correct to* four *decimal figures.*

$\left(\textit{Rough Estimate.} \quad \dfrac{62 \times 0{\cdot}07}{400}, \text{ or } \dfrac{4{\cdot}34}{400}, \text{ or } 0{\cdot}0108. \right)$

The given expression $= \dfrac{0{\cdot}0061924 \times \mathbf{7}{\cdot}046}{\mathbf{4}{\cdot}01235}$.

$$\begin{array}{r|l} 0{\cdot}0061 & 9\cancel{24} \\ & \underline{7{\cdot}046} \\ \hline 0{\cdot}0433 & 4 \\ 2 & 4 \\ & 4 \\ \hline \end{array}$$

$$\begin{array}{rr} 4{\cdot}0_{\backslash}1_{\backslash}\cancel{235}\;) & \underline{0{\cdot}0436\;2}\;(\;0{\cdot}0108\ldots \\ & \underline{35\;0}\phantom{\;(\;0{\cdot}0108\ldots} \\ & 3\;0\phantom{\;(\;0{\cdot}0108\ldots} \end{array}$$

Here the combined processes of multiplication and division are roughly equivalent to multiplication by $\frac{7}{4}$. Hence to obtain a final result true to *four* decimal figures, we retain *five* decimal figures in the multiplicand. The fifth decimal figure in the quotient being clearly greater than 5, the final result true to four decimal places is 0·0109.

Miscellaneous Examples on Contracted Work.

[*The harder of these examples may be left till the pupil has learned the use of Logarithms. It is intended that some examples should be worked by Contracted Decimal processes, and others by Logarithmic Tables. In some instances it will be instructive to use both methods for the sake of comparison.*]

EXAMPLES X. e.

Find the following products *correct to the second decimal figure* :

1. $0{\cdot}2394 \times 2{\cdot}31 \times 1{\cdot}57$.

2. $18{\cdot}602 \times 0{\cdot}1457 \times 1{\cdot}356$.

3. $36{\cdot}73 \times 0{\cdot}073 \times 0{\cdot}2367$.

4. $242{\cdot}3 \times 0{\cdot}01872 \times 0{\cdot}9713$.

Find the value of the following expressions *correct to the nearest hundredth* :

5. $\dfrac{21{\cdot}434 \times 3{\cdot}721}{8{\cdot}532}$.

6. $\dfrac{44{\cdot}852 \times 14{\cdot}73}{98{\cdot}153}$.

7. $24{\cdot}368 \times \dfrac{0{\cdot}0394}{0{\cdot}897}$.

8. $\dfrac{0{\cdot}7859}{4{\cdot}628}$ of $3{\cdot}0103$.

From the following equations find the value of x *correct to the nearest unit* :

9. $\dfrac{x}{98{\cdot}0035} = 0{\cdot}42074 \times 5{\cdot}312$.

10. $\dfrac{x}{1{\cdot}0398 \times 10{\cdot}326} = 65{\cdot}471$.

11. $x \times 9{\cdot}47 = 1{\cdot}01359 \times 3214$.

12. $\dfrac{48{\cdot}3}{39{\cdot}01} \times x = 238$.

Find correct to four significant digits :

13. $\dfrac{69{\cdot}7324}{5{\cdot}7002 \times 6{\cdot}08175}$. 14. $\dfrac{348}{502}$ of 19 millions.

15. A sovereign weighs 123·27 grains of standard gold, and a Japanese 20 yen gold piece weighs 257·21 grains. Find the value in British money (correct to the nearest penny) of the 20 yen piece.

16. If 10 dollars are worth £2. 1*s.* $1\frac{1}{2}$*d.*, find the equivalent of £1 in dollars. [Decimalize £2. 1*s.* $1\frac{1}{2}$*d.*, and obtain a final result true to *four* decimal figures.]

17. A merchant in New York buys goods in Geneva to the value of 4004 francs. Find the value of the goods in dollars, when the exchange between London and New York is 4·8632 dollars to £1; and the exchange between London and Geneva is 25·21 francs to £1. [Give your result true to the 2nd decimal figure, *i.e.* the nearest cent.]

18. One gallon of water weighs 10 lbs. One litre of water weighs 1 kilogram. One kilogram = 2·2046 lbs. *nearly.* Find the equivalent of 1 gallon in terms of litres. To how many figures do you consider the result is to be trusted ?

Find the following powers within *one-thousandth* of the true value :

19. $(1{\cdot}41421)^3$. 20. $(1{\cdot}26001)^3$.

21. If $a = 0{\cdot}00731$, find the values of a^2 and a^3, correct to *five* decimal places.

Shew that within this degree of accuracy $1 + a + a^2$ and $1 + a + a^2 + a^3$ yield the same result.

[Observe that if a denotes any decimal less than ·01 or $\frac{1}{100}$, then a^2 is less than ·0001 or $\frac{1}{10,000}$; and a^3 is less than ·000001 or $\frac{1}{1,000,000}$.]

22. If $a = 0{\cdot}00316$, and $b = 0{\cdot}00082$, find the value of

(i) $a^2 + ab + b^2$, to the nearest *millionth.*

(ii) $\dfrac{ab}{a+b}$, with an error less than $\dfrac{1}{10^4}$.

Find, to the nearest unit, the value of

23. $122{\cdot}735 \times 510{\cdot}34 \times \dfrac{111}{10^4}$. 24. $\dfrac{23{\cdot}789 \times 16{\cdot}8954}{0{\cdot}0055 \times 124{\cdot}91}$.

25. Having given 1 metre = 3·2808 feet, find to the nearest hundredth the number of square feet in one square metre.

26. An Are is the area of a square on a side of 10 metres; and 1 metre = 1·0936... yards. Express 1 Are in terms of square yards correct to the nearest hundredth.

27. Find, correct to the second decimal place, the number of acres in a hectare, using the following data:

1 Ha. = $(100)^2$ sq. metres; 1 acre = 4840 sq. yds.; 1 metre = 1·0936 yds.

28. Find the value of 1 cubic inch in terms of cubic centimetres, having given 1 inch = 25·400 millimetres. The result should be obtained to five significant figures only.

29. Express a speed of 1 *mile an hour* in terms of *feet per second.* Hence find a constant decimal multiplier (correct to four decimal figures) which will convert a speed given in *miles per hour* into *feet per second.* Illustrate by an example.

30. The work done by a force in raising 1 pound through 1 foot is called a *foot-pound*; and that done in raising 1 kilogram through 1 metre is called a *kilogram-metre.*

Express 1 kilogram-metre in terms of foot-pounds correct to four significant digits; and hence name a constant decimal multiplier which will convert work expressed in kilogram-metres into foot-pounds.

Given 1 metre = 1·0936 yards;
1 kilogram = 2·2046 pounds.

Approximate Data. Limits of Errors.

200. Addition and Subtraction. If the decimals whose sum or difference is required are known to be only *approximate*, care must be taken not to attempt a greater degree of accuracy in the result than is justified by the data.

EXAMPLE. *The sides of a triangle are known to measure* 5·37 in., 4·62 in., *and* 6·78 in., *true to the nearest* hundredth. *Between what limits must the perimeter lie?*

Here the 1st side must lie between 5·365″ and 5·375″;
... 2nd side .... 4·615″ and 4·625″;
... 3rd side .... 6·775″ and 6·785″;
∴ the perimeter .... 16·755″ and 16·785″;

Or the work might be arranged thus: taking the positive signs to give the upper and the negative signs to give the lower limits.

5·37 ± ·005
4·62 ± ·005
6·78 ± ·005
∴ limits of perimeter are 16·77 ± ·015

That is, the perimeter = 16·77″, with a possible error (in excess or defect) not exceeding ·015. Hence we see that if the given lengths are added as they stand the last decimal figure is not to be trusted.

201. In taking the *difference* between two approximate quantities the maximum result is obtained when from the *greatest* possible value of the 1st quantity we subtract the *least* possible value of the 2nd quantity. While the *minimum* is got when from the *least* value of the 1st we take the *greatest* value of the 2nd.

(*Examples X. f.* 1–11 *may be taken here.*)

202. Multiplication. In cases of multiplication where one or both of the given factors must be regarded as approximate, it is most important to ascertain that the degree of accuracy attempted in the product is not greater than the data justify. Some guiding principles are given below.

When an approximate decimal is multiplied by any number greater than unity, the error is *increased* in the product.

For example: when an approximate decimal, given correct to a specified number of places, is multiplied by 10, 100, 1000, ..., the number of places to which the product is correct is reduced by *one*, by *two*, by *three*, ... respectively.

Thus if 60·31 represents a length in metres, known to be true only to the nearest *hundredth* (or centimetre), then

60·31 metres × 10 = 603·1 m., true to the nearest *tenth* or *dm.*

60·31 metres × 100 = 6031 m., true to the nearest *unit* or *metre.*

203. Hence if an approximate decimal (correct for instance to the nearest hundredth) is multiplied by any *definite* number, integral or decimal, between 1 and 10, the product may be trusted to *one* decimal place; and if multiplied by any such number between 10 and 100, the product may be trusted to the *nearest unit.*

This however is not necessarily true if the multiplier is itself *approximate.* The following example shews the distinction.

EXAMPLE. *Multiply* 60·31 (*true to the* 2nd *decimal place*) *by* 3·14, *regarding the multiplier* (i) *as* definite, (ii) *as* true to the 2nd decimal figure.

In each case we give the multiplication in full, underlining approximate or doubtful figures.

(i)
$$\begin{array}{r} 60{\cdot}3\underline{1} \\ 3{\cdot}14 \\ \hline 180{\cdot}9\underline{3} \\ 6{\cdot}03\underline{1} \\ 2{\cdot}412\underline{4} \\ \hline 189{\cdot}3\underline{7}\ldots \end{array}$$

(ii)
$$\begin{array}{r} 60{\cdot}3\underline{1} \\ 3{\cdot}1\underline{4} \\ \hline 180{\cdot}9\underline{3} \\ 6{\cdot}03\underline{1} \\ \underline{2}{\cdot}\underline{4}\underline{1}\underline{2}\underline{4} \\ \hline 18\underline{9}{\cdot}\underline{3}\underline{7}\ldots \end{array}$$

In (i) the doubtful figures arise only from the approximate figure $\underline{1}$ in the multiplicand.

In (ii) the last figure $\underline{4}$ of the multiplier is approximate, so that the last partial product is doubtful *throughout.*

Thus (i) yields the result 189·4, true to the nearest *tenth.*

(ii) yields .... 189· , of which the last figure, though possibly correct, is open to doubt.

NOTES. 1. The distinction may be further explained by observing that in (i) the main error is roughly 3 times the error in the *multiplicand*; while in (ii) we have in addition 60 times the error in the approximate *multiplier.*

2. From an examination of the above results it may be concluded that *the product of two approximate quantities can never be correct to more* (though possibly to fewer) *significant figures than there are in the less correct of the factors.*

204. By thus underlining, as they arise in the work, any doubtful figures due to the approximate figures of the given factors, we may always tell roughly how far the product is to be trusted. This is enough in practice to prevent any serious misconception as to the degree of accuracy obtainable. If however the actual limits of error are required, they may be found thus.

EXAMPLE 1. *Find approximately the limits of error in the product of* 60·31 *and* 3·14, *each factor being true to the* 2nd *decimal figure.*

Here $60{\cdot}3\underline{1} \times 3{\cdot}1\underline{4} = 18\underline{9}{\cdot}\underline{4}$, the underlined figures in the product being doubtful. [Art. 203, Ex.]

Now, by Art. 187, the real value of the required product may be anything between

$$60{\cdot}315 \times 3{\cdot}145, \text{ or nearly } 189{\cdot}7,$$
$$\text{and } 60{\cdot}305 \times 3{\cdot}135, \text{ or nearly } 189{\cdot}1.$$

That is, the result 189·4 is liable to an error of about ± ·3.

NOTE. The limits of error may be approximately obtained without actually performing the multiplication of the greatest and least values.

Take the upper values, and in multiplying write down *only those figures which arise from the terminal fives.* We thus get a possible correction of about + ·32; and we infer a possible correction of about − ·32 for the lower limit.

$$\begin{array}{r|l} 60{\cdot}31 & \mathbf{5} \\ & 3{\cdot}14\mathbf{5} \\ \hline {\cdot}01 & 5 \\ {\cdot}30 & \underline{2} \\ \hline {\cdot}32 & \end{array}$$

Example 2. *Given that the equatorial diameter of the earth is* 7926 *miles* (*correct only to the nearest mile*) *and that* the circumference of a circle = diameter × π, *where* $\pi = 3{\cdot}1416$ (*correct to four decimal figures*); *find the length of the equator as nearly only as the data justify.*

Since the units' figure of the multiplicand is approximate, it is useless to carry the work beyond that figure.

Underlining as doubtful those digits which arise from approximate figures in the data, we see that the equator is nearly equal to 24900 miles, *the units' figure being open to doubt.*

$$\begin{array}{r@{}l}
792\underline{6} & \\
3 & \cdot 141\underline{6} \\
\hline
2377\underline{8} & \\
792 & \cdot \underline{6} \\
317 & \cdot 0 \\
7 & \cdot 9 \\
\underline{4} & \cdot \underline{7} \\
\hline
2490\underline{0} & \cdot
\end{array}$$

By the method given in the Note to Example 1, it may be shewn that (if nothing more is known as to the values of the diameter and π than the figures given above) the result is liable to an error not exceeding ±2 miles (nearly).

(*Examples X. f.* 12–26 *may be taken here.*)

205. Division. When an approximate decimal is divided by any number greater than unity, the error is *diminished* in the quotient.

For example: when an approximate decimal, given correct to a specified number of figures, is divided by 10, 100, 1000, ..., the number of decimal figures to which the quotient is correct is increased by *one*, by *two*, by *three*, ..., respectively.

Thus if 6031 metres represent a length true only to the nearest unit, then

6031 metres ÷ 10 = 603·1 metres, to the nearest *tenth*, or *dm.*

6031 metres ÷ 100 = 60·31 metres, to the nearest *hundredth*, or *cm.*

Hence 6031 metres (correct only to the nearest metre) when divided by any *definite* number, integral or decimal, between 10 and 100 gives a quotient true to *one* decimal place: and so on.

If the *divisor* is also approximate, sufficient information as to the degree of accuracy obtainable is usually to be got by underlining as doubtful all figures in the work which arise from the approximate figures in the data.

EXAMPLE. *Divide* 413·8 *by* 7·03, *each number being only true to the last significant digit.*

Here the quotient obviously becomes doubtful at the 3rd significant figure (resulting from the partial dividend $6\underline{0}\underline{6}0$), and it would be useless therefore to continue the work beyond that point.

$$\begin{array}{r@{\,}l} & \overline{58{\cdot}8}\ldots = \textit{Quotient.} \\ 7{\cdot}03\,) & 413{\cdot}8\underline{0}\,\underline{0}\ldots \\ & 351\ \underline{5} \\ \cline{2-2} & 62\ \underline{3}\,\underline{0} \\ & 56\ 2\underline{4} \\ \cline{2-2} & 6\ \underline{0}\,\underline{6}\,\underline{0} \\ & 5\ 62\underline{4} \\ \cline{2-2} \end{array}$$

NOTE. The upper limit of the quotient is got by giving to the *dividend* its *greatest* possible value, and to the *divisor* its *least* value.

That is, upper limit $= \dfrac{413{\cdot}85}{7{\cdot}025} = 58{\cdot}91\ldots.$

Similarly, lower limit $= \dfrac{413{\cdot}75}{7{\cdot}035} = 58{\cdot}81.$

(*Examples X. f.* 27–35 *may be taken here.*)

EXAMPLES X. f.

(*Addition and Subtraction.*)

1. The sides of a triangle are approximately 5·42″, 4·06″, and 3·60″; each measurement being correct to the nearest hundredth of an inch. Between what limits must the perimeter lie?

How could the perimeter be most nearly represented with *three significant figures?*

2. In a round voyage a steamer makes four passages of 2030 miles, 1450 miles, 3380 miles, and 940 miles respectively, the length of each passage being given only *to the nearest ten miles.* Within what limits of error is it possible to ascertain the total length of the voyage?

3. Add together 6·07 m., 8·36 m., and 7·40 m., each length being correct to the nearest cm., and name the greatest possible error in excess or defect.

4. Find the greatest and least possible value of

$$16{\cdot}037 + 0{\cdot}973 + 2{\cdot}001,$$

if each decimal is liable to an error not exceeding *one-thousandth.*

5. Add together 4·016, 0·101, 2·892, and 3·000. If each decimal is correct to the 3rd place, to how many decimal places may the sum be trusted?

6. From a line 7·63″ long, a part 5·06″ in length is cut off, each measurement being true to the *nearest hundredth.* Between what limits must the remainder lie?

7. Each of the following measurements can be relied upon as correct to *three* significant digits only. Find their sum as correctly as the data justify.

42·300 metres, 5·260 metres, and 0·942 metre.

Simplify the following, stating between what limits the results must lie, if each decimal is correct to the 3rd decimal figure.

8. 4·680 – 2·073.

9. 18·009 – 11·900.

10. 2·308 + 3·002 – 5·304.

11. 7·000 – 2·220 – 2·200.

(*Multiplication.*)

12. Along a straight line I set off ten lengths, each of 4·7 cm. correct only to the nearest millimetre. If I consider the total length measured as 47 cm., to what error is my estimate liable?

13. The standard weight of a brick is 6·8 lbs. correct to the nearest *tenth* of 1 lb. Within what degree of accuracy can you estimate the weight of

(i) 100 bricks? (ii) 1000 bricks? (iii) 721 bricks?

14. Between what limits must the product 4·7 × 6·3 lie,

(i) if each factor is true only to the nearest *tenth*;

(ii) if each factor is liable to an error of ±0·1?

15. Between what limits must the following rectangular areas lie, the length and breadth being known only approximately, as follows:

(i) Length = 5·3 cm., breadth = 4·2 cm., correct *to nearest millimetre*?

(ii) Length = 6·0 cm., breadth = 5·0 cm., ,, ,, ,,

(iii) Length = 29 feet, breadth = 21 feet, correct within ±1 ft.?

(iv) Length = 510 yds, breadth = 440 yards, each dimension being liable to an error of ±10 yards?

Find the following products, not attempting greater accuracy than the data justify:

16. 5·8 × 3·7, each factor true *to the nearest tenth.*

17. 7·32 × 6·04, each factor true *to the nearest hundredth.*

18. 50·2 × 4·08, each factor true *to 3 significant digits.*

19. 0·0617 × 33·5, each factor true *to 3 significant digits.*

20. 4·26 × 0·508, the 1st factor true within ±·01, the 2nd factor within ±·001.

21. From the formula *circumference* = *diameter* × π, where $\pi = 3{\cdot}1416$, *correct to four decimal figures*, find the circumference of a circle 431 feet in diameter. To what degree of accuracy can the result be given?

22. Find the total atmospheric pressure on a sphere 10 inches in diameter, having given that the surface of such a sphere contains 314·16 sq. inches, and that the pressure of the atmosphere is 14·707 lbs. per sq. inch.

Considering both data as only approximate (correct to the last figure given) how far can the result be trusted?

23. Find the total weight of 1000 spherical leaden bullets each 1 inch in diameter, not attempting greater accuracy than the following appróximate data justify. Each bullet contains 0·524 cu. inch, and the weight of 1 cu. inch of lead is 0·408 lb.

24. Find the greatest and least possible weight of an oak beam containing 15 cubic feet, from the following *approximate* data. One cubic foot of water weighs 62·4 lbs. Oak weighs 0·85 of its bulk of water.

25. Find as nearly as possible the cost of making a road from the following rough data: estimated length of road 80 miles; estimated cost per mile £350. The estimate of length is liable to a 5 per cent. error in excess or defect, and the estimate of cost to a 10 per cent. error.

26. Find the product of 82·7 and 0·06435, where each factor is true only to the last significant figure.

Use this example to illustrate the following principle:

> "*The product of two approximate quantities can never be correct to more* (though possibly to fewer) *significant figures than there are in the less correct of the factors.*"

[Take as multiplicand the less correct factor, i.e. that with fewer significant digits; and draw your conclusion from the *first partial product.*]

(*Division.*)

27. A given length is represented by 754 metres, correct only to the nearest *metre*. To what units respectively may the following lengths be assumed correct?

(i) 754 m. × 100. (ii) 754 m. ÷ 100.
(iii) 754 m. × 1000. (iv) 754 m. ÷ 1000.

To how many *significant digits* is each of these lengths correct? Shew that while the *absolute* error is changed by multiplication or division, the *relative* error is unchanged.

28. Read off £3. 14*s*. $2\frac{3}{4}$*d.* as the decimal of £1 to six decimal figures. In each of the following cases determine how many decimal figures must be used to obtain a result true to the nearest farthing:

When £3. 14*s*. $2\frac{3}{4}$*d.* is (i) *divided* by 10; (ii) *multiplied* by 100; (iii) *divided* by 100; (iv) *multiplied* by 1000.

Write down the result in each case.

29. One cubic foot of distilled water weighs approximately 62·425 lbs. Assuming this to be true only to the last figure given, find in lbs. the weight of 1 cubic inch to as many decimal figures as can be correctly calculated.

Find also the weight of 1 cubic inch of water in grains. Does the datum suffice to give a result true to the nearest grain?

30. Given that 1 ton = 1016 kilograms (correct only to the nearest unit), obtain the equivalent of 1 lb. in grams, not attempting greater accuracy than the datum justifies.

31. Given that 1 mile = 1609 metres (true only to the nearest metre), find with such accuracy as the datum admits

(i) the value of 1 inch in millimetres;

(ii) the number of square metres in a square mile.

32. In the following cases dividend and divisor are to be considered *approximate* (true only to the last significant digit). Obtain the quotients, not proceeding beyond the first doubtful figure.

(i) 16·68 ÷ 3·74. (ii) 463·28 ÷ 31·07.

(iii) 9462800 ÷ 4070. (iv) 843·7 ÷ 0·00372.

Use these cases to illustrate the following principle:

> "*When one approximate quantity is divided by another, the quotient can never be true to more significant figures than there are in the less correct of the given quantities.*"

33. From the formula *circumference* = *diameter* × π, where π = 3·1416 (correct only to four decimal figures), find, with such accuracy as the data admit, the length of the diameter of a circle of which

(i) the circumference = 7·28 metres, true only to the nearest *cm.*

(ii) the circumference = 24900 miles, true only to the nearest *hundred miles.*

34. Light travels at the approximate rate of 186,000 miles per second. The distance of the Earth from the Sun may be taken as 92·8 millions of miles. Understanding both data as only correct to three significant figures, find how long light takes to come from the Sun to the Earth; and indicate the error to which your result is liable.

35. Given that 1 cubic foot of water weighs 62·425 lbs.,

and that 1 cubic inch of silver weighs 0·377 lb.,

find as nearly as the data justify the *specific gravity* of silver (i.e. the number of times that silver weighs its own bulk of water).

CHAPTER XI.

SOME APPLICATIONS OF DECIMAL PROCESSES.

Decimals of Compound Quantities. Ratios. Percentages.

206. *To express one compound quantity as the decimal of another compound quantity*, we reduce both to the *same denomination*, and divide the first by the second. Whether to reduce both quantities to the *highest* common denomination, or to the *lowest*, or to some intermediate one, is a matter of convenience to be decided in each particular case.

EXAMPLE 1. *Express* £4. 2*s*. 6*d*. *as the decimal of* £6. 13*s*. 4*d*.

Here it is best to bring both quantities up to pounds and fractions of a pound.

$$\frac{£4.\ 2s.\ 6d.}{£6.\ 13s.\ 4d.}=\frac{£4\frac{1}{8}}{£6\frac{2}{3}}=\frac{33}{8}\times\frac{3}{20}=0{\cdot}61875.$$

EXAMPLE 2. *Express the ratio of* £5. 16*s*. 2*d*. *to* £71. 11*s*. 5*d*. *in* decimal *form, and hence find what* percentage *the first quantity is of the second* (*correct to* 3 *significant digits*).

Decimalizing each of the given sums at sight, we have by contracted division:

$$\frac{£5.\ 16s.\ 2d.}{£71.\ 11s.\ 5d.}=\frac{5{\cdot}8083\ldots}{71{\cdot}571}$$

$$=\underline{0{\cdot}0812.}$$

$$\begin{array}{r|r}
\multicolumn{1}{r}{} & 0{\cdot}08115 = \textit{Quotient.} \\ \cline{2-2}
7{\cdot}{}_{\backslash}1{}_{\backslash}5{}_{\backslash}7 & 0{\cdot}58083 \\ \cline{2-2}
\multicolumn{1}{r}{} & 827 \\ \cline{2-2}
\multicolumn{1}{r}{} & 111 \\ \cline{2-2}
\multicolumn{1}{r}{} & 39
\end{array}$$

Let x denote the required rate per cent.

Then $$\frac{x}{100}=\text{given ratio}=0{\cdot}0812.$$

$$\therefore\ x=8{\cdot}12;$$

so that the first sum is 8·12 % of the second.

And we observe that a ratio expressed in *decimal form* can be at once read off as a *percentage* without further work.

EXAMPLE 3. *In the year* 1905 *the Great Northern Railway's passenger receipts were as follows:*

Third Class.	Second Class.	First Class.
£1,376,527	£21,239	£183,156.

Express the receipts in each class as a percentage of the total passenger receipts, correct to three *significant digits.*

By addition, the *total* receipts = £1,580,922.

Let the 3rd class receipts be x per cent. of the total.

$$\text{Then } \frac{x}{100}=\frac{1,376,527}{1,580,922}=\frac{1,376,527}{1,581,000} \text{ (nearly)}$$

$$=\frac{1376{\cdot}527}{1581}=0{\cdot}871\,;$$

$$\therefore\ x=87{\cdot}1.$$

Since the result is required to 3 significant digits we retain 4 significant digits in the divisor. [Art. 197.]

$$\begin{array}{r|l} & 0{\cdot}871 = \textit{Quotient.} \\ 15{,}8{,}1 & 1376{\cdot}5 \\ & \underline{\ \ 1117} \\ & \ \ \ \ \ 10 \end{array}$$

So that the 3rd class receipts are 87·1 % of the total.

Similarly the other percentages may be found.

207. *To find the value of a decimal of a compound quantity*, we reduce the compound quantity to a single denomination, and multiply it by the decimal. In some cases it is best to reduce to the highest denomination, in others to the lowest.

EXAMPLE 1. *Find* 7·324 *of* £5. 5*s. to the nearest penny.*

Here £5. 5*s.* = £$5\frac{1}{4}$.

$$\begin{array}{r|r} & £\ \ \ \ \\ & 7{\cdot}324 \\ & \underline{5\tfrac{1}{4}} \\ & 36{\cdot}620 \\ \tfrac{1}{4} & \underline{1{\cdot}831} \\ & 38{\cdot}451 \end{array}$$

= £38. 9*s.* 0*d.*

to the nearest penny.

EXAMPLE 2. *Find* 4·48 *of* 10*s.* 7*d. to the nearest penny.*

Here 10*s.* 7*d.* = 127 pence.

$$\begin{array}{r} 127 \text{ pence} \\ \underline{4{\cdot}48\ \ \ \ \ \ \ \ \ } \\ 508{\cdot}\ \ \ \ \ \ \ \ \ \ \ \\ 50{\cdot}8\ \ \ \ \ \ \ \ \ \\ \underline{10{\cdot}16\ \ \ \ \ \ \ \ \ } \\ 568{\cdot}96 \text{ pence} \end{array}$$

= 569 pence,

to the nearest penny,

= £2. 7*s.* 5*d.*

EXAMPLE 3. *Find, correct to the nearest penny, the value of* £16. 11*s*. $2\frac{1}{2}$*d*. × 3·0245.

Decimalizing at sight, we have £16. 11*s*. $2\frac{1}{2}$*d*. = £16·560416..., and we want a result (in £'s) true to the 3rd decimal figure.

$$\begin{array}{r|l}
\multicolumn{1}{c}{\text{£.}} & \\
1_{\backslash}6_{\backslash}\cdot5_{\backslash}6_{\backslash}0_{\backslash} & 4\ \cancel{2} \\
 & 3\cdot0245 \\ \hline
49\cdot681 & 3 \\
\cdot331 & 2 \\
66 & 2 \\
8 & 2 \\ \hline
\multicolumn{1}{r}{\underline{50\cdot087}} & \multicolumn{1}{l}{=\text{£}50.\ 1s.\ 9d.,\ \text{to the nearest penny.}}
\end{array}$$

EXAMPLE 4. *Find, correct to the nearest penny,* $7\frac{3}{8}$ *per cent. of* £115. 10*s*. 8*d*.

$$\text{Here } 7\tfrac{3}{8} \text{ per cent. of £115. 10}s.\ 8d. = \text{£}115\cdot533\ldots \times \frac{7\frac{3}{8}}{100}$$
$$= \text{£}1\cdot15533\ldots \times 7\tfrac{3}{8}.$$

$$\begin{array}{c|r|l}
 & \multicolumn{1}{c}{\text{£.}} & \\
 & 1\cdot155 & 3\cancel{3} \\
 & & 7\frac{3}{8} \\ \hline
 & 8\cdot087 & 3 \\
\frac{1}{4} & \cdot288 & 8 \\
\frac{1}{8} & \cdot144 & 4 \\ \hline
 & \multicolumn{1}{r}{\underline{8\cdot521}} & \multicolumn{1}{l}{=\text{£}8.\ 10s.\ 5d.,\ \text{to the nearest penny.}}
\end{array}$$

[The multiplication must give a result true to the 3rd decimal figure.]

EXAMPLES XI. a.

Express correct to *three* significant figures:

1. £2. 6*s*. 8*d*. as the decimal of £2. 10*s*.
2. 2*s*. $7\frac{1}{2}$*d*. .... £3. 10*s*.
3. £12. 15*s*. 6*d*. .... £2. 3*s*. 9*d*.
4. 3 cwt. 1 qr. 14 lbs. .... 11 cwt. 2 qrs.

Reduce the following ratios to *decimal form*, and then read off each as a *percentage*.

5. 6*s*. 8*d*. to £2. 13*s*. 4*d*.   6. 1*s*. $5\frac{1}{2}$*d*. to 2*s*. $7\frac{1}{4}$*d*.
7. 31 tons 10 cwt. to 56 tons 5 cwt.
8. 2 ac. 1 r. 1 p. to 2 ac. 3 r. 35 p.

Express decimally, correct to *three* significant figures :

9. £19. 5*s.* as a *percentage* of £87.
10. £48. 15*s.* .... of £1712.
11. £389 .... of £11231.
12. £947. 10*s.* .... of £27843.
13. £9. 12*s.* 6*d.* .... of £127. 10*s.*
14. £5. 14*s.* 10*d.* .... of £57. 8*s.* 2*d.*

Find, to the nearest penny, the value of the following :

15. 0·36 of £2. 10*s.* **16.** 2·125 of £3. 4*s.*
17. 8·24 of £3. 2*s.* 6*d.* **18.** 5·84 of £2. 3*s.* 9*d.*
19. £2. 2*s.* 1*d.* × 4·36. **20.** £384. 7*s.* 5*d.* × 0·008351.

Working in decimals, find, to the nearest penny, the value of the following :

21. 4 % of £565. **22.** 3 % of £7821.
23. $4\frac{1}{2}$ % of £472. 8*s.* **24.** $6\frac{1}{4}$ % of £1289. 10*s.*
25. $5\frac{1}{2}$ % of £17. 2*s.* 8*d.* **26.** $4\frac{1}{2}$ % of £423. 11*s.*
27. $3\frac{3}{8}$ % of £724. **28.** $4\frac{3}{4}$ % of £81. 10*s.* 3*d.*

Express as percentages (correct to the 3rd significant digit) the ratios of :

29. 73 fr. 41 c. to 1981 fr. **30.** Km. 43·200 to 98473 metres.
31. 2 cwt. 1 qr. 25 lbs. to $\frac{1}{4}$ ton. **32.** 41 yds. 3 in. to 50 yds. 1 ft.
33. 47,109 to 896,431. **34.** 634,218 to 2,826,431.

35. In a town of 361,420 inhabitants there occur in one year 6144 deaths. At what rate is this *per mille*? Answer to the nearest integer.

36. A company, having a paid up capital of £5,250,000, sets aside in one year £183,750 from its profits for dividend. What rate per cent. does it pay ?

37. The gross revenue of a Railway Company averages £6,072,231 ; and the working expenses average 62 per cent. of this sum. How much would the company save in a year if it could bring its working expenses down to 50 per cent. of the total receipts ? Answer to the nearest pound.

38. It is arranged that the three partners in a firm shall receive respectively 30 %, 25 %, and 20 % of the annual profits, and that the rest shall be divided between the Reserve Fund and the Employés' Pension Fund in the ratio of 2 to 1. For the year 1906 the total profits were £47,821. 18*s.* How much of this went to each partner and to each fund ?

39. The area of Ireland under flax and potatoes in 1855 and in 1901 is given below. Find, to the nearest whole number, the decrease of area under each crop as a percentage of the area in 1855.

	Acres under flax.	Acres under potatoes.
1855 - - -	97,075	982,301
1901 - - -	55,442	635,321

40. The following table shews the average price of a quarter of wheat and of barley for the years 1902 and 1903. Express the prices for 1903 as percentages (to the nearest tenth) of those for 1902.

If a thousand pounds' worth of wheat and a thousand pounds' worth of barley had been stored in 1902, what would each have been worth in 1903, to the nearest pound?

	Wheat.		Barley.	
	s.	*d.*	*s.*	*d.*
1902 - - -	28	1	25	8
1903 - - -	26	9	22	8

Practice Decimalized.

208. The use of aliquot parts in shortening the work of compound multiplication was explained in Chapter VI. In all but the simple instances there given the actual work should be kept in decimals, as shewn in the following Examples.

EXAMPLE. *Find, to the nearest penny, the cost of* 305$\frac{1}{2}$ *tons at* £7. 8*s.* 4*d.* *per ton.*

	£.	
	305·5	=cost of 305$\frac{1}{2}$ tons at **£1** per ton.
	7	
	2138·5	
5*s.* $=\frac{1}{4}$ of £1	76·375	
3*s.* 4*d.* $=\frac{1}{6}$ of £1	50·916	6
	2265·792	=£2262. 15*s.* 10*d.* to the nearest penny.

209. **Simple Practice** is the process of finding by aliquot parts the cost of a *simple* quantity, when the cost of one unit is given.

For instance: *Find the cost of* 305$\frac{1}{2}$ *tons at* £7. 8*s.* 4*d.* *per ton.*

Compound Practice finds the cost of a *compound* quantity, when the cost of one of its denominations (usually the highest) is given.

For instance: *Find the rent of* 24 ac. 2 r. 16 p. *at* £1. 17*s.* 6*d.* *per acre.*

In compound practice one of the given quantities should be decimalized, and the other treated by aliquot parts.

Example 1. *Find, to the nearest penny, the rent of* 24 ac. 2 r. 16 p. *at* £1. 17*s.* 6*d. per acre.*

Here it is best to decimalize 24 ac. 2 r. 16 p., and take £1. 17*s.* 6*d.* by aliquot parts (£2 *less* 2*s.* 6*d.*, or £$\frac{1}{8}$).

Now 24 ac. 2 r. 16 p. = 24·6 acres.

$$\begin{array}{r|l} 40 & 16{\cdot}0 \text{ p.} \\ \hline 4 & 2{\cdot}4 \text{ r.} \\ \hline & {\cdot}6 \text{ ac.} \end{array}$$

$$\begin{array}{r|rl} & \text{£.} & \\ & 24{\cdot}6 & = \text{cost at £1 per acre.} \\ & 2 & \\ \hline & 49{\cdot}2 & \\ \textit{less } 2s.\ 6d., \text{ or £}\frac{1}{8} & 3{\cdot}075 & \\ \hline & 46{\cdot}125 & = \text{£}46.\ 2s.\ 6d. \end{array}$$

Note. In this example we might have decimalized £1. 17*s.* 6*d.* and treated 24 ac. 2 r. 16 p. by aliquot parts, but the solution would not have been so short. That quantity should be decimalized which falls less readily into aliquot parts.

Example 2. *A bankrupt whose debts amount to* £2587. 12*s.* 3*d. can pay a dividend of* 14*s.* 4$\frac{1}{2}$*d. in the* £. *Find the total value of his assets available for distribution.*

£2587. 12*s.* 3*d.* = £2587·6125.

$$\begin{array}{rl|r|l} & & \text{£.} & \\ & & 2587{\cdot}612 & 5 = \text{value at £1 in the £.} \\ \hline 10s. & = \frac{1}{2} \text{ of £1} & 1293{\cdot}806 & 3 \\ 3s.\ 4d. & = \frac{1}{3} \text{ of } 10s. & 431{\cdot}268 & 8 \\ 10d. & = \frac{1}{4} \text{ of } 3s.\ 4d. & 107{\cdot}817 & 2 \\ 2\frac{1}{2}d. & = \frac{1}{4} \text{ of } 10d. & 26{\cdot}954 & 3 \\ \hline & & 1859{\cdot}847 & = \text{£}1859.\ 16s.\ 11d., \text{ to the nearest penny.} \end{array}$$

210. If neither of the Compound quantities can be conveniently represented in aliquot parts, Practice does not give the best solution. In such cases both quantities should be decimalized and the result obtained by contracted multiplication.

Example. *Find the cost of* 3 tons 0 cwt. 54$\frac{3}{4}$ lbs. *at* £16. 11*s.* 2$\frac{1}{2}$*d. per ton.*

Here £16. 11*s.* 2$\frac{1}{2}$*d.* = £16·560416... ;

and 3 tons 0 cwt. 54$\frac{3}{4}$ lbs. = 3·02444... tons.

Hence, by contracted multiplication, we find the value of £16·5604 × 3·02444, correct to the 3rd decimal figure (see Art. 207, Ex. 3), obtaining £50·085, or £50. 1*s.* 8*d.*

$$\begin{array}{r|l} 112\left\{\begin{array}{r} 4 \\ 4 \\ 7 \end{array}\right. & \begin{array}{l} 54{\cdot}75 \quad \text{lbs.} \\ \hline 13{\cdot}6875 \\ \hline 3{\cdot}4219 \end{array} \\ \hline 20 & 0{\cdot}48884 \text{ cwt.} \\ \hline & {\cdot}02444 \text{ ton.} \end{array}$$

211. From the last Example it will be seen that a *Decimal System* of money, weights, and measures dispenses altogether with the method of Practice.

EXAMPLE. *Find the weight of* 30 Km. 7 Hm. 60 m. *of copper wire, running* 27 Kg. 640 gm. *to the kilometre.*

Each compound quantity is decimalized at sight:

thus 30 Km. 7 Hm. 60 m. = 30·76 Km.
and 27 Kg. 640 gm. = 27·64 Kg.

$$\begin{array}{r|l}
\text{Kilograms.} & \\
27{\cdot}64 & \\
30{\cdot}76 & \\
\hline
829{\cdot}2 & \\
19{\cdot}348 & \\
1{\cdot}658 & 4 \\
\hline
\underline{850{\cdot}206} & = 850 \text{ Kg. } 206 \text{ gm.}
\end{array}$$

EXAMPLES XI. b.

Working in decimals, find by Practice (to the nearest penny) the value of:

1. 227 at £9. 13*s*. 4*d*.
2. 309 at £2. 18*s*. 4*d*.
3. 417½ tons at £2. 12*s*. 9*d*. per ton.
4. 160¼ acres at £25. 6*s*. 3*d*. per acre.
5. 437·6 oz. of standard gold at £3. 17*s*. 10½*d*. per oz.

Find by the method of aliquot parts (in each case to the nearest penny):

6. £3. 12*s*. 4*d*. × 5·0265.
7. 16·308 of £11. 17*s*. 3*d*.

By decimalizing one compound quantity, and treating the other by aliquot parts, find, to the nearest penny, the value of:

8. 1153 ac. 2 r. at £1. 12*s*. 6*d*. per acre.
9. 500 ac. 3 r. 31 p. at £1. 6*s*. 8*d*. per acre.
10. 23 cwt. 3 qrs. 14 lbs. at £2. 11*s*. 6*d*. per cwt.
11. 2 tons 12 cwt. 50 lbs. at £1. 3*s*. 4*d*. per ton.
12. 345 ac. 2 r. 15 p. at £2. 2*s*. 6*d*. per acre.
13. 1684 tons 8 cwt. 100 lbs. at £1. 12*s*. 6*d*. per ton.
14. 13 cwt. 3 qrs. 4 lbs. at £2. 18*s*. 4*d*. per cwt.
15. 2 gals. 3 qts. 1 pt. at £2. 5*s*. 8*d*. per gallon.
16. 2 fur. 40 yds. at £3. 13*s*. 4*d*. per mile.
17. 7 yds. 1 ft. 9 in. at £1. 14*s*. 3*d*. per yard.

Find the dividend on:

18. £4287. 17*s.* 6*d.* at 14*s.* 4*d.* in the £.

19. £2073. 6*s.* 3*d.* at 11*s.* 8*d.* in the £.

20. £1710. 14*s.* 6*d.* at 13*s.* $4\frac{1}{2}$*d.* in the £.

By decimalizing both compound quantities and using contracted multiplication, find (to the nearest penny or the nearest centime) the value of:

21. 7 cwt. 2 qrs. 24 lbs. at £1. 7*s.* 5*d.* per cwt.

22. 19. ac. 3 r. 24 p. at £2. 1*s.* 5*d.* per acre.

23. 30 tons 8 cwt. 1 qr. at £1. 3*s.* 10*d.* per ton.

24. 27·48 metres at 5 fr. 9 c. per metre.

25. 5 Km. 473 m. at 105 fr. 60 c. per hectometre.

26. 39 Kg. 4 Dg. at 17 fr. 40 c. per kilogram.

27. What are the available assets of a bankrupt who can pay 9*s.* $4\frac{1}{2}$*d.* in the £ on debts amounting to £15845?

28. How much will be required to pay the wages of 1232 workmen for one week at the average rate of £1. 3*s.* $4\frac{1}{2}$*d.* each?

29. A watch loses 8 min. 16 sec. in 6 hours: find its loss in 17 days 9 hours 45 min.

30. Each edge of a cube of copper measures 1 dm. 7 cm.: find its weight to the nearest gram, having given: 1 cub. dm. of water weighs 1 kilogram, and the specific gravity of copper is 8·788 times that of water.

Further Applications. Formulae.

212. We conclude the Chapter with a set of miscellaneous examples further illustrating the application of approximate Decimal processes.

EXAMPLE 1. *A copper wire of cross-section* 1 *square inch can sustain a weight of* 26 *tons. If the breaking strain of a wire of any given metal is proportional to the cross-section, find to the nearest* 10 *kilograms what weight can be sustained by a similar wire whose cross-section is* 1 *square centimetre.*

Given 1 *inch* = 2·5400 *cm.*
1 *Kg.* = 2·2046 *lbs.*

Since 1 in. = 2·5400 cm.

∴ 1 *square* inch = $(2{\cdot}5400)^2$ square cm.
= 6·4516 sq. cm.

$$\begin{array}{r} 2{\cdot}54 \\ 2{\cdot}54 \\ \hline 5{\cdot}08 \\ 1{\cdot}270 \\ {\cdot}1016 \\ \hline 6{\cdot}4516 \end{array}$$

Thus 6·4516 sq. cm. sustain 26 × 2240 lbs.

$$\therefore \qquad 1 \text{ sq. cm. } \ldots\ldots\ldots \frac{26 \times 2240}{6\cdot4516} \text{ lbs., or } \frac{58240}{6\cdot4516 \times 2\cdot2046} \text{ Kg.}$$

The divisor is nearly 14; so that the required result will have *four integral* digits.

And this result has to be evaluated to the nearest 10 units; hence, to *three* significant digits.

$$\begin{array}{r|l} & \overline{9027} = \textit{Quotient.} \\ 6_{\backslash}\cdot4_{\backslash}5_{\backslash}2 & 58240 \\ \cline{2-2} & 172 \\ \cline{2-2} & 43 \end{array} \qquad\qquad \begin{array}{r|l} & \overline{409}\ldots = \textit{Quotient.} \\ 2\cdot2_{\backslash}0_{\backslash}5 & 9027 \\ \cline{2-2} & 207 \\ \cline{2-2} & 9 \end{array}$$

Thus req^{d} result = 4090 Kg. to the nearest 10 Kg.

EXAMPLE 2. *The horse-power of a pumping engine necessary to raise* Q cubic feet *of water per* minute *to a height of* H feet *is given by the formula*

$$\text{horse-power} = 0\cdot0023 \times H \times Q.$$

Find the horse-power required to raise 10,000 gallons *from a depth of* 80 *fathoms in* 1 *hour.*

Given 1 *fathom* = 6 *feet.* 1 *gallon* = 277·5 *cu. inches* (*to nearest tenth*).

Here $H = 80 \times 6 = 480$ ft.

And 10,000 gallons = 2775000 cu. in. (to nearest 1000)

= 1606 cu. feet (to nearest unit).

Also 1606 cu. ft. per *hour*

$= \frac{1606}{60}$, or 26·8 cu. ft. per *minute* (to nearest tenth).

So that $Q = 26\cdot8$.

Thus req^{d} horse-power $= 0\cdot0023 \times 480 \times 26\cdot8$

$= 2\cdot3 \times 4\cdot8 \times 2\cdot68$,

the last digits in the 1st and 3rd factors being only *approximate*.

$$\begin{array}{r} 2\cdot\underline{3} \\ 4\cdot8 \\ \hline 9\cdot\underline{2} \\ 1\cdot8\,\underline{4} \\ \hline 11\cdot\underline{0}_{\backslash}\underline{4} \\ 2\cdot6\underline{8} \\ \hline 22\cdot\underline{1} \\ 6\cdot6 \\ \cdot\underline{9} \\ \hline \underline{\underline{30\cdot}} \end{array}$$

Multiplying out, and underlining doubtful figures as explained in Art. 202, we find that the required horse-power is **30**, probably correct to the nearest unit.

EXAMPLES XI. c.

In these Examples the following data are to be used when required:

1 *metre* = 39·3701 *inches.*
1 *kilogram* = 2·2046 *lbs.*
1 *gram* = 15·43 *grains.*
1 *litre* = 1·7600 *pints.*

1 *inch* = 2·5400 *centimetres.*
1 *lb., or* 7000 *grains* = 0·4536 *kilogram.*
1 *gallon, or vol. of* 10 *lbs. of water* = 4·5460 *litres.*

1. What is the cost, to the nearest penny, of 5 litres 12 centilitres of wine at 6 shillings the litre?

2. Find, to the nearest pound, the value of 5000 tons of wheat at 6*s*. $2\frac{1}{2}$*d*. per cental. (A cental is 100 lbs.)

3. Express 1 gallon in terms of litres correct to the second decimal figure. [Use 1 litre = 1·7600 pints.] Hence find to the nearest *tenth* how many litres there are in 2·3 gallons.

4. A kilometre is 0·6214 of 1 mile. How many yards to the nearest unit is that?

5. Find the cost of 9 gals. 3 qts. of spirit at £2. 15*s*. per gallon.

6. The following Table shews the principal silver coins of the countries named with their average equivalents in English money. Fill up the column designed to shew the equivalent of £1 in terms of each coin correct to the second decimal figure.

Country.	Coin.	Value in English Money.	Number equivalent to £1.
France - -	Franc	$9\frac{1}{2}$*d*.	
Germany - -	Mark	$11\frac{3}{4}$*d*.	
United States -	Dollar	4*s*. $1\frac{1}{2}$*d*.	
Russia - -	Rouble	2*s*. $1\frac{1}{2}$*d*.	
India - - -	Rupee	1*s*. 4*d*.	

7. A litre of air weighs 1·293 grams and contains 23 per cent. by weight of oxygen. What is the weight of oxygen in a hectolitre of air?

8. Find the value of 14 Kg. 300 gm. of tea at 3*s*. 6*d*. per kilogram, giving the answer to the nearest penny.

9. Find the value of x in each of the following statements:

(i) 30 centimetres = x inches, to the nearest hundredth.

(ii) 15 miles an hour = x feet per second.

(iii) 35·6 feet of a certain wire go to the pound: this is equivalent to x pounds per mile.

10. Find correct to the nearest *tenth* the value of x which makes $\frac{2x-3{\cdot}2}{1{\cdot}7}$ equal to $\frac{x-1{\cdot}5}{0{\cdot}4}$.

Find in £. *s.* *d.* the value of

11. 0·275 of £1 + 12·75 of 2*s.*

12. 2·0875 of £3 + 0·18 of £2. 10*s.* − 1·76 of 6*s.* 3*d.*

13. 21·75 of 7*s.* 6*d.* + 0·3625 of 10*s.* + 13·75 of 1*s.*

14. Express in feet and inches

0·00875 of 100 yards + 0·0046875 of 1 mile.

15. From the formula $(a+b)^2=a^2+2ab+b^2$, find to the nearest unit the value of $(7531{\cdot}05)^2$, having given $(7531)^2=56715961$. Use no superfluous figures in your work.

16. In the first quarter of 1903, the British Isles exported 42,424,700 lbs. of cotton yarn, of which

British India	took	8,265,400 lbs.
Holland	- -	6,817,000 ,,
Germany	- -	6,481,400 ,,

Give the export to each of these countries as a percentage (to the nearest integer) of the whole export.

17. In the first four months of 1906 the Indian Government sold Bills amounting to 97,984,311 rupees, obtaining £6,537,578 in exchange. Find the value of a rupee in English money to the nearest tenth of a penny. N.B. Use no more figures than are necessary to obtain a result to the indicated degree of accuracy.

18. From the table given below find what percentage of the population were parliamentary electors in England and Wales, in Scotland, in Ireland, and in the United Kingdom as a whole. Neglect the changes in population since 1901.

Find also what population is on the average represented by a Member of Parliament in England and Wales, in Scotland, and in Ireland.

	Population in 1901.	Number of Electors in 1906.	Members of Parliament.
England and Wales -	32,527,843	5,824,884	495
Scotland - - -	4,472,103	750,401	72
Ireland - - -	4,458,775	691,423	103

19. The mean atmospheric pressure is 14·7 lbs. per square inch: express this in kilograms per square centimetre, correct to 10 grams.

20. The sum of £160,860 was paid in one year for licenses to employ male servants, the license costing 15 shillings for each servant. Taking the whole population as 42,500,000, find to the nearest unit how many *per mille* were employed as men servants.

21. One metre of copper wire (2 millimetres thick) is found to weigh 27·64 grams. Find, as accurately as the data admit, the weight of 1 yard of the same wire (i) in grams, (ii) in ounces.

How many metres of such wire go to the kilogram? And how many yards to the pound?

22. The number of feet (s) through which a body, falling from rest, descends in t seconds under the action of gravity (neglecting the resistance of the air) is given by the formula

$$s=\tfrac{1}{2}gt^2,$$

where $g=32{\cdot}2$ correct to the 3rd significant digit.

A stone dropped from the edge of an overhanging cliff reaches the water in $4\frac{1}{2}$ seconds. Find the height of the cliff, not attempting greater accuracy than the data justify.

23. A railway embankment H feet high, A feet wide at the top, and B feet wide at the bottom contains $\frac{\mathsf{A}+\mathsf{B}}{2}\times\mathsf{H}\times\mathsf{L}$ cubic feet of earth in a length of L feet. If 18 cubic feet of earth weigh a ton, what is the weight (to the nearest *hundred tons*) of 200 feet of an embankment 20 feet high, 14 feet wide at the top, and 37 feet wide at the bottom?

24. The greatest load which a beam, supported at its ends, will bear at its middle point is given by the formula

$$\mathsf{W}=\frac{\mathsf{B}\times\mathsf{D}^2}{\mathsf{L}}\times k;$$

where B=the breadth of the beam in *inches*; D=depth in *inches*; L=length in *feet*; W=load in *hundred-weight*; and k is a constant depending on the material of the beam.

Find to the nearest hundred-weight the greatest load that can be sustained by an oak beam, 20 feet long, 7 inches broad, 10·5 inches deep; having given that, *for English oak*, $k=5{\cdot}0$.

25. A force pump delivers a column of water 2 feet high and 3 inches in diameter at each stroke. If the pump works at the rate of 30 strokes a minute, and starts working at 10 a.m., when will it fill a tank 12 feet long, 7 feet wide, and 5 feet deep?

N.B. A column of water A feet high and D feet in diameter contains $\mathsf{A}\times\mathsf{D}^2\times0{\cdot}7854$ cubic feet.

26. The pressure of water on a given area at a given depth may be found from the formula

$$\mathsf{P}=0{\cdot}4335\times\mathsf{H}\times\mathsf{A};$$

where P denotes the pressure in *pounds*, H the depth in *feet*, and A the area in *square inches.*

Find, to the nearest pound, the pressure on the circular end of a cylinder, 9 inches in diameter, submerged at a depth of 10 feet; having given that the area of a circle, d inches in diameter, contains $d^2\times 0{\cdot}7854$ square inches.

27. The "Indicated Horse-power" of a steam-engine is found from the formula

$$\textit{horse-power}=\frac{d^2\times 0{\cdot}7854\times s\,.\,p\,.\,2r}{33{,}000};$$

where $d=$ the diameter of the cylinder in *inches,*
$s=$ the length of piston-stroke in *feet,*
$p=$ the average steam-pressure in lbs. per square inch,
$r=$ the number of revolutions per minute.

In an engine whose cylinder is 16 inches in diameter, and length of stroke $2\frac{1}{2}$ feet, find the indicated horse-power (to the nearest unit) when the steam-pressure is 40 lbs. per square inch, and the number of revolutions 30 per minute.

28. A sheet of paper is found on measurement to be 1·36 metres long and 0·75 metre wide; find its area. If each *linear* measurement is liable to an error of one per cent., find, to the nearest square centimetre, the greatest possible error in the area in excess or defect; and express this error as a percentage of the approximate result.

29. To prevent a train from leaving the metals as it passes round a curve, the outer rail is raised. The following rule has been given for finding the necessary elevation:

$$\textit{Elevation of outer rail in } \text{inches}=\mathsf{B}\times\frac{\mathsf{V}^2}{\mathsf{R}\times 1{\cdot}25};$$

where $\mathsf{B}=$ the gauge of the line in *feet*; $\mathsf{R}=$ the radius of the curve in *feet*, and $\mathsf{V}=$ the speed allowed in *miles per hour.*

Taking the ordinary British gauge of 4 ft. $8\frac{1}{2}$ in., and a curve of 425 feet radius, find to the nearest tenth of an inch the proper elevation of the outer rail, in order that a train may pass over it safely at 20 miles an hour.

30. The following table gives the population and birth-rate per 1000 for the three greatest cities of the United Kingdom.

	Population.	Birth-rate.
London - -	4,613,812	28·4
Glasgow - -	786,897	31·9
Liverpool -	716,810	33·4

Find the birth-rate for the three cities taken together.

CHAPTER XII.

EASY PROBLEMS.

213. Examples on Work and Time. When we have to compare the work of several agents, it is necessary first to ascertain the amount of work each can do *in the same time*, usually one day, one hour, or one minute, according to the condition of the problem.

EXAMPLE 1. *A can do in* 10 *days a piece of work which B could do in* 15 *days: how long would they take if both worked together?*

A does $\frac{1}{10}$ of the work in **one** day,

B ... $\frac{1}{15}$ .... **one** day;

$\therefore$ together they can do $\frac{1}{10}+\frac{1}{15}$, or $\frac{1}{6}$ of the work in 1 day.

That is, they would require 6 days to do the *whole* work.

NOTE. The pupil should reason out this example with letters instead of numbers. Thus if A and B can do the work in m and n days respectively, it may be shewn that when working together they will take $\dfrac{mn}{m+n}$ days.

EXAMPLE 2. *A cistern can be filled by a supply-pipe in* 5 *minutes, and emptied by a waste-pipe in* 8 *minutes. If both pipes are opened when the cistern is empty, how long will it be before the cistern is full? Also, if the cistern can hold* 120 *gallons, how long would it be before exactly* 36 *gallons had run in?*

In 1 minute $\frac{1}{5}$ of the whole *runs in*

and 1 .... $\frac{1}{8}$ .... *runs out.*

$\therefore$ after 1 minute the contents of the cistern $=\frac{1}{5}-\frac{1}{8}$, or $\frac{3}{40}$ of the whole.

$\therefore$ the number of minutes required to fill the cistern $=1\div\frac{3}{40}=\frac{40}{3}=13\frac{1}{3}$.

Again, after 1 minute the cistern contains $\frac{3}{40}$ of 120 gals., or 9 gals.; so that 4 minutes would elapse before the contents were 36 gals.

EXAMPLE 3. *Three men A, B, C, working alone can do a certain work in 6, 8, and 12 days respectively. B and C work together for 2 days, and then A takes C's place: how soon will the work be finished?*

A, B, C respectively do $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{12}$ of the work in 1 day;

$\therefore$ after 2 days B and C will have done $\left(\frac{2}{8}+\frac{2}{12}\right)$ of the work

$=\frac{5}{12}$ of the work;

therefore $\frac{7}{12}$ of the work remains to be done by A and B together.

Now A and B together do $\frac{1}{6}+\frac{1}{8}$, or $\frac{7}{24}$ of the work in 1 day;

$\therefore$ the required time $=\left(\frac{7}{12}\div\frac{7}{24}\right)$ days $=2$ days.

EXAMPLES XII. a.

1. A can do a piece of work in 6 days, and B can do it in 12 days: how long will they take if both work together?

2. A bath can be filled by one pipe in 4 min. and emptied by another in 5 min. If both are turned on together, in how long will the bath be full?

3. How long would two men together take to dig a trench which they could do in 5 hours and $7\frac{1}{2}$ hours respectively when working alone?

4. A can dig a trench in 2 days, B in 3, C in 6. If all three work together, how long will they take?

5. A can mow a field in 9 hours, B in 6 hours: if they begin working together at 6 a.m., when will the work be finished?

6. A cistern can be filled by a supply-pipe in 25 min. and emptied by a waste-pipe in three-quarters of an hour. If both pipes are opened together, in what time will the cistern be full?

7. Two pipes can fill a cistern in 3 and 4 hours respectively, and a waste-pipe can empty it in 6 hours. If the cistern is empty, and all the pipes are opened together, in how many hours will the cistern be half full?

8. A cistern can be filled by one tap in m minutes, and emptied by another in n minutes. If both taps are turned on when the cistern is empty, what fraction of its contents will be filled in 1 min.? In how many minutes will the cistern be full?

9. Three men A, B, and C do a piece of work together in 10 days. If A would have taken 30 days, and B 45 days to do it alone, in how many days would C do it?

10. A can do in 10 days what B can do in 12 days; how long would they take together? How much more does A do than B in 1 day?

11. A can do a piece of work in $2a$ days, B in a days, and C in 4 days. How long will the work take if all three work together?

If B can do the work in 5 days, work this example numerically, an verify your second answer by substituting 5 for a in the first.

12. A and B together could build a wall in 25 days. They wor together for 15 days and then A finishes it alone in 20 days. Ho long would each take separately?

13. A and B can do a piece of work in 12 days; after workin 2 days they are assisted by C who works at the same rate as A, an the work is finished in $6\frac{1}{4}$ days more: in how many days would alone do the work?

14. After doing $\frac{4}{15}$ of a work in 20 days, A calls in B, and with his help finishes the work in 22 days. In what time could either hav done it alone?

15. A and B can finish a piece of work in $1\frac{1}{2}$ days, A and C in 2 days, and B and C in 3 days. If 6*s.* be paid for the whole work, what daily wages must be paid to each workman?

16. Water enters a tank by a tap A which can fill it in 3 hours, and leaves it by two taps B and C which can empty it in 4 hours and 6 hours respectively.

(i) If the cistern is empty, and taps A and B are turned on, how soon will it be full?

(ii) If the cistern is full, and all three taps are turned on, how soon will it be empty?

Also work this example *generally*, supposing a hours, b hours, and c hours are the times required by the three pipes, and then verify the numerical answers by putting $a=3$, $b=4$, $c=6$.

214. Examples on Time and Distance. The speed or velocity of a moving body is measured, when uniform, by the distance traversed in a unit of time.

Thus when we say that the velocity of a uniformly moving body is v, we mean that the body traverses v units of distance in 1 unit of time; for instance, 60 *miles* in 1 *hour*, or 1760 *yards* in 1 *minute*, or 88 *feet* in 1 *second*, according to the units of distance and time which we adopt.

If a body moving uniformly travels v feet in 1 minute,

it will travel vt feet in t minutes;

so that, if s denotes the distance travelled (in feet), we have

$$s=vt.$$

And so for other units.

When we say that a variable speed *at any given instant* is 60 miles per hour, we mean that, if the body continued to move with the speed it then has, it would cover 60 miles in 1 hour, 1760 yards in 1 minute, and so on.

215. A velocity expressed in miles per hour may easily be expressed in other units.

EXAMPLE 1. *A body travels at the rate of* 15 *miles per hour; express its velocity in feet per second.*

15 *miles* per *hour* is equivalent to $(15 \times 1760 \times 3)$ *ft.* per *hour*,

that is, $\frac{15 \times 1760 \times 3}{60 \times 60}$, or 22 feet per second.

EXAMPLE 2. *A train running at uniform speed of* 45 *miles per hour is* 8 *minutes in travelling from one station to the next. Find their distance apart.*

In 8 min. the train will travel $(45 \times \frac{8}{60})$ miles.

$\therefore$ the number of miles between the stations $= 45 \times \frac{8}{60} = 6$.

216. Consider the two following cases:

(i) Two trains A, B travelling *in the same direction* at 45 and 30 miles an hour respectively.

(ii) Two trains with the same velocities as before travelling *in opposite directions.*

In (i) A is gaining $45 - 30$, or 15 miles on B in every hour; therefore 15 mi. per hour is the rate of A's *approach* or *separation* according as A is behind or in front of B.

In (ii) the distance between the two trains is being diminished by $45 + 30$, or 75 miles in every hour; that is, their *rate of approach* is 75 mi. per hr.

217. The rate of approach or separation of two moving bodies is called the **relative velocity** of either with respect to the other, and must be carefully distinguished from the *actual* velocity either of them has.

EXAMPLE 1. *A passenger train starts from Bristol at* 35 *miles per hour at the same time that a goods train starts from Huddersfield at* 25 *miles per hour. If the stations are* 180 *miles apart, and there are no stoppages, when and where will the trains meet?*

The *rate of approach* is $35 + 25$, or 60 mi. per hr.; but at starting they were 180 miles apart;

$\therefore$ they will be together in $\frac{180}{60}$, or 3 hours.

In 3 hours the passenger train will have gone (35×3) miles.

$\therefore$ they meet 105 miles from Bristol.

Example 2. *A train whose length is* 176 *feet is moving at the rate of* 45 *miles per hour, and overtakes a second train,* 154 *feet long, which is moving on a parallel line at the rate of* 30 *miles per hour. Find how long the first train will take* (i) *to pass a milestone,* (ii) *to pass a passenger in the second train,* (iii) *to pass the second train.*

$$45 \text{ mi. per hr.} = \frac{45 \times 1760 \times 3}{60 \times 60} \text{ ft. per sec.}$$

$$= 66 \text{ ft. per sec.}$$

and $$15 \text{ mi. per hr.} = 22 \text{ ft. per sec.}$$

These are respectively the *actual velocity* of the first train, and the *velocity of its approach* to the second.

(i) The time taken to pass a milestone is the time the first train takes to travel *a distance equal to its own length* with its *actual* velocity.

$$\therefore \text{ time required} = \tfrac{176}{66} \text{ secs.} = 2\tfrac{2}{3} \text{ secs.}$$

(ii) Since the passenger is moving at the same rate as the second train, the time taken is the time the first train takes to travel *a distance equal to its own length* with its *velocity of approach.*

$$\therefore \text{ time required} = \tfrac{176}{22} \text{ secs.} = 8 \text{ secs.}$$

(iii) The time taken by the first train just to clear the second train is the time it takes to travel *a distance equal to the sum of the lengths of the two trains* with its *velocity of approach.*

$$\therefore \text{ time required} = \frac{154 + 176}{22} \text{ secs.} = 15 \text{ secs.}$$

EXAMPLES XII. b.

Express the following speeds in feet per second :

1. 30 mi. per hr.

2. 50 mi. per hr.

3. $7\frac{1}{2}$ mi. per hr.

4. 1 mi. 7 fur. per hr.

Express in miles per hour :

5. 22 ft. per sec.

6. $58\frac{2}{3}$ ft. per sec.

7. 88 yds. per min.

8. 165 yds. per min.

9. How many miles per hour does a man walk who passes through a street 528 yards long in 6 minutes ?

10. How long does a bicyclist, riding 15 mi. an hour, take to cross a viaduct 110 yards in length ?

11. Find the length of a bridge which a man riding 9 mi. an hour can cross in $7\frac{1}{2}$ seconds.

12. How long will a train 154 ft. long, travelling 35 mi. an hour, take to pass a milestone?

13. How many miles per hour does a train travel if it is 176 ft. long and takes 4 seconds to pass a certain point?

14. How long will a train 150 ft. long, travelling at 45 mi. an hour, take to pass through a station whose platform is 60 yards long?

15. Find the length of a viaduct which a train, 252 feet long, travelling at 30 mi. an hour, can cross in 21 seconds.

16. Two men walk to meet each other from two places A and B, 32 miles apart. If their speeds are respectively $3\frac{1}{2}$ and $4\frac{1}{2}$ mi. an hour, when and where will they meet?

17. *A* and *B* are travelling on the same road in opposite directions, *A* at the rate of $5\frac{1}{2}$ mi. an hour, and *B* at the rate of $4\frac{1}{2}$ mi. an hour. If at starting they are 25 miles apart, after how long will they pass each other, and how far from *B*'s starting place?

18. *A* and *B* start walking in the same direction from two places 9 miles apart. If they walk at 4 and $2\frac{1}{2}$ mi. an hour respectively, when does *A* overtake *B*?

19. Two friends walking at $3\frac{1}{2}$ and $4\frac{1}{2}$ mi. an hour respectively start at 10 a.m. from two towns 30 miles apart. At what time will they meet, and how far will each have walked?

20. A pedestrian starts to walk at 3 mi. an hour from A to B, a distance of 36 miles, at 8 a.m. A cyclist starts an hour later to ride from B to A, at 8 mi. an hour. At what time will they meet and how far from A?

21. How long will two trains whose lengths are 185 ft. and 211 ft., travelling in opposite directions, at 25 and 35 miles per hour respectively, take to pass each other?

22. A steamboat 344 ft. long, going at the rate of 25 miles an hour, overtakes a sailing vessel 196 ft. long, going at the rate of $11\frac{1}{2}$ miles per hour. In what time will the steamboat pass the sailing vessel?

23. Two marching columns, which are respectively 170 yards and 150 yards in length, pass each other in opposite directions at the respective rates of 4 miles and $3\frac{1}{2}$ miles an hour. How long after the meeting of their front ranks will their rear ranks be 1000 yards apart?

24. A passenger train, moving at the rate of 45 miles an hour, overtakes a mineral train, $1\frac{1}{2}$ times as long and moving on a parallel line at the rate of 27 miles an hour, and passes it completely in 25 seconds. How long would the passenger train take to pass completely through a station 165 yards in length?

218. Averages. The **average** or **mean** of a series of quantities (of the same kind) is the result obtained by adding them together, and dividing the sum by the number of the quantities.

EXAMPLE 1. *In six successive years the rain-fall at a certain place was* 31·2 *inches,* 27·8 *inches,* 32·4 *inches,* 26·9 *inches,* 29·6 *inches, and* 30·8 *inches. Find the average rain-fall for these years.*

The total fall for the six years

$$=31{\cdot}2+27{\cdot}8+32{\cdot}4+26{\cdot}9+29{\cdot}6+30{\cdot}8=178{\cdot}7 \text{ inches.}$$

Hence the average fall $=\frac{178{\cdot}7}{6}=29{\cdot}8$ inches (to the nearest tenth).

Thus the same total would have been reached, if in each of the six years the *average* instead of the *actual* rain-fall had occurred.

EXAMPLE 2. *If* a *lbs. of tea at* x *shillings per pound,* b *lbs. at* y *shillings per pound,* c *lbs. at* z *shillings per pound are mixed together, what is the value of the mixture per pound?*

The total *amount* of tea $=a+b+c$ pounds;

the total *value* of the tea $=ax+by+cz$ shillings;

$\therefore$ the value of **one** pound $=\frac{ax+by+cz}{a+b+c}$ shillings.

Thus the value per pound of the mixture is the *mean* value of the several amounts which enter into it.

EXAMPLES XII. c.

1. What is the average height of five brothers whose several heights are 5 ft. 10 in., 5 ft. $7\frac{1}{2}$ in., 5 ft. 4 in., 4 ft. $10\frac{1}{2}$ in., and 4 ft. 2 in.?

2. The Allan Line in 1904 owned 26 steamers having an aggregate tonnage of 126,185 tons. Find the average tonnage of the Company's boats in that year. In 1905 the *Virginian* and *Victorian*, each of 10,754 tons, were added; by how much did this accession of tonnage increase the average? In each case give the result to the nearest ton.

3. The average monthly profits of a firm from January to April were £416. 8*s.* 6*d.*; the profits for May and June were £437. 2*s.* and £489. 4*s.* respectively. Find the average monthly profits for the half-year.

4. If 12 lbs. of tea at 2*s.* 4*d.* per pound, 10 lbs. at 2*s.* 6*d.* per pound, and 8 lbs. at 2*s.* 9*d.* per pound are mixed together, find the value of the mixture per pound.

5. The price of copper per ton on the 1st day of each month from January to June was as follows: £89. 5*s.*, £91, £93. 15*s.*, £96. 2*s.* 6*d.*, £98, £99. 17*s.* 6*d.* Find the average of these prices.

If the average price (similarly reckoned) for the whole year was £99, find the average price from July to December.

6. In an examination the average mark of the first seven candidates was 92 and of the first eight 90·5: find the number of marks gained by the eighth candidate.

7. In five successive minutes from its start a goods-train runs 88 yards, 147 yards, 220 yards, 352 yards, and 586 yards; and for the next five minutes maintains an average speed of 33 miles an hour. What is the whole distance covered, and at what average speed?

8. The average age of 125 boys in the Fifth Forms of a school was 16·4 years; and the average age of the 35 boys in the Sixth Form was 17·7 years. Find (to the nearest tenth) the average age of the Sixth and Fifth Forms taken together.

9. In one month a cricketer played 20 innings. His average for the first *nine* was a runs, and for the last *eleven* b runs. Find his average for the month; and work out the result (true to one decimal figure) when $a=11·7$, and $b=23·3$.

10. The Hamburg American steamer *Deutschland* can make the passage from Cherbourg to New York, a distance of 3514 miles, in 5 days 12 hours. Find her average day's run in miles, and her average sea-speed in *knots*, *i.e.* nautical miles per hour. [69 miles = 60 knots, nearly.]

Her runs for the first four days having been 640 miles, 622 miles, 597 miles, and 655 miles, what must be her average speed in knots for the rest of the passage, in order that she may make the time above mentioned? [Your results are not to be carried beyond the nearest *tenth* of a knot.]

11. The rain-fall of a certain town in the months of April, May, and June 1907 was 1·90, 0·94, and 1·72 inches respectively; being in April 0·77 in. above, and in May and June 1·46 in. and 1·78 in. respectively below, the average of those months for the previous six years. Find the average for these three months both for 1907, and for the whole 7 years, to two places of decimals.

12. A train from the moment of its start gradually gathers speed in such a way that for the first 5 minutes its rate is increased every minute by 6 miles an hour.

Tabulate the distances (in yards) which would be traversed in each minute if the speed throughout it were (i) that at the beginning of the minute, (ii) that at the end of the minute.

Estimate the whole distance covered by adding the two columns and taking the average of the totals.

The true distance run is given by the formula

$$s=\tfrac{1}{2}ft^2,$$

where s = the required number of yards, f = the number of yards per minute added to the speed in each minute (176), and t = the number of minutes (5). Find s; and compare your average result with the true distance.

219. Miscellaneous Examples.

EXAMPLE 1. *At what times between 4 and 5 o'clock will the hands of a clock be* (i) *together,* (ii) *separated by* 13 *minute-spaces?*

[In every hour the minute-hand traverses 60 minute-spaces while the hour-hand traverses 5 of such spaces. Thus in every 60 minutes of time the minute-hand will gain 55 minute-spaces on the hour-hand.]

(i) At 4 o'clock the hour-hand is 20 minute-spaces ahead of the minute-hand, so that they will be together when the minute-hand has gained 20 minute-spaces.

Now the min.-hand gains 55 min.-spaces in every $60'$;

that is, ... 20 .... $60' \times \frac{20}{55}$, or $21\frac{9}{11}'$.

Thus the required time is $21\frac{9}{11}'$ past 4.

(ii) This will happen *twice*, namely when the minute-hand is 13 minute-spaces behind and before the hour-hand.

In the first case, the minute-hand has to gain $20-13$, or 7 minute-spaces on the hour-hand.

$\therefore$ the required time is $60' \times \frac{7}{55}$, or $7\frac{7}{11}'$ past 4.

In the second case, the minute-hand has to gain $20+13$, or 33 minute-spaces on the hour-hand.

$\therefore$ the required time is $60' \times \frac{33}{55}$, or $36'$ past 4.

EXAMPLE 2. *In a race of* 200 *yds. B can give a start of* 10 *yds. to A, and C give a start of* 20 *yds. to B; how much start could C give A?*

While *B* runs 200 yds., *A* runs 190 yds. .... (i),
.... 180 ......, *C* ... 200 .... (ii).

To compare *A* and *C*, multiply by 9 in (i) and by 10 in (ii), thus making *B*'s number of yards the same in each, viz. 1800.

Thus while *B* runs 1800, $\begin{cases} A \text{ runs } 190 \times 9, \text{ or } 1710 \text{ yds.,} \\ C \text{ runs } 200 \times 10, \text{ or } 2000 \text{ yds.} \end{cases}$

$\therefore$ *C* runs 200 yds. while *A* runs 171;

that is, *C* can give 29 yds. start to *A*.

EXAMPLE 3. *In what ratio must coffee worth* 2*s.* 2*d. per lb. be mixed with coffee worth* 1*s.* 4*d. per lb., that the mixture may be worth* 1*s.* 8*d. per lb.?*

Suppose x lbs. at 26*d.* are mixed with y lbs. at 16*d.*

Then we have $26x+16y=20(x+y)$;

$$\therefore\ 6x=4y,$$

$$\text{so that } \frac{x}{y}=\frac{2}{3}.$$

Hence the values of x and y are 2 and 3, or any numbers which have the ratio of 2 to 3.

EXAMPLES XII. d.

Find when the hands of a clock are (i) together, (ii) opposite to each other:

1. Between 3 and 4 o'clock. **2.** Between 9 and 10 o'clock.

Find at what times the hands are at right angles:

3. Between 6 and 7 o'clock. **4.** Between 5 and 6 o'clock.

5. At what times between 7 and 8 o'clock will there be 12 minute-spaces between the two hands of a watch?

6. At what times between 2 and 3 o'clock will the hands of a watch be separated by one minute-space?

7. It is between 5 and 6 o'clock, and the distance between the two hands is 3 minute-spaces: what is the time?

8. *A* gives *B* a start of 50 yards; if he runs 11 yards to *B*'s 9, how far has *B* run before he is overtaken?

9. In what ratio must tea worth 2*s.* 8*d.* per lb. be mixed with tea worth 1*s.* 9*d.* per lb. in order that the mixture may be worth 2*s.* per lb.?

10. A wine-merchant buys two kinds of spirits at 14*s.* 6*d.* and 17*s.* 6*d.* per gallon; he blends them and sells the mixture at 16*s.* 8*d.* per gallon without gain or loss. In what ratio was the mixture made?

11. Beer at 10*d.* per gallon is mixed with another kind at 11*d.* per gallon; how many gallons of each must an 18-gallon cask of the mixture contain, if it is worth 15*s.* 10*d.*?

12. *A* can give *B* a start of 20 yds., and *C* a start of 27 yds., in a walking race of a quarter of a mile. How much can *B* give *C* in 120 yds.?

13. In a race of 1 mile *A* beats *B* by 32 yds., and *C* by 48 yds. By how much can *B* beat *C* in 1080 yds.?

14. In 100 yds., *A* gives *B* a start of 10 yds., and *C* one of 15 yds. How much should *B* give *C* in 150 yds.?

15. In a game of billiards *A* can give *B* 20 points in 100; *B* can give *C* 5 points in 100. How many points can *A* give *C*?

16. In a game of 500 up, *A* can give 50 points to *B*, and 68 to *C*; how many points can *B* give *C*?

17. *A* and *B* are out shooting grouse: *A* fires 7 shots to *B*'s 3, but *A* kills one in four, while *B* kills one in two. When *B* has missed 36 shots, how many birds has *A* killed?

18. *A* can beat *B* by 5 yds. in 100; *C* can beat *B* by $14\frac{1}{2}$ yds. in 100. By how much will *C* beat *A* in a mile race, the rates of walking remaining uniform?

19. A circular running path is 726 yds. round. Two men start from the same point, and walk in opposite directions, one at the rate of $3\frac{3}{4}$ mi., and the other at the rate of $4\frac{1}{2}$ mi. per hour. When and where will they meet for the first time?

20. *A* can run 100 yds. in $10\frac{1}{2}$ secs. and *B* can run 100 yds. in $11\frac{1}{5}$ secs. What start must be given to *B* to make the race a dead heat?

21. *A* runs $6\frac{2}{3}$ yds. while *B* runs 7 yds.; *B* runs $16\frac{1}{2}$ yds. while *C* runs 15 yds. If *A* can run a mile in 5 min. 15 secs., what time will *C* take to do it?

22. *A* and *B* start from the same point to run in opposite directions round a circular race-course 550 yds. in circumference, *A* not starting till *B* has run 100 yds. They pass each other when *A* has run 250 yds. Which will come first to the starting point, and at what distance will they then be apart?

23. A ship 44 miles from the shore springs a leak which admits $2\frac{1}{4}$ tons of water in $5\frac{1}{2}$ minutes; 92 tons would suffice to sink her, but the pumps can throw out 12 tons in an hour. Find the average rate of sailing that she may just reach the shore as she begins to sink.

24. *A* and *B* ran a race which lasted a minute and a half. *A* gave *B* a start of 10 yds. and beat him by 1 yd. *A* ran 40 yds. while *B* ran 39 yds. Find the length of the course, and the rates of running of *A* and *B*.

25. A train 88 yds. long overtook a man walking along the line at the rate of 4 mi. per hour, and passed him completely in 10 seconds; what was the rate of the train? If it afterwards overtook a second man, and passed him in 9 seconds, at what rate was he walking?

26. *A* starts 3 minutes after *B* for a place $4\frac{1}{2}$ miles distant. *B*, on reaching his destination immediately returns, and after walking a mile meets *A*. If *A*'s speed be a mile in 18 minutes, what is *B*'s speed?

27. *A* and *B* start at the same time from London to Blisworth, *A* walking 4 miles an hour, and *B* riding 9 miles an hour. *B* reaches Blisworth in 4 hours, and immediately rides back to London. After 3 hours' rest he starts again for Blisworth at the same rate. How far from London will he overtake *A*, who has in the meantime rested for 6 hours?

220. We shall now give some problems which may best be solved by means of equations.

Example 1. *A is two years younger than B; seven years ago five-sixths of A's age was greater than three-tenths of B's age by 9 years; find their present ages.*

Let x years represent A's age, then B's age is $(x+2)$ years. Seven years ago A's age was $(x-7)$ years, and B's age was $(x+2-7)$ years, or $(x-5)$ years.

Hence $\frac{5}{6}(x-7)-\frac{3}{10}(x-5)=9.$

We can *clear this equation of fractions* by multiplying both sides by 30, which is the L.C.M. of the denominators 6 and 10; thus

$$30\times\tfrac{5}{6}(x-7)-30\times\tfrac{3}{10}(x-5)=9\times30$$

or

$$25(x-7)-9(x-5)=270;$$

$$25x-175-9x+45=270;$$

$$\therefore\ 16x=400;$$

$$\therefore\ x=25.$$

Thus A's age is 25 years, and B's age is 27 years.

Example 2. *A man can walk from his house to a railway station and back in a certain time at 4 mi. an hour. If he walks out at 3 mi. an hour and returns at 5 mi. an hour, he takes 10 minutes longer for the double journey. Find the distance between his house and the station.*

Let x be the required distance in miles.

At 4 mi. per hr. he will *go* and *return* in $\dfrac{2x}{4}$ hours, or $\dfrac{x}{2}$ hours.

At 3 .... *go* in $\dfrac{x}{3}$ hours,

and at 5 .... *return* in $\dfrac{x}{5}$ hours.

Also 10 min. $=\frac{1}{6}$ of an hour;

$$\therefore\ \frac{x}{3}+\frac{x}{5}=\frac{x}{2}+\frac{1}{6}.$$

Clearing of fractions, we have

$$10x+6x=15x+5;$$

whence $\qquad x=5.$

Thus the required distance is 5 miles.

EXAMPLE 3. *At noon A starts to ride at 8 mi. an hour; two hours later B starts after him on a bicycle at 12 mi. an hour. How far will A have ridden before he is overtaken by B? Find also at what times A and B will be 8 miles apart.*

Let x represent the number of hours A has ridden before he is overtaken; then B has ridden for $x-2$ hours.

A rides $8x$ miles in x hours,

B ...... $12(x-2)$ .... $x-2$ hours.

And when B overtakes A he has covered the same distance as A;

$$\therefore\ 12(x-2)=8x;$$

$$\text{whence}\qquad x=6.$$

$\therefore$ A has ridden for 6 hours, and has covered 48 miles.

For the second part of the question, if x represents the required number of hours after noon, we have by similar reasoning

$$12(x-2)=8x\pm 8,$$

where in the last term the upper or lower sign is to be taken according as B is 8 miles *ahead of* or *behind* A. In the former case $x=8$, and in the latter $x=4$.

Thus the required times are 4 p.m. and 8 p.m.

EXAMPLES XII. e.

(*Some of the following examples admit of easy solution without the use of algebraical symbols. The method of solution is left to the pupil's discretion.*)

1. B's age exceeds A's by 3 years, and two-thirds of A's age is less than five-sixths of B's by 10 years; what are their ages?

2. A is 9 years younger than B, and 6 years older than C; three-fourths of A's age, four-fifths of B's, and one-half of C's together amount to 37 years: find their ages.

3. If silk costs six times as much as linen, and I spend £9. 8*s.* in buying 23 yards of silk and 50 yards of linen, find the cost of each per yard.

4. How many pounds of tea at 1*s.* 6*d.* and at 2*s.* 6*d.* per lb. must be mixed to make a box of 200 lbs. worth altogether £18?

5. A roll of cloth was bought at 5*s.* 6*d.* a yard, and another roll, 25 yards longer, at 5*s.* a yard; the two together cost £100. 15*s.*: how many yards were there in each roll?

6. How many books can be bought for £5, if 17 cost as much over £2 as 7 of them cost under a sovereign?

[Let x shillings represent *the price of each book*, then

$$17x - 40 = 20 - 7x.]$$

7. If the price of 16 eggs is as much under half-a-crown as the price of 12 exceeds 5*d.*, how many can be obtained for 3*s.* 9*d.*?

8. By buying oranges at 15 for a shilling and selling them at a dozen for 15*d.* a man gained 13*s.* 6*d.*; find the number of oranges.

9. I bought a certain number of articles at five for sixpence; if they had been eleven for one shilling, I should have spent sixpence less: how many did I buy?

10. Two persons start at noon from towns 60 miles apart. One walks at the rate of four miles an hour, but stops $2\frac{1}{2}$ hours on the way; the other walks at the rate of 3 miles an hour without stopping: when and where will they meet?

11. A man can walk from A to B and back in a certain time at the rate of 4 miles an hour; if he walks $3\frac{1}{2}$ miles an hour from A to B, and $4\frac{1}{2}$ miles an hour from B to A, he requires $3\frac{1}{3}$ minutes longer for the double journey: what is the distance from A to B?

12. B has 5 miles start of A, but travels at the rate of only 3 miles an hour, while A travels at the rate of $4\frac{1}{2}$ miles an hour; where will A overtake B, and how long will he take to do it?

13. A boy walks to school at the rate of $3\frac{1}{2}$ miles an hour, and is one minute late; if he had walked at the rate of $3\frac{1}{3}$ miles per hour he would have been 3 minutes late: find the distance to the school.

14. A boy walks to school at the rate of $3\frac{1}{2}$ miles per hour, and is 4 minutes late; the next day he increases his pace by a quarter of a mile per hour, and is 2 minutes late: find the distance to the school.

15. Two trains start at the same time from Sheffield and Leicester and proceed towards each other at the rates of 30 and 50 miles per hour respectively. When they meet it is found that one train has run $14\frac{1}{2}$ miles more than the other. Find the distance between Sheffield and Leicester.

16. Two cyclists start from the same place to ride in the same direction. A starts at noon at 8 mi. an hour, and B starts at 1.30 p.m. at 10 miles an hour. How far will A have ridden before he is overtaken by B? Find also at what times A and B will be 5 miles apart.

17. If P and Q represent two towns 35 miles apart, and if A walks from P to Q at $3\frac{1}{2}$ mi. an hour while B walks from Q to P at $2\frac{1}{2}$ mi. an hour, both starting at 9 a.m., when will they be 5 mi. apart?

18. Two men ride towards each other from two places 60 miles apart, one at 12 mi. an hour, and the other at 9 mi. an hour. Find when they are first 18 miles apart. How must your equation be altered so as to find the time when they are 18 miles apart after meeting?

19. A, B, and C set out to walk from Bath to Bristol at 5, 6, and 4 mi. an hour respectively. C starts 3 minutes before, and B 7 minutes after A. Find (i) when and where A overtakes C; (ii) when and where B overtakes A.

20. I row against a stream flowing $1\frac{1}{2}$ mi. an hour to a certain point, and then turn back, stopping 2 mi. short of the place whence I originally started. If the whole time occupied in rowing is 2 hrs. 10 mins., and my uniform speed in still water is $4\frac{1}{2}$ mi. an hour, find how far upstream I went.

21. A cyclist has to ride 75 miles. He rides for a time at 9 mi. an hour and then alters his speed to 15 mi. an hour, covering the distance in 7 hours. At what time did he change his speed?

22. A dairyman buys milk at 5*d.* per quart: how must he mix it with water so that he can sell it at $4\frac{1}{2}$*d.* per quart without loss?

23. If 18 gallons of spirits at 18*s.* 6*d.* per gallon are mixed with 14 gallons at 21*s.* per gallon, how much water must be added to reduce the value to 16*s.* 6*d.* per gallon?

24. A can run 50 yds. while B runs 45 yds.: if B has 5 minutes start in a race, what time will A take to get level with B?

25. A person swimming in a stream which runs $1\frac{1}{3}$ mi. an hour finds that in a given time he can swim five times as far with the stream as he can against it: at what rate does he swim?

26. Three trains A, B, C travel on the railway from Bristol to Hull, a distance of 220 miles, at the rate of 25, 20, 30 miles per hour respectively; A and B leave Bristol at 7 a.m. and 8.15 a.m. respectively, and C leaves Hull at 10.30 a.m.: when and where will A be equidistant from B and C?

27. An electric train is ten minutes late when it performs its usual journey at 10 mi. an hour, but only five minutes late when it travels at 12 mi. an hour: find the length of its journey.

[Let x mi. represent the distance, and let a be the number of hours the train takes when running to time; then

$$a=\frac{x}{10}-\frac{1}{6}; \text{ also } a=\frac{x}{12}-\frac{1}{12}.]$$

MISCELLANEOUS EXAMPLES III.

1. On a certain day the average price of wheat was 28*s.* 5*d.* per 480 lbs., of barley 24*s.* 5*d.* per 400 lbs., of oats 17*s.* 11*d.* per 312 lbs. Find, correct to the nearest penny, the price of each per cwt. (C.S.)

2. Eleven persons contributed a certain sum. Nine of them gave £5 apiece, and the other two gave £5 and £10 more respectively than the average subscription of the eleven subscribers. What sums did the two persons give?

3. The tax on tobacco at the average rate of 4*s.* for 1 lb. produced £13,184,767 in the year ending March 31st, 1905. Find, to the nearest ton, the weight of tobacco taxed.

4. Find, to the nearest penny, the price per lb., in English money, of tea which costs in France 3·50 francs per kilogram, assuming that 1 Kg. = 2·204 lbs., £1 = 25·34 fr.

5. A labourer who was earning, wet or fine, £1 a week, took work at 7*d.* an hour. He worked 8 hours a day and $5\frac{1}{2}$ days a week; but in the course of 52 weeks he lost 15 days' work through rain. What did he gain in the 52 weeks by the change?

6. Find the value of

$$35{\cdot}705682 \times 581{\cdot}35823 \div 82{\cdot}05972$$

to two places of decimals, using contracted methods.

7. Give, to the nearest penny, the sum wanted to complete the following letter: "Dear Sir, we hand you herewith London draft for   , being the equivalent in English money of 15 dollars, at the rate of 4·875 dollars to the pound sterling." (C.S.)

8. A boy had to multiply £5·786495 by 317; but he disregarded all figures in the multiplicand after the third decimal place. Find, to the nearest farthing, what the consequent error was. (C.S.)

9. A bicyclist rode from A to B, a distance of $25\frac{1}{2}$ miles, in 2 hours 15 minutes. What was his average speed in feet per second? Give your answer to the nearest whole number.

The bicyclist returned from B to A by another route, by which the distance was 28 miles, and at an average speed which was five-sixths of the average speed on the outward journey. How long (to the nearest minute) did the return journey occupy? (C.S.)

10. From the following data, find (to the nearest whole number) how many gallons of beer were drunk per head, and (to the nearest shilling) how much per head was contributed to the revenue in this way:

Year.	Population.	Barrels of beer drunk. 1 barrel = 36 gallons.	Revenue from beer in £.
1904–5	43,331,000	35,862,068	13,123,679

(C.S.)

11. Multiply 1·003659 by 273·3, correct to four decimal places. Find the numerical value, when $x=11$, of

$$x^3-10{\cdot}9x^2+9{\cdot}4x-102.$$

12. Taking 70 yards as equivalent to 64 metres, shew that a man going 2 metres per second will walk 18 miles in 4 hrs. 1 m. 23 s.

13. *A* can give *B* 36 yards in a race of 300 yards, and *A* can give *C* 33 yards in 330; find what start *C* should allow *B* in a race of 300 yards.

14. In travelling by train the knocks heard indicate the passage of the carriage from one rail to the next. Verify the truth of the following general statement for the particular case of 34 miles an hour: that, when the length of each rail is 22 feet, the number of knocks heard in a quarter of a minute is the number of miles per hour the train is travelling.

15. A man paid this year for income tax £20. 12*s.* 6*d.* at 11*d.* in the £, and the year before £26. 11*s.* 3*d.* at 1*s.* 3*d.* in the £. If he obtains the same increase of salary next year and the income tax is 10*d.* in the £, what will his income tax amount to?

16. A man has to go by train from A to B and back each day (Sunday excepted) for 40 weeks every year. A return ticket costs 4*s.* 6*d.*, except on Saturday, when it costs 3*s.* 1*d.* What will he save annually by taking a season ticket which costs £25 a year?

17. Multiply 1·423056... by 32·6231... correct to 3 places of decimals, and divide 43·7846... by 6·4395... correct to 2 places of decimals.

18. Five men can do a piece of work in 14 days; if they have two boys to help them they can do it in 12 days; find what proportion of a man's work a boy can do.

19. A crew, rowing 32 strokes a minute, rows a mile in 7 min. 35 secs.; find, correct to an inch, the distance travelled each stroke.

20. Find the sum of $\frac{4}{9}$ of £17. 14*s.* 9*d.*, 4757 farthings, and £17·675.

21. If 25 francs = £1, and a gram = 0·002204 lbs., find in French money the price of a kilogram of an article which costs a shilling an ounce.

22. The claims against a bankrupt company are £5000 claimed by the bank for an overdraft, £4000 claimed by *A*, and £11,000 by other creditors. Moreover, *A* has guaranteed the bank against loss. The available assets are £4000. The case is taken into court, where *A*'s claim against the company is reduced to £2000, and the other claims are allowed in full. How much more is *A* out of pocket after discharging his guarantee than he would have been if the court had not reduced his claim? (C.S.)

23. High water at Cork is 3 hours later, and at Boulogne $2\frac{1}{2}$ hours earlier, than at London Bridge. High water at Boulogne on Aug. 23 was at 10.3 a.m. (French time). Find the local time of the corresponding high water at Cork, given that English time is 10 minutes behind French time and 25 minutes ahead of Irish time. (C.S.)

24. Platinum costs £4. 18*s.* 10*d.* per Troy ounce. What will a platinum crucible weighing 21·6 grams cost, the cost of workmanship being neglected? [A Troy ounce is 31·1 grams.]

25. A man 30 years of age has accumulated £3000 and has a salary of £600 a year, of which he spends £300 and saves £300. He determines to stop work (and give up his salary) as soon as his savings would enable him to continue spending £300 a year until the age of 90. How long must he continue to work? *N.B.*—No interest on savings is to be reckoned. (C.S.)

26. A train 210 yards long, going 25 miles an hour, overtakes another train 318 yards long, going 17 miles an hour in the same direction on a neighbouring parallel line; how long will the first train take in passing the second one?

27. A man buys 500 yards of copper wire at 2*d.* per yard, and sells it in France at 30 centimes per metre. Find his profit in English money, correct to a penny, having given

£1 = 25·34 francs, and 1 yd. = 0·914 metre.

28. A man lives at a place from which there are two railway lines to London, belonging to different companies. On three days of the week he goes and returns by the same line; on the other three week days he is obliged to travel to London by this line, and return by the other; and on Sundays he does not travel at all. The charges by either line are 1*s.* for a third-class single ticket, 1*s.* 10*d.* for a third-class return. A season ticket for a year costs £20. 8*s.* by the railway he usually travels over. If he has 5 weeks' holiday during the year, and does not travel by either line during that time, will it pay him to take a season ticket? (C.S.)

29. For estimating the rental value of property a valuer charges 5 per cent. on the first £200 and $2\frac{1}{2}$ per cent. on the remainder of the estimated rental value. What will his charge be for valuing property which he estimates at a rental value of £1735?

30. On the Great Western Railway, among the receipts one year were £356,926 derived from first-class passengers, and £472,793 from second-class passengers. The rates on which the fares are based are 2*d.* and $1\frac{1}{4}$*d.* per mile respectively. If the company had abolished the second class, and had reduced the rate for first class to $1\frac{1}{2}$*d.*, find how much the company would have gained or lost, assuming that all the second-class passengers would have travelled first class. (C.S.)

31. I know the average weight of five metal bars to be 2 kilograms 75 grams, and the weights of four of them to be 3 kilograms 8 grams, 2 kilograms 92 grams, 2 kilograms 14 grams, and 1 kilogram 9 grams, respectively. All these weights being correct to the nearest gram, between what limits must the weight of the fifth bar lie? (C.S.)

32. The population of a certain town is now 77,793. The annual birth-rate is 30·8 per 1000, and death-rate 21·7 and the number of new settlers is equal to the number of people that leave the town. If these rates do not change, what will be the population three years hence to the nearest hundred? (C.S.)

33. A journey of 132 miles can be performed either by river steamer or by train. The charge by steamer is at the rate of 1 shilling for 20 miles, and by train at the rate of 1 penny per mile. I cannot afford to pay more than 8 shillings for the journey. What is the least distance which I must travel by steamer? (C.S.)

34. *A* and *B* in one boat challenge *C* and *D* in another boat to a single sculling match of 50 miles. *A* and *B* take it in turns to row stretches of 6 miles, *A* beginning. *C* and *D* take it in turns to row for an hour, *C* beginning. If *A* and *C* can each row $5\frac{1}{2}$ miles an hour, while *B* and *D* can only row 5 miles an hour, which boat wins and by how much? (C.S.)

35. Three boys, *P*, *Q*, *R*, set out for a town some miles away, and use a tandem bicycle to get there sooner. Each walks 4 miles an hour, and any two on the bicycle go 10 miles an hour. *P* sets off on foot, *Q* and *R* on the bicycle. *Q* and *R* ride 2 miles, then *Q* goes on foot while *R* waits for *P*. How long does *R* wait, and how far does *Q* walk while he is waiting? When *P* arrives, *P* and *R* ride on till they overtake *Q*. At what distance does this happen? And at what time from the start? (C.S.)

PART II.

CHAPTER XIII.

SQUARE ROOT.

221. THE terms **square root** and **cube root** have been defined in Chap IV., and it was there explained how to find the square or cube root of certain numbers which can be readily expressed in factors. The pupil is recommended here to revise Arts. 84 and 85.

The principal object of the present chapter is to explain a general process for finding square roots. The process by which roots of numbers are found is known as **evolution** or the **extraction of roots.**

222. The following table should be carefully studied. The pupil will find it useful to extend it for himself.

Number	1	2	3	4	5	6	7	8	9	10	11	12	...
Square	1	4	9	16	25	36	49	64	81	100	121	144	...

Here we notice

(i) That a number consisting of *one* digit has either *one* or *two* digits in its square.

(ii) That none of the numbers from 1 to 9 has a square ending in 2, 3, 7, or 8. And since every number must end in one of the nine digits or zero, it follows that *no square number* can end with 2, 3, 7, or 8.

Thus the square of every number must end in 0, 1, 4, 5, 6, or 9.

(iii) Every integer is the square root of some other integer, but only a few numbers have an exact integral square root. For example, 40 lies between 36 and 49, and its square root lies between 6 and 7, and is not integral.

223. A number which has an exact square root, either integral or fractional, is called a **perfect square.**

Thus 64, 169 are perfect squares whose square roots are 8 and 13 respectively.

Also $\frac{64}{169}$ is a perfect square whose root is $\frac{8}{13}$.

It will be seen later that an approximate square root of any number which is not a perfect square can always be found to any required degree of accuracy.

224. From the table in Art. 222, it is seen that a number of 1 digit has either 1 or 2 digits in its square.

Again since $10^2=100$, $100^2=10000$, $1000^2=1000000$, and so on, it is evident that

a number of 2 digits, that is a number lying between 9 and 100, has either 3 or 4 digits in its square;

a number of 3 digits, that is a number lying between 99 and 1000, has either 5 or 6 digits in its square; and so on.

Thus for every additional digit in a number there are two additional digits in the square of the number. If then in any given number whose square root is required we separate the digits into *periods of two beginning from the right*, the number of digits in the square root will be equal to the number of such periods. If the given number consists of an odd number of digits the last period will contain only one digit. Thus the square roots of the numbers 34ˎ81 and 4ˎ15ˎ75ˎ21 will have 2 and 4 digits respectively.

225. *The square of the sum of two numbers is equal to the sum of the squares of the two numbers + twice the product of the two numbers.*

Thus if a and b are any two numbers,

$$(a+b)^2=a^2+2ab+b^2; \qquad \text{[Art. 57]}$$

$$\therefore\ (a+b)^2-a^2=(2a+b)b.$$

Similarly $$(40+6)^2=40^2+\textit{twice}\ (40\times 6)+6^2;$$

$$\therefore\ \text{also}\ 46^2-40^2=(\textit{twice}\ 40+6)\times 6.$$

In the same way, we have

$$329^2=(320+9)^2$$
$$=320^2+\textit{twice}\ (320\times 9)+9^2,$$

or $$329^2-320^2=(\textit{twice}\ 320+9)\times 9.$$

Example 1. *To find the square root of* 2116.

Supposing we know the result to be 46, we have to devise a process for finding the digits 4 and 6.

$$\begin{array}{rl}
 & 21\,16\ (40+6 \\
 & 16\,00 \\ \cline{2-2}
\left.\begin{array}{r}40\times 2\\ +6\end{array}\right\}=86 & \left|\begin{array}{l}5\,16\\ 5\,16\end{array}\right. \\ \cline{2-2}
\end{array}$$

Marking off the digits in periods of two, beginning from the right, we see that the root will consist of 2 digits. Also 2116 lies between 40^2 and 50^2. And since

$$46^2-40^2=(\text{twice } 40+6)\times 6,$$

we see that after subtracting 40^2 (or 1600) from 2116, the remainder when divided by (twice 40+6) will give the second digit of the root. Part of this divisor is the digit we are seeking to complete the root; but the more important part is twice the number 40 already found. Hence we use this part as an approximate divisor, and the next digit of the root is obtained by trial. In practice we take 51 as a 'trial dividend,' instead of the full remainder 516, and 8 as 'trial divisor.'

The next example shews how the process may be extended, if necessary, beyond the second figure of the root.

Example 2. *Find the square root of* 108241.

(i)

$$\begin{array}{rl}
 & 10\,82\,41\ (\,300+20+9 \\
 & 9\,00\,00 \\ \cline{2-2}
\left.\begin{array}{r}300\times 2\\ +20\end{array}\right\}=620 & \left|\begin{array}{r}1\,82\,41\\ 1\,24\,00\end{array}\right. \\ \cline{2-2}
\left.\begin{array}{r}320\times 2\\ +9\end{array}\right\}=649 & \left|\begin{array}{r}58\,41\\ 58\,41\end{array}\right. \\ \cline{2-2}
\end{array}$$

(ii)

$$\begin{array}{rl}
 & 10\,82\,41\ (\,329 \\
 & 9 \\ \cline{2-2}
62 & \left|\begin{array}{r}182\\ 124\end{array}\right. \\ \cline{2-2}
649 & \left|\begin{array}{r}5841\\ 5841\end{array}\right. \\ \cline{2-2}
\end{array}$$

∴ the required square root is 329.

(i) shews the work in full; (ii) is the practical arrangement, shortened (as in long division) by the omission of *place* ciphers.

In (ii) the process is as follows. After marking off the periods from the right, the highest square below 10 (the last period) is 9; its square root is 3, which is placed as first digit of the root. After subtracting the square of 3, the first remainder is 1, to which we affix the period 82, and the dividend is 182. We use 18 as 'trial dividend' and *twice* 3, or 6, as 'trial divisor.' This suggests 3 as the digit for the tens' place. This proves too large and we select 2. The next remainder is 58, to which we affix the period 41. The portion of the root obtained so far is 32. Hence, the new 'trial divisor' is 32 × 2, or 64. Using 584 as 'trial dividend' we obtain the final digit 9.

Reason for the process. In (i), at the first stage we subtract 300^2; at the second we subtract from the remainder (twice $300+20)\times 20$, that is (twice $300\times 20+20^2$); so that up to this point we have subtracted 300^2+ twice $(300\times 20)+20^2$, which is equal to $(300+20)^2$ or 320^2.

From the last remainder we subtract (twice $320+9)\times 9$.

So that in all we subtract in successive operations

320^2+ twice $(320\times 9)+9^2$, which is equal to $(320+9)^2$ or 329^2.

There is no remainder. Thus 329 is the square root.

EXAMPLE 3. *Find the square root of* 13010449.

```
         (i)
     13,01,04,49 ( 3607
      9
66   | 401
     | 396
720  |   50449
7207 |   50449
```

```
         (ii)
     13,01,04,49 ( 3607
66   | 401
7207 |    50449
```

The required square root is 3607.

Here after two digits have been obtained we find that the 'trial divisor' 72 will not divide into 50; thus the next digit in the root is 0. We place this in the root and also in the divisor, and bring down the next period.

(ii) Shews how the work may be further shortened by the Italian method.

EXAMPLES XIII. a.

Find the square root of

1. 361.
2. 529.
3. 841.
4. 961.
5. 1369.
6. 1849.
7. 2304.
8. 3481.
9. 3364.
10. 5329.
11. 37249.
12. 53824.
13. 19600.
14. 95481.
15. 549081.
16. 501264.
17. 819025.
18. 2819041.
19. 18671041.
20. 13704804.
21. 1006009.
22. 420865225.
23. 8617223241.
24. 1157428441.

25. From the work in Ex. 3, Art. 225, shew that the square of 3600 is less than 13010449 by 50449.

26. By finding part of the square root of 174345616, find by how much 174345616 exceeds (i) $(13000)^2$, (ii) $(13200)^2$.

27. Given that 184041 is the square of 429, find as shortly as possible the square root of 18464209.

226. Square root of a decimal. The principle of place value of successive digits enables us to use the foregoing method to extract the square root of a decimal number. By the principles of multiplication of decimals, it is evident that the square of any decimal must always contain an even number of decimal digits, and that the number of decimal digits in the square will be double the number of such digits in the square root.

For example: $(\cdot2)^2=\cdot04$, $(\cdot8)^2=\cdot64$, $(\cdot13)^2=\cdot0169$,
$(\cdot05)^2=\cdot0025$, $(2\cdot8)^2=7\cdot84$, $(6\cdot3)^2=39\cdot69$.

Hence in finding the square root of a decimal we begin by marking off periods of two digits to right and left *starting from the decimal point.* Each period will furnish one digit to the square root.

Example. *Find the square root of*

(i) 18·4041; (ii) ·00367236.

(i)

```
       18,·40,41 ( 4·29
       16
 82  |  240
     |  164
 849 |   7641
     |   7641
```

The square root = 4·29.

(ii)

```
       ·00,36,72,36 ( ·0606
           |  36
           |  36
 120       |     7236
 1206      |     7236
```

The square root = ·0606.

In (i) the square root is obviously greater than 4 and less than 5. After obtaining the first digit of the root we place the decimal point, and the rest of the work proceeds exactly as in the case of integers.

In (ii) as the first period of the decimal consists of two ciphers, the corresponding first digit in the root will be a cipher after the decimal point. Bringing down the next period we have 6 for the corresponding digit in the root, and no remainder. Bringing down the third period we have 12 for 'trial divisor' and 7 for 'trial dividend.' Hence the next digit in the root is 0.

EXAMPLES XIII. b.

Find the square root of the following decimals:

1. 11·56.
2. 20·25.
3. ·6241.
4. ·0289.
5. 7·3441.
6. 41·2164.
7. 49·1401.
8. 998·56.
9. 99·8001.
10. 1218·7081.
11. 5·774409.
12. ·00139876.

Find the square root of the following decimals:

13. 9·504889. **14.** 557·196025. **15.** ·0001595169.

16. 304·607209. **17.** 2704·416016. **18.** 67620·8016.

19. Shew from the Example of Art. 226 that $(4{\cdot}2)^2$ is less than 18·4041 by ·7641.

20. Find by how much 5·354596 exceeds $(2{\cdot}31)^2$.

227. If a number is not a perfect square its square root can be found to any required number of figures, but the process can never terminate.

EXAMPLE 1. *Find the first four figures of the square root of* 5·23.

$$\begin{array}{l|r}
 & 5_{\backslash}{\cdot}23_{\backslash}00_{\backslash}00_{\backslash}\ldots\ (2{\cdot}286\ldots \\
 & 4 \\ \hline
42 & 123 \\
 & 84 \\ \hline
448 & 3900 \\
 & 3584 \\ \hline
4566 & 31600 \\
 & 27396 \\ \hline
 & 4204
\end{array}$$

After bringing down the last significant figure, two ciphers are brought down at each stage, and appended to the remainder at that stage. Hence no new figure in the root can produce a 'divisor' which will bring the process to an end.

The required square root $=2{\cdot}286\ldots$.

NOTE 1. By observing the remainders at the different stages of work we have the following results:

$$5{\cdot}23-(2{\cdot}2)^2={\cdot}39,\quad 5{\cdot}23-(2{\cdot}28)^2={\cdot}0316,$$

$$5{\cdot}23-(2{\cdot}286)^2={\cdot}004204,\ \text{and so on.}$$

And as these remainders are getting smaller and smaller, it follows that with each new figure in the root we obtain a result which is a closer approximation to the true value than the result at the preceding stage.

NOTE 2. In finding the approximate square root of a decimal it is particularly important to observe the rule for pointing the digits in pairs *from the decimal point.* Thus the square roots of

·4, 2·5, 8·1, and ·121

are *not* ·2, ·5, ·9, and ·11;

for by writing the decimals in the form

$${\cdot}40_{\backslash}00_{\backslash}\ldots,\quad 2_{\backslash}{\cdot}50_{\backslash}00_{\backslash}\ldots,\quad 8_{\backslash}{\cdot}10_{\backslash}00_{\backslash}\ldots,\quad {\cdot}12_{\backslash}10_{\backslash}00_{\backslash}\ldots,$$

it is evident that the first digits of the roots are ·6, 1, 2, and ·3 respectively.

EXAMPLE 2. *Find the first 7 figures of the square root of 2.*

(i)

	2,·00,00... (1·414213...	
	1	
24	100	
	96	
281	400	
	281	
2824	11900	
	11296	
2828⋮2	604	00*
	565	64
2828⋮41	38	3600
	28	2841
2828⋮423	10	075900
	8	485269
	1	590631

(ii)

	2,·00,00,... (1·414
24	100
281	400
2824	11900
$2,8,2\cancel{8}$	604 (213
	38
	10
	2

Thus the required square root $=1·414213....$

In (i) the work is given in full; in (ii) the Italian method has been used.

At the stage marked * *four* digits of the root have been obtained, and the 'trial divisor' consists of *four* digits, viz. 2828. The remainder at this stage is 604, and if instead of bringing down a new period, and appending a new digit to the divisor, we divide 604 by 282(8), cutting off the last digit and using the contracted method, we can obtain *three* new digits of the root, as shewn in (ii), and it is clear that the work is merely the shortened form of that shewn to the left of the vertical line in (i).

This example illustrates a general rule which may be enunciated as follows:

When a certain number of digits of a square root have been found by the ordinary rule, one less *than the number already found can be obtained by contracted division.*

For a general proof of the rule, the student is referred to Hall and Knight's *Elementary Algebra*, Art. 220.

228. Though it is impossible to find an exact number which when multiplied by itself is equal to 2, by carrying on the work of the preceding example far enough we can find a decimal number whose square can be made to differ from 2 by as little as we please. [Compare Art. 227, Note 1.] This is what is meant when we speak of "the square root of 2."

EXAMPLES XIII. c.

Find the square root of the following, correct to three places of decimals.

[*If necessary the fourth decimal digit must be examined in order to correct the third.*]

1. 3. **2.** 5. **3.** 6. **4.** 10.
5. 7. **6.** 2·5. **7.** 0·47. **8.** 0·07.
9. 0·51. **10.** 0·051. **11.** 0·144. **12.** 235·6.
13. 133·05. **14.** 17·453. **15.** 290. **16.** $0{\cdot}57\dot{1}$.

[*The square root of a fraction*, if the denominator is a perfect square, *is found by taking the square root of the numerator and denominator separately.*

Thus $\sqrt{\frac{9}{25}}=\frac{3}{5}$, because $\frac{3}{5}\times\frac{3}{5}=\frac{9}{25}$.

A mixed number must first be expressed as an improper fraction.]

Find the square root of

17. $3\frac{1}{16}$. **18.** $1\frac{25}{144}$. **19.** $7\frac{18}{49}$. **20.** $\frac{361}{576}$.
21. $37\frac{212}{841}$. **22.** $\frac{5329}{95481}$. **23.** $\frac{11881}{41209}$. **24.** $1125\frac{711}{729}$.

229. In the case of a fraction whose denominator is not a perfect square, we may either convert the fraction into a decimal as a first step, or multiply numerator and denominator by such a number as will make the denominator a perfect square.

EXAMPLE. *Find the square root of* (i) $2\frac{45}{59}$; (ii) $1\frac{5}{18}$.

$$\text{(i)}\quad \sqrt{2\tfrac{45}{59}}=\sqrt{2{\cdot}7627\ldots}=1{\cdot}6621\ldots.$$

$$\text{(ii)}\quad \sqrt{1\tfrac{5}{18}}=\sqrt{\frac{23}{18}}=\sqrt{\frac{23\times2}{18\times2}}=\frac{\sqrt{46}}{6}=\frac{6{\cdot}78233\ldots}{6}=1{\cdot}13038\ldots.$$

The first method is usually to be preferred unless the denominator is small.

EXAMPLES XIII. c. (*Continued.*)

Find correct to three places of decimals the square root of

25. $\frac{1}{8}$. **26.** $\frac{7}{5}$. **27.** $\frac{5}{7}$. **28.** $4\frac{3}{13}$.
29. $1\frac{1}{17}$. **30.** $\frac{8}{11}$. **31.** $\frac{1}{0{\cdot}4}$. **32.** $\frac{2{\cdot}5}{0{\cdot}16}$.

Evaluate the following square roots, using the method of Ex. 2, Art. 227, to shorten the work.

33. $\sqrt{\cdot 0565}$ to 5 places.

34. $\sqrt{\cdot 036}$ to 6 places.

35. $\sqrt{\frac{7}{19}}$ to 5 places.

36. $\sqrt{3\frac{2}{29}}$ to 6 places.

Surds.

230. If the root of a number cannot be exactly obtained, the root is called a **surd** or **an irrational number.**

Thus $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ are surds, and they are said to be **incommensurable**, since the decimal part of such quantities cannot be absolutely expressed as a fraction of the unit.

231. It will be found that $\sqrt{5}=2\cdot 236068\ldots$, that is $\sqrt{5}$ lies between $2\cdot 23606$ and $2\cdot 23607$; and therefore the error in using either of these quantities instead of $\sqrt{5}$ is less than $\cdot 00001$. By taking the root to a greater number of decimal places we can approximate still nearer to the true value.

232. Expressions involving surds, such as $\frac{1}{\sqrt{3}}$, $\frac{4}{\sqrt{5}-1}$, $\frac{\sqrt{3}+1}{\sqrt{3}-1}$, $\frac{1}{2+\sqrt{3}}$, are of frequent occurrence. If in dealing with such expressions we were at once to substitute approximate numerical values for the surds, we should often be involved in tedious and unnecessary labour. To avoid this it is advisable to postpone substitution until the surd expressions have been simplified as far as possible by means of the following formulæ.

(i) $\sqrt{a}\times\sqrt{b}=\sqrt{ab}$. Thus $\sqrt{2}\times\sqrt{3}=\sqrt{6}$.

(ii) $\sqrt{a^2\times b}=\sqrt{a^2}\times\sqrt{b}=a\sqrt{b}$. Thus $\sqrt{18}=\sqrt{9\times 2}=3\sqrt{2}$.

(iii) $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})=a-b$.

Thus $(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})=5-3=2$.

EXAMPLE. *Find the value of* (i) $\frac{1}{\sqrt{5}}$; (ii) $\frac{\sqrt{7}}{\sqrt{7}-\sqrt{5}}$, *each correct to five figures.*

In each of these examples the first step is to get rid of surds in the *denominator.* Moreover in (ii) it will be noticed that if the denominator is not freed of surds we shall require the values of $\sqrt{7}$ and $\sqrt{5}$, and shall then have to divide one long decimal by another.

$$\text{(i)}\ \frac{1}{\sqrt{5}}=\frac{1}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}=\frac{\sqrt{5}}{5}=\frac{2\cdot 236068\ldots}{5}=\cdot 44721\ldots.$$

$$\text{(ii)}\ \frac{\sqrt{7}}{\sqrt{7}-\sqrt{5}}=\frac{\sqrt{7}}{\sqrt{7}-\sqrt{5}}\times\frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}+\sqrt{5}}=\frac{\sqrt{7}\times\sqrt{7}+\sqrt{7}\times\sqrt{5}}{(\sqrt{7})^2-(\sqrt{5})^2}$$

$$=\frac{7+\sqrt{35}}{7-5}=\frac{7+\sqrt{35}}{2}$$

$$=\frac{7+5{\cdot}91607\ldots}{2}$$

$$=\frac{12{\cdot}91607\ldots}{2}$$

$$=6{\cdot}4580\ldots.$$

233. Cube, fourth, and higher roots. It will be seen in a subsequent chapter that the extraction of roots can be very easily effected by the aid of logarithms. For this reason, and because cube and higher roots are rarely required, we do not here give any specific method for their extraction.

234. Since the fourth power of a number is the square of its square, the fourth root of a number is the square root of its square root.

Thus $3^4=81=9^2$; and 9 is the square of 3,

and $\sqrt[4]{81}=3=\sqrt{9}$; and 9 is the square root of 81.

EXAMPLES XIII. d.

Find the value of the following surd expressions, correct to four places of decimals :

1. $\frac{2}{\sqrt{3}}$.
2. $\frac{15}{\sqrt{5}}$.
3. $\frac{2}{\sqrt{8}}$.
4. $\frac{8}{\sqrt{12}}$.
5. $\frac{48}{\sqrt{6}}$.
6. $\frac{\sqrt{2}}{\sqrt{3}}$.
7. $\frac{10}{\sqrt{7}}$.
8. $\frac{25}{\sqrt{11}}$.
9. $\frac{1}{2\sqrt{3}}$.
10. $\frac{1}{\sqrt{24}}$.
11. $\frac{1}{2+\sqrt{3}}$.
12. $\frac{1}{\sqrt{3}-1}$.
13. $\frac{4}{\sqrt{5}-1}$.
14. $\frac{\sqrt{3}-1}{\sqrt{3}+1}$.
15. $\frac{24}{7-\sqrt{5}}$.
16. $\frac{\sqrt{5}+3}{\sqrt{5}-2}$.

Find the fourth root of

17. 331776.
18. 108243216.
19. ·000006765201.

Evaluate to 3 significant digits

20. $\sqrt[4]{53{\cdot}686}$.
21. $\sqrt[4]{1723{\cdot}58}$.

235. Miscellaneous Examples involving square root.

EXAMPLE 1. *A rectangular garden, the length of which is four times its breadth, has an area of 2 ac. 321 sq. yds.; find its length and breadth to the nearest yard.*

Let the sides be denoted by x and $4x$ yards respectively;

then the area $= x \times 4x$ sq. yds.

But the area $= 10041$ sq. yds.;

$\therefore \; 4x^2 = 10041$;

whence $x = 50{\cdot}1\ldots$

and $4x = 200{\cdot}4\ldots$.

```
4 | 100 41
   ------
     25,10,·25 ( 50·1
     25
1001  ┌──
      │  1025
      │  1001
      │  ────
      │    24
```

Thus the required lengths are 50 yards, and 200 yards.

EXAMPLE 2. *A square field contains 10 acres. Find the cost of running a fence round it at 3s. 6d. a yard. Also find to the nearest penny, the cost of a chain, at 1s. 4d. a yard, which will reach from one corner to the opposite one.*

(i) If the length of a side $= x$ yards, we have

$$x^2 = 10 \times 4840; \quad \text{whence } x = 220.$$

$$\therefore \text{ cost of fence} = \frac{3\frac{1}{2} \times 220 \times 4}{20} \text{ pounds}$$

$$= £154.$$

(ii) Let the diagonal of the square be denoted by d yards.

Then by a well-known property of a right-angled triangle,

$$d^2 = 220^2 + 220^2 = 220^2 \times 2;$$

$$\therefore \; d = 220\sqrt{2}.$$

$$\therefore \text{ required cost} = £(1\tfrac{1}{3} \times 11\sqrt{2}) = £(1\tfrac{1}{3} \times 11 \times 1{\cdot}41421\ldots)$$

[Art. 227, Ex. 2.]

$$= £20{\cdot}742, \text{ to 3 places of decimals,}$$

$$= £20.\ 14s.\ 10d.$$

NOTES. (i) If the sides of a right-angled triangle are a, b, c units respectively (the side c being opposite the right angle), $c^2 = a^2 + b^2$.

(ii) If the side of a square is a units, its diagonal is $a\sqrt{2}$ of such units.

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EXAMPLES XIII. e.

1. The adjacent sides of a rectangular field are 161 yds. and 240 yds.; find the length of a diagonal.

2. A rectangular courtyard measures 137 ft. diagonally from corner to corner; if one side is 88 ft., find the other.

3. The area of a square field falls short of 10 acres by 439 sq. yds.; find the length of each side.

4. Find, to the nearest yard, the length of each side of a square field whose area is 439 ac. 33 p.

5. The sides of a rectangle are 577 m. and 450 m. respectively; find to the nearest metre, the side and diagonal of a square of equal area.

6. Find the values of x which satisfy the following equations:

(i) $\frac{x}{54}=\frac{1944}{x}$; (ii) $\frac{1625}{x}=\frac{x}{65}$; (iii) $\frac{x}{23{\cdot}03}=\frac{16{\cdot}92}{x}$.

7. The area of a square field is 1 ac. 785 sq. yds. Find the cost of fencing it round at 2*s*. 7*d*. per yard.

8. How long will it take to walk round a square field whose area is 160 ac., at the rate of 4 mi. per hour.

9. The length of a rectangular courtyard is 3 times its breadth: if its area is 2715 sq. yds., find the length of its sides to the nearest tenth of a yard.

10. The cost of levelling and turfing a square cricket field at £175. 9*s*. 4*d*. per acre is £987. Find the cost of surrounding it with railings at 3*s*. 2*d*. per yard.

11. The cost of paving a square court with stone slabs, each 18 inches square, at 2*s*. 3*d*. a slab, is £16. 4*s*. What is the length of each side of the court?

12. There is a square enclosure of 10 acres; a man walks at the rate of 3 mi. an hour along one side, along a diagonal, along another side, and so returns along the other diagonal to the starting point. How many minutes does it take him?

13. ABC is a triangular field of which the base BC = 150 yards, and the height (that is, the perpendicular drawn from A to BC) is 88 yards. Find the area of the triangle; and also find to the tenth of a yard the side of a square field of equal area.

[N.B. The area of a triangle is half that of a rectangle on the same base and of the same height.]

14. The area of a circle in square centimetres is given by the formula $A = \pi r^2$, where r denotes the radius in centimetres, and π is a constant multiplier approximately equal to 3·1416.

(i) Find to the nearest sq. mm. the area of a circle whose diameter is 20 cm.

(ii) Find to the nearest mm. the side of a square whose area is equal to that of the circle.

15. If a, b, c denote the sides of a triangle, its area may be found from the formula

$$\textit{area of triangle} = \tfrac{1}{4}\sqrt{(a+b+c)\times(b+c-a)\times(c+a-b)\times(a+b-c)}.$$

Find the areas of the triangles whose sides are

(i) 11 inches, 9 inches, 8 inches;

(ii) all equal to 10 inches.

[The answers are to be correct to three significant figures.]

16. From the formula for the area of a circle given in Ex. 14, we get $r = \sqrt{\frac{A}{\pi}}$. Use the formula in this shape to find the radius of a circle whose area is equal to that of a square on a side of 10 cm. [Answer to three significant digits.]

17. When 10 litres of water have been poured into a certain cylindrical jar, the water stands 8 cm. deep. Find the internal diameter of the jar, having given

$$\textit{volume of water in jar} = h \times d^2 \times 0{\cdot}7854,$$

where h = the depth of the water in cm., and d denotes the diameter. [Answer to three significant figures.]

18. When the temperature is t degrees Centigrade, the velocity of sound in air is given *in metres per second* by the formula

$$\textit{velocity} = 332{\cdot}4 \times \sqrt{1 + {\cdot}00366 \times t}.$$

Find, to the nearest metre, the velocity of sound at a temperature of 20° C.

CHAPTER XIV.

AREAS AND VOLUMES.

236. Square Measure. The Tables of Square Measure and the easier applications of them have been given in Chapter II. The pupil is recommended to revise Arts. 30–33 before proceeding to the examples of greater variety and difficulty in the present chapter.

237. Rectangular Areas. In Arts. 33 it was shewn that

area *of rectangle* = **length** × **breadth,**

in the case when the length and breadth were expressed in terms of the same unit by *whole numbers*, but the formula holds good when the length and breadth are fractional.

This may be illustrated as follows :

Suppose the length and breadth are 3·2 cm. and 2·4 cm. ; we shall shew that the area is (3·2 × 2·4) sq. cm.

For
$$\text{length} = 3{\cdot}2 \text{ cm.} = 32 \text{ mm.},$$
$$\text{breadth} = 2{\cdot}4 \text{ cm.} = 24 \text{ mm.}$$
$$\therefore\ \text{area} = (32 \times 24) \text{ sq. mm.} = \frac{32 \times 24}{10^2} \text{ sq. cm.}$$
$$= (3{\cdot}2 \times 2{\cdot}4) \text{ sq. cm.}$$

Thus for *any* rectangle, if the *length* = l units, and the breadth = b units *of the same kind*, the area (A) *in corresponding units of square measure* is given by the formula

$$A = l \times b.$$

The numbers denoted by l and b are called the **dimensions** of the rectangle. The sum of the sides of the rectangle is called its perimeter.

Thus the perimeter $= 2(l + b)$.

238. From the formula $A = l \times b$, we derive

$$l = A \div b, \text{ or } \frac{A}{b}, \quad \text{and} \quad b = A \div l, \text{ or } \frac{A}{l};$$

so that any one of the three quantities involved may be found when the numerical values of the other two are given.

NOTE. In applying these formulæ care must be taken that the units of length and area correspond. Thus in the formula $b = A \div l$, an area given in *square feet* must be divided by a length *expressed in linear feet*; an area in *square metres* by a length *in linear metres*. The results will be the breadth in linear feet and linear metres respectively.

EXAMPLE 1. *Find the cost of painting a ceiling* 18 *ft.* 8 *in. long by* 13 *ft.* 6 *in. wide, at* 1*s.* 3*d. per square yard.*

Here

$$\text{length} = 18 \text{ ft. } 8 \text{ in.} = 18\tfrac{2}{3} \text{ ft.},$$
$$\text{breadth} = 13 \text{ ft. } 6 \text{ in.} = 13\tfrac{1}{2} \text{ ft.}$$
$$\therefore \ \text{area} = (18\tfrac{2}{3} \times 13\tfrac{1}{2}) \text{ square ft.}$$
$$= (\tfrac{56}{3} \times \tfrac{27}{2} \times \tfrac{1}{9}) \text{ square yards.}$$

And the cost is $1\frac{1}{4}$*s.* per square yard,

$$\therefore \ \text{the total cost} = \left(\frac{\overset{7}{\cancel{56}}}{\cancel{3}} \times \frac{\cancel{27}}{\cancel{2}} \times \frac{1}{\cancel{9}} \times \frac{5}{\cancel{4}}\right) \text{ shillings}$$
$$= 35s. = £1.\ 15s.$$

NOTE. As the *area* is not asked for, no simplification of the fractions is required until the last step.

EXAMPLE 2. *The area of a rectangular strip of cloth is* 0·0136 *of a square metre, and its length is* 85 *centimetres; find its breadth.*

Here

$$\text{area} = 0{\cdot}0136 \text{ sq. metre}$$
$$= 136 \text{ sq. centimetres,}$$

and

$$\text{length} = 85 \text{ centimetres;}$$
$$\therefore \ \text{breadth} = \tfrac{136}{85} \text{ cm.}$$
$$= 1{\cdot}6 \text{ cm.}$$

EXAMPLE 3. *The area of a rectangular field is* 4 *ac.* 2 *r.* 9 *p., and its length is* 12 *chains* 15 *links; find its breadth.*

First express 4 ac. 2 r. 9 p. as a decimal of an acre.

$$4 \text{ ac. } 2 \text{ r. } 9 \text{ p.} = 4{\cdot}55625 \text{ acres.}$$

$$\begin{array}{r|l} 40 & 9{\cdot}0 \text{ poles} \\ 4 & 2{\cdot}225 \text{ roods} \\ & {\cdot}55625 \text{ acre} \end{array}$$

that is,

$$\text{area} = 45{\cdot}5625 \text{ sq. chains.}$$

And

$$\text{length} = 12{\cdot}15 \text{ chains;}$$
$$\therefore \ \text{breadth} = (45{\cdot}5625 \div 12{\cdot}15) \text{ chains}$$
$$= 3{\cdot}75 \text{ chains}$$
$$= 3 \text{ chains } 75 \text{ links.}$$

EXAMPLES XIV. a.

Express in square feet the areas of rectangles having the following dimensions:

1. $10\frac{1}{2}$ ft., $9\frac{1}{3}$ ft. **2.** $14\frac{2}{3}$ ft., $9\frac{3}{4}$ ft. **3.** 7 ft., 72 in.
4. 6·4 ft., $3\frac{1}{8}$ ft. **5.** $6\frac{2}{3}$ yds., 8·1 ft. **6.** 135 in., $10\frac{1}{3}$ ft.

Express in square metres the areas of rectangles with the following dimensions:

7. 12 m., 8·25 m. **8.** 10·25 m., 4·64 m. **9.** 2·54 Km., 0·04 m.

Find the lengths of the rectangles in which

10. Area = 504 sq. yds., breadth = 28 ft. [Ans. in yds.]
11. Area = 77 sq. cm., breadth = 4·4 cm. [Ans. in cm.]
12. Area = $367\frac{1}{2}$ sq. ft., breadth = $8\frac{3}{4}$ yds. [Ans. in ft.]
13. Area = 18 sq. ft. 128 sq. in., breadth = 3 ft. 4 in.

Express in sq. yds., sq. ft., and sq. in., the areas of the following squares, having given

14. Each side = 7 ft. 6 in. **15.** Each side = 5 ft. 8 in.
16. Perimeter = 13 ft. **17.** Perimeter = 21 ft. 4 in.

18. Find the cost of painting a rectangular screen 6 ft. 4 in. long by 4 ft. 8 in. wide at 9*d.* per sq. ft.

19. Find the cost of polishing the floor of a ball-room, 26 ft. 8 in. long and 20 ft. 3 in. wide, at 1*s.* 6*d.* per sq. yd.

20. Find the cost of matting for a room 7·2 metres long and 5·5 metres broad at 75 centimes per square metre.

21. Find, to the nearest penny, the cost of painting a surface 8·5 metres long by 4·6 metres wide at 9*d.* per square metre.

22. It cost £1 to paint a door 7 ft. 6 in. high and 5 ft. 4 in. wide. What was the cost per sq. ft.?

23. How much per sq. yd. was spent on the decoration of a ceilin 21 ft. 4 in. long and 20 ft. 3 in. wide if the whole cost was £12?

[*The Tables in Arts.* 34–36 *should be revised before attemptin Examples* 24–34.]

Find the areas of the following rectangles, giving the result i acres:

24. Length = 20 chains, breadth = 8 chains 25 links.
25. Length = 99 yards, breadth = 4 chains 20 links.

26. The side of a square field measures 6 chains 75 links: find its area in acres, roods, and poles.

27. The perimeter of a square field is 87 chains: find its area in acres, roods, and poles.

28. Find the breadth of a rectangular field which is 3600 links in length, and which has an area of $76\frac{1}{2}$ acres.

29. The acreage of a rectangular field is 37·6607, and the breadth is 16 chains 6 links: find its breadth.

30. The side of a square field is 25 chains: what rent must be paid for it at the rate of £2. 10*s.* per acre.

31. Express (i) in hectares, (ii) approximately in acres, the area of a rectangular plot of ground of which the length and breadth are respectively 805 metres and 74 metres.

32. Find the rent of a rectangular field measuring 200 metres by 85 metres at £12. 10*s.* per hectare; and find approximately the corresponding cost per acre.

33. The rent of a plot of building land is 8250 fr., this being at the rate of 1250 fr. per hectare. If the length is 750 metres, find the breadth.

34. A rectangular piece of ground is 26 chains 16 links in length, and its breadth is five-sixths of its length. Find, to the nearest penny, what rent should be paid for it at the rate of £2. 12*s.* 4*d.* per acre.

239. EXAMPLE 1. *Find, to the nearest penny, the cost of paving a rectangular courtyard $94\frac{1}{2}$ ft. long and 56 ft. wide with tiles 14 inches by 8 inches, at £2. 5s. per thousand.*

$$\text{Area of courtyard} = \left(94\tfrac{1}{2} \times 56\right) \text{ sq. ft.},$$

$$\text{area of one tile} = \left(\tfrac{14}{12} \times \tfrac{8}{12}\right) \text{ sq. ft.} = \tfrac{7}{9} \text{ sq. ft.}$$

Let x = the number of tiles required, then since the area to be covered is equal to the area of the material used, we have

$$\tfrac{7}{9} \times x = \tfrac{189}{2} \times 56\ ;$$

$$\therefore\ x = \frac{189}{\cancel{2}} \times \overset{4}{\cancel{56}} \times \frac{9}{\cancel{7}}$$

$$= 6804.$$

$$\therefore\ \text{cost at } £2\tfrac{1}{4} \text{ per thousand} = £6{\cdot}804 \times 2\tfrac{1}{4}$$

$$= £15{\cdot}309$$

$$= £15.\ 6s.\ 2d.$$

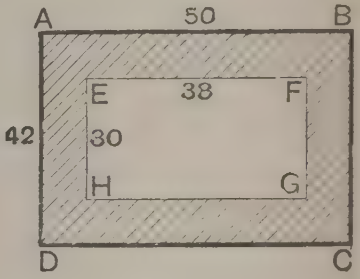
$$\begin{array}{r} 6{\cdot}804 \\ \underline{2\frac{1}{4}} \\ 13{\cdot}608 \\ \underline{1{\cdot}701} \\ \underline{\underline{15{\cdot}309}} \end{array}$$

NOTE. In this case it will be found on trial that the length and breadth of the area to be covered are exact multiples of the dimensions of the tiles. Thus 6804 represents an exact number of tiles which can be fitted together to cover the given area without subdivision. In practice this will not always be the case, and in the examples which follow we shall, where necessary, suppose some of the tiles, etc. to be divided.

EXAMPLE 2. *A quadrangle, 50 feet long by 42 feet broad, contains a rectangular lawn surrounded by a gravel path of uniform width. If the width of the path is 6 feet, find the dimensions of the lawn, and the area of the path.*

Let ABCD represent the quadrangle, and EFGH the lawn.

Then EF is less than AB by *twice* the width of the path; and EH is less than AD by *twice* the width of the path.



That is, $\quad EF = (50 - 6 \times 2)$ ft. $= 38$ ft.

and $\quad EH = (42 - 6 \times 2)$ ft. $= 30$ ft.

And the area of the path = area ABCD − area EFGH

$= (50 \times 42)$ sq. ft. $- (38 \times 30)$ sq. ft.

$= 960$ sq. ft.

EXAMPLES XIV. a. *(Continued.)*

35. How many tiles each 9 in. long by 6 in. wide will be required to pave a court which is 24 ft. 6 in. long and 15 ft. 9 in. wide?

36. Find the cost of flooring a square room whose side is 20 feet with bricks 8 in. long and 4 in. wide, the bricks being worth 9*d*. a score.

37. If 1638 tiles, 7 in. long and 5 in. wide, are used for a hall 16 ft. 3 in. wide, what is its length?

38. What was the width of flagstones 1 ft. 6 in. long, 288 of which were required exactly to cover a courtyard 30 ft. long and 18 ft. wide?

39. In what way must tiles $4\frac{1}{4}$ in. long by $3\frac{3}{4}$ in. wide be laid so as exactly to cover the floor of a hall 38 ft. $1\frac{1}{2}$ in. long by 20 ft. $2\frac{1}{4}$ in. wide? If the total cost of the flooring amounts to £193. 3*s*. 4*d*. what is the cost of the tiles per dozen?

40. A rectangular plot 64 ft. 6 in. long and 30 ft. 9 in. wide is to be laid with sods of turf 15 in. long by 9 in. wide. If the turfs are placed in the most convenient manner, how many will be used in all, and how many of these will have to be divided?

41. Square turfs, whose sides are 16 inches, are laid on a rectangular surface 40 ft. 8 in. long and 24 ft. wide. How many turfs will be required? How many can be laid *whole*, and how many cut *in half*? Illustrate by a diagram on squared paper.

(*Examples* 42–49 *should be illustrated by diagrams.*)

42. Find in square yards the area of a path 6 feet wide surrounding a lawn whose length is 30 yds. and breadth 24 yds.

43. A carpet laid down in the centre of a room leaves a margin 2 ft. wide uncovered, the dimensions of the room being 25 ft. by 18 ft. Find the cost of staining the uncovered margin at $1\frac{1}{2}d.$ per sq. ft.

44. How many tiles 6 in. square will be required to pave a footpath 4 ft. wide carried round the outside of a grass-plot 25 yds. long by 13 yds. broad?

45. A square garden each side of which is 20 yds. is traversed centrally by two paths at right angles to one another, and is thus divided into 4 beds. If the paths are respectively 8 ft. and 6 ft. wide, find the area of each bed.

46. In a courtyard 67 ft. 6 in. long and 42 ft. 9 in. wide there is a foot-way, 5 ft. 6 in. wide, running the whole length of the yard. What is the cost of paving the whole, the price per sq. yd. for the foot-way being 3*s.* 6*d.*, and for the remainder 3*s.*?

47. A carpet 28 ft. long and 22 ft. wide is laid in the centre of a room 30 ft. long by 26 ft. wide. If the carpet costs 1*s.* 6*d.* per square yard, and the rest of the floor is stained at the rate of 2*s.* 3*d.* per dozen square yards, find the total cost.

48. The floor of a hall 48 ft. long by 36 ft. broad is partly to be tiled and the rest boarded, the tiled portion forming a uniform margin 2 yards wide round the hall. The tiles measure 9 inches by 8 inches each, and the boards 6 feet by 6 inches: how many of each are required?

49. A rectangular court is 50 yds. long and 30 yds. wide. It has paths, 6 ft. in width, joining the middle points of opposite sides, and also a paved corridor of the same width running all round the outside of it. The remainder is covered with grass. If the cost of the pavement for the paths and corridor is 1*s.* 8*d.* per square foot, and the cost of the grass 3*s.* per square yard, find the whole cost of laying out the court and corridor.

240. Carpeting of Floors. Carpet is usually sold in rolls of uniform width at so much per *linear* yard. To carpet a given floor it is necessary to cut from such a roll a sufficient number of strips (each having the *same length* as the floor), and then place them side by side to form an area of carpet equal to the area of the floor. It is evident, however, that unless the width of the floor is an exact multiple of the width of the roll, the requisite amount of carpet will not be represented by an exact number of strips. In other words, part of one strip will be wasted.

EXAMPLE 1. *Find the cost of carpet* 30 *in. wide at* 4*s.* 2*d. a yard for a room whose length is* 20 *ft. and width* 14 *ft. How many square feet of carpet will be wasted?*

Here width of room = 14 ft., and width of the roll = $2\frac{1}{2}$ ft.;

$$\therefore \text{ number of strips, each 20 ft. long, } = \frac{14}{2\frac{1}{2}} = 5\tfrac{3}{5}.$$

Thus **6** strips will be required, two-fifths of the last strip being wasted.

$$\begin{aligned}\therefore \text{ length of carpet used} &= 20 \text{ ft.} \times 6 = 120 \text{ ft.}\\ &= 40 \text{ yds.}\\ \therefore \text{ the required cost} &= 4s.\ 2d. \times 40\\ &= £8.\ 6s.\ 8d.\end{aligned}$$

$$\text{The waste} = \tfrac{2}{5} \text{ of a strip.}$$

$$\begin{aligned}\therefore \text{ number of sq. ft. wasted} &= \tfrac{2}{5} \times 20 \times 2\tfrac{1}{2}\\ &= 20.\end{aligned}$$

NOTE. If the breadth of the room is an exact multiple of the breadth of the carpet, we may proceed more directly as follows:

Suppose the room is l ft. long and b ft. broad, and that the carpet is a ft. broad. Then if x is the number of feet in the required length, we have at once

$$a \times x = l \times b;$$

$$\therefore \; x = \frac{l \times b}{a}.$$

EXAMPLE 2. *To carpet a room* 18 *ft. long by* 11 *ft.* 3 *in. wide with carpet* 27 *in. wide cost* £5. 17*s.* 6*d.* *If* 12*s.* 6*d. of this was paid for making and laying the carpet, what was the price per yard of the carpet?*

Here width of room = $11\frac{1}{4}$ ft., and width of carpet = $2\frac{1}{4}$ ft.

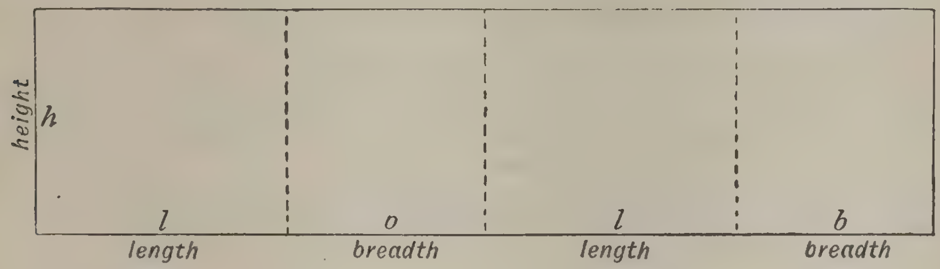
$$\begin{aligned}\therefore \text{ number of strips} &= 11\tfrac{1}{4} \div 2\tfrac{1}{4} = 5.\\ \therefore \text{ length of carpet} &= 18 \text{ ft.} \times 5 = 30 \text{ yds.}\end{aligned}$$

Now cost of carpet (before making) = £5. 5*s.*;

$$\begin{aligned}\therefore \text{ price of carpet per yd.} &= \tfrac{105}{30} \text{ shillings}\\ &= 3s.\ 6d.\end{aligned}$$

241. Papering the Walls of a Room. Wall-paper is generally made in pieces, 12 yards in length, and of uniform width (usually 21 inches), and is sold at so much *per piece.* To find the number of pieces required for a room of given dimensions we should divide the area of the walls by the area of each piece. But results so obtained are of little value, as in practice doors, windows, and fireplace have to be allowed for; moreover, there is always some waste in cutting the paper and "matching the pattern."

242. Consider a room whose length, breadth, and height are respectively l feet, b feet, and h feet. Then if we suppose the walls to be hinged at the corners and capable of being opened out, we obtain one continuous rectangle as represented in the diagram below:



Hence it appears that the four walls together have an area equal to that of a single rectangle whose width is h ft. (viz. the height of the room), and whose length is $(2l+2b)$ ft. (viz. the perimeter of the floor).

Hence *area of four walls = perimeter × height*

$$= 2(l+b) \times h \text{ square ft.}$$

Example 1. *A room is* 17 *ft.* 5 *in. long,* 14 *ft.* 7 *in. wide and* 11 *ft.* 6 *in. high. Supposing that doors, windows, etc. occupy* 76 *sq. ft., and that the waste amounts to* 2 *'pieces,' find the cost of the requisite amount of paper at* 3*s.* 6*d. per piece of* 12 *yds.*

Here $l+b=32$ ft., and $h=11\frac{1}{2}$ ft.

$$\therefore \text{ area of walls} = \left(2 \times 32 \times \tfrac{23}{2}\right) \text{ sq. ft.}$$
$$= 736 \text{ sq. ft.}$$

$$\text{Area } \textit{to be papered} = (736 - 76) \text{ sq. ft.}$$
$$= 660 \text{ sq. ft.}$$

$$\text{Area of each piece} = \left(12 \times 3 \times \tfrac{21}{12}\right) \text{ sq. ft.} = 63 \text{ sq. ft.}$$

$$\therefore \text{ number of pieces} = (660 \div 63) + 2, \text{ allowing for waste,}$$
$$= 12\tfrac{10}{21}.$$

But since the paper is sold by the piece, **13** pieces would have to be paid for, and the cost would be 3*s.* 6*d.* × 13, or £2. 5*s.* 6*d.*

EXAMPLE 2. *Find the cost of painting the four sides and the bottom of a tank 2 yds. 1 ft. long, 4 ft. wide, and $4\frac{1}{2}$ ft. deep at 4d. per square foot.*

Area of four vertical sides = perimeter × depth ;

that is, area of sides $=(2 \times 11 \times \frac{9}{2})$ sq. ft. $=99$ sq. ft.

Area of bottom $=(7 \times 4)$ sq. ft. $=28$ sq. ft.

Total area to be painted $=(99+28)$ sq. ft. $=127$ sq. ft.

$\therefore$ cost of painting $=(127 \times \frac{1}{3})$ shillings

$=42\frac{1}{3}$ shillings

$=$ £2. 2*s.* 4*d.*

EXAMPLES XIV. b.

1. How many yards of carpet 2 ft. 6 in. wide will be required for a room 20 ft. long and 15 ft. wide?

2. What length of carpet 27 in. wide will be required to cover the floor of a room 17 ft. long and 11 ft. 3 in. wide?

3. It takes exactly 29 yards 1 ft. of carpet $\frac{3}{4}$ yd. wide to cover a floor 14 ft. 8 in. long; find the width of the floor.

4. Find the cost of carpeting a room 25 ft. long and 18 ft. wide, with carpet 2 ft. 3 in. wide at 6*s.* per yard.

5. How many strips of carpet 27 in. wide will be required for a room 20 ft. 3 in. long and 16 ft. wide, and what will be the cost at 3*s.* 4*d.* per yard? What fraction of the last strip will be wasted?

6. Find the cost of carpet 30 in. wide necessary to cover the floor of a room 15 ft. long and 12 ft. wide, the carpet costing 3*s.* 4*d.* per yard. Shew that $7\frac{1}{2}$ sq. ft. will be wasted.

7. What length of carpet 27 in. wide will be required for a room 18 ft. long and 12 ft. 9 in. wide, and what will be its cost at 5*s.* 6*d.* per yard? How many square feet of carpet will be wasted?

8. Find the area of the walls of the rooms whose dimensions are

(i) Length 16 ft. 8 in., width 15 ft. 4 in., height 12 ft. 9 in.

(ii) Length 20 ft. 3 in., width 16 ft. 5 in., height 15 ft. 9 in.

9. Find the cost of painting the walls of a room 12 ft. 9 in. long, 11 ft. 3 in. wide, and 10 ft. high at 1*s.* 6*d.* per square yard, without allowing for doors and windows.

10. Find the cost of colouring the walls and ceiling of a room whose height is 14 ft., length 15 ft., and breadth 12 ft., the walls costing 9*d.* per square yard, and the ceiling $4\frac{1}{2}$*d.* per square yard.

11. Find the external surface of the closed rectangular boxes, the outer dimensions of which are

(i) Length 6 ft., breadth 4 ft., depth 3 ft. 6 in.
(ii) Length 3 ft. 6 in., breadth 2 ft. 4 in., depth 1 ft. 3 in.

12. The external dimensions of a closed rectangular box are 9 ft., 4 ft. 6 in., and 3 ft. Find the cost of painting it outside at 2*d.* per square yard.

13. Supposing the paper saved from doors, windows, etc. makes up for the waste, how many pieces 12 yds. long and 27 in. wide will be required for a room 18 ft. 9 in. long, 11 ft. 3 in. wide, and 13 ft. 6 in. high?

14. How many pieces of paper, 12 yds. long, and 21 in. wide, will be required for a room 16 ft. 5 in. long, 13 ft. 7 in. wide, and 11 ft. 3 in. high, allowing 55 sq. ft. for windows, etc., and 2 extra pieces for waste?

15. A room is 18 ft. square, and 11 ft. high: doors, windows, etc. occupy 90 sq. ft., and 1 piece of paper is wasted. Find the cost of papering the walls with paper 27 in. wide, at 6*s.* per piece of 12 yds.

16. A room 16 ft. 9 in. long, 10 ft. 3 in. wide and 11 ft. high has a door 7 ft. by 3 ft., two windows each 4 ft. by 5 ft., and a fireplace 4 ft. by $3\frac{1}{2}$ ft. Find the cost of painting the walls at 9*d.* per square yard.

17. The cost of paper for a room 15 ft. 7 in. long by 14 ft. 5 in. wide is £1. 15*s.* If the paper is 30 in. wide and costs 5*s.* per piece of 12 yds. find the height of the room, assuming that the waste is balanced by the amount of paper saved by windows, etc.

18. Find the number of square feet of lead required to line an open cistern whose inside dimensions are as follows: length 5 ft. 3 in., width 3 ft., depth 1 ft. 10 in.

19. Find the cost of painting the outside of a box (with lid) whose length is 3 ft. 9 in., width 3 ft. 4 in., and depth 3 ft., at the rate of 4*d.* per square yard.

20. What will be the cost, to the nearest franc, of paper for the two walls of a passage which is 21·85 metres long, 10·4 metres high, at 4 fr. 35 c. per piece of 8 metres in length and 0·45 metre in width?

21. If it costs £21. 6*s.* 5*d.*, at 8*d.* per square foot, to paint the walls of a room 19 ft. 6 in. long, 14 ft. 9 in. wide, and 11 ft. 3 in. high, how many square feet are occupied by windows, etc.?

EXAMPLES XIV. c.

(*Miscellaneous.*)

[*In Examples* 1–7 *the following approximate equivalents may be used:*

1 *metre* $=39\frac{3}{8}$ *in.;* 1 *square metre* $=1550$ *square inches;*

1 *kilometre* $=\frac{5}{8}$ *mile;* 1 *hectare* $=2\frac{1}{2}$ *acres.*]

1. A rectangular floor is 8·25 m. in length and 6 m. in width: find

(i) its approximate area in square feet;

(ii) the cost of staining it at $1\frac{1}{2}d.$ per square metre;

(iii) how much would be saved by staining it at 1*d.* per square yard.

2. The area of a rectangular courtyard is 235·84 sq. m. and its length is 16 m. 75 cm.; find its breadth (i) in metres, (ii) approximately in feet and inches.

3. What is the width of a road (to the nearest tenth of a yard), if its area is 92 ares per kilometre of length?

4. A belt of wood is 3·8 Km. in length, and has an average width of 42·35 metres; find the cost to a shooting tenant who rents it at £2. 10*s.* per acre. [Answer to the nearest penny.]

5. The length, breadth, and height of a room are 5·65 m., 4·35 m., and 4·5 m. respectively. Allowing 18 square metres for doors, windows, and fireplace, find the cost of painting the walls at $4\frac{1}{2}d.$ per square yard.

6. On a map drawn to the scale of an inch to a mile a certain estate covers an area of 2·44 square inches. Find the real area of the estate in acres.

7. The map of a district is drawn on a scale of 1·6 cm. to a mile: what area on the map will represent a lake 2000 hectares in extent?

8. It is proposed to cover with equal square tiles a floor 17 ft. 6 in. long by 15 ft. 9 in. broad; find in how many ways this can be done when the side of the tile contains an integral number of inches, and find the number required in each case.

9. It costs £1. 2*s.* 6*d.* to paint the ceiling of a passage which is 20 ft. 3 in. long and 15 ft. wide. If the passage is $10\frac{1}{2}$ ft. high, what would be the additional cost of painting the walls at the same rate?

10. A passage 40 ft. long, 8 ft. wide, and 10 ft. high. has two doors 7 ft. by 4 ft., and a window 5 ft. 6 in. by 2 ft. Find the cost of colouring the walls and ceiling at $4\frac{1}{2}d.$ per square yard.

11. Assuming the area of a triangle to be given by the formula

$$area = \tfrac{1}{2}(base) \times height,$$

find the area of a triangle in which the base is 25 chains and the height is 3 chains 21 links.

12. The area of a triangle standing on a base of 15 m. 45 cm. is 90·61 sq. metres, find the height.

13. Assuming the area of a trapezium (*i.e.* a quadrilateral with a pair of parallel sides) to be given by the formula

$$area = \tfrac{1}{2}(sum\ of\ parallel\ sides) \times (distance\ between\ them),$$

find, to the nearest square metre, the area of a trapezium when the parallel sides are 34·72 m. and 27·08 m., and the distance between them is 20·5 m.

14. A quadrilateral field has two parallel sides measuring 13 chains 46 links and 11 chains 54 links, the perpendicular distance between them being 6 chains 20 links. If the rent is £38. 15*s.*, what is the rent per acre?

15. The area of a circle of diameter d is given approximately by the formula

$$A = d^2 \times 0{\cdot}7854.$$

Use this formula to find

(i) the area of a circle whose diameter is 3·6 metres, (to the nearest hundredth of a square metre);

(ii) the cost of gravel, at 4*d.* per square yard, for a circular court whose diameter is 43 feet.

16. Find, to the nearest tenth of a square foot, the area of a circular ring whose external and internal radii are 27 feet and 23 feet respectively.

17. A circular lawn 190 feet in diameter is surrounded by a path 10 feet wide; find the area of the path to the nearest square yard.

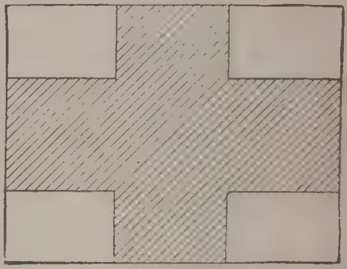
18. A rectangular hall 24 ft. 9 in. long and 13 ft. 6 in. wide has a semi-circular verandah at one end. Find, to the nearest penny, the cost of laying wooden blocks at 3*s.* 4*d.* a square yard for the hall, and stone at 4*s.* a square yard for the verandah.

19. A rectangular tennis-ground is 110 yds. long and 55 yds. wide. Find the expense of sowing it with grass-seed at the rate of $3\frac{1}{2}$ bushels per acre, the price of the seed being £1. 1*s*. 4*d*. per bushel.

20. A square field contains $2\frac{1}{2}$ acres; what is the cost of making a path 3 yds. wide inside the field round the boundary at 1*s*. 6*d*. per square yard?

21. The number of acres in the county of Suffolk is 931,000. What is the area which represents this county on a map of England which is drawn to the scale of 1 inch to 10 miles? Give your answer to the nearest square inch, and draw a rectangle of this area. (C.S.)

22. Find the number of square feet in the shaded and unshaded areas represented in the adjoining diagram, on the supposition that it is drawn on a scale of 1 cm. to 5 ft. Make any necessary measurements.



23. The sides of a rectangular field are as 2 : 1, and its area is an acre; find the length of its diagonal.

24. A rectangular garden, whose length is twice its breadth, contains 2244·5 sq. metres. How much would it cost to surround it with a fence at 13 fr. 25 c. per metre?

25. The area of a gravel path 8 ft. wide, surrounding a square park is 4 acres. Find the area of the park, including the path, to the nearest tenth of an acre.

26. The difference between the areas of two square fields is 5 acres. If the length of a side of the smaller is 71 yds. find the length of a side of the larger.

27. The length and breadth of a room are measured by means of a walking stick, and are estimated at 15 ft. and $12\frac{1}{2}$ ft. In making the calculation it is assumed that the stick is 3 ft. long. The actual length is 2 ft. 9 in. Find the errors in the measurements of the room arising from this mistake, and the error in the area to the nearest square foot. (C.S.)

28. A street improvement scheme involves purchasing the houses on one side of the street, and the land on which they stand along a frontage of 352 ft. and to an average depth of 53 ft. The houses cost £87700, and the land costs £3. 7*s*. per sq. ft. The roadway is then widened 9 ft., and remaking this extra piece costs 8*s*. per sq. yd. The remaining land is sold again at £5. 13*s*. per sq. ft. Find the net cost of the improvement. (C.S.)

Cubic Measure.

243. The Tables of Cubic Measure and some easy applications of them have been given in Chap. II. [See Arts. 37–40.] It is there shewn that the volume of a **rectangular block**, or **cuboid**, is given by the formula

Volume = length × breadth × height,

where by *length*, *breadth*, and *height* we mean the number of *linear* units in each, and by *volume* the number of *corresponding* units of cubic measure.

Note. This rule was proved in the case where the length, breadth, and height were expressed by whole numbers. By reasoning similar to that of Art. 237, it may be shewn to hold good also when the dimensions are fractional.

244. Let V be the number of cubic units in the volume of a cuboid, and let the number of units in length, breadth, and height be denoted by l, b, and h respectively. Also let A represent the number of square units in the base, then $\mathsf{A}=l\times b$.

The formula of the last article gives

$$\begin{aligned}\mathsf{V}&=l\times b\times h\\&=\mathsf{A}\times h.\end{aligned}$$

Thus we have

volume of cuboid = (*area of base*) × *height.*

Note. The formula *volume* = (*area of base*) × *height* has been proved on the supposition that the base is *rectangular*; but it holds equally (see *School Geometry*, p. 393) for a solid figure constructed in like manner on a base of any shape whatever. The truth of this will be assumed in some of the examples which follow.

245. From the formula $\mathsf{V}=l\times b\times h$ we derive

$$\text{(i) } l=\frac{\mathsf{V}}{b\times h};\quad \text{(ii) } b=\frac{\mathsf{V}}{l\times h};\quad \text{(iii) } h=\frac{\mathsf{V}}{l\times b};$$

so that any one of the four quantities in the formula can be found when the other three are known.

Note. In using these formulæ care must be taken that the units of length and volume correspond. Thus in the formula $b=\frac{\mathsf{V}}{l\times h}$, if V is in cubic metres, l and h must each be in linear metres, and the result b will be in linear metres.

EXAMPLE 1. *A rectangular tank measures internally* 8 *ft. in length,* 6 *ft. in breadth, and* 2 *ft.* 4 *in. in depth. How many cubic feet of water will it contain? And how many gallons, supposing* 1 *cubic ft. is equivalent to* $6\frac{1}{4}$ *gallons?*

$$\begin{aligned}\text{The } \textit{volume} \text{ of the tank} &= \textit{length} \times \textit{breadth} \times \textit{depth}\\ &= (8 \times 6 \times 2\tfrac{1}{3}) \text{ cubic ft.}\\ &= 112 \text{ cubic ft.}\end{aligned}$$

$$\begin{aligned}\therefore \text{ the } \textit{capacity} \text{ of the tank} &= 6\tfrac{1}{4} \text{ gallons} \times 112\\ &= 700 \text{ gallons.}\end{aligned}$$

EXAMPLE 2. *Find the height of a rectangular solid whose volume is* 7 *cu. ft.* 864 *cu. in., length* 4 *ft., and breadth* 1 *ft.* 3 *in.*

Here volume = 7 cu. ft. 864 cu. in. = $7\frac{1}{2}$ cu. ft.

$$\therefore \text{ from the formula } h = \frac{\mathsf{V}}{l \times b},$$

$$\begin{aligned}\text{the required height} &= \frac{7\frac{1}{2}}{4 \times 1\frac{1}{4}} \text{ ft.}\\ &= 1 \text{ ft. } 6 \text{ in.}\end{aligned}$$

EXAMPLE 3. *During a rainfall of* 25 *mm., how many litres of water fall per hectare? And, approximately, how many gallons per acre?*

[*Assume* 1 *Ha.* = $2\frac{1}{2}$ *acres,* 1 *litre* = $1\frac{3}{4}$ *pints, approximately.*]

Volume of water = *area* × *height*. [See Art. 244, NOTE.]

$$\begin{aligned}1 \text{ Ha.} &= 10000 \text{ square metres}\\ &= 10000 \times 10^2 \text{ square decimetres,}\end{aligned}$$

$$25 \text{ mm.} = \frac{25}{10^2} \text{ decimetres.}$$

$$\begin{aligned}\therefore \text{ vol. of water} &= \left(10000 \times 10^2 \times \frac{25}{10^2}\right) \text{ cubic decimetres}\\ &= 250{,}000 \text{ cu. dm.}\\ &= 250{,}000 \text{ litres.}\end{aligned}$$

Again 1 Ha. = $2\frac{1}{2}$ acres, approximately.

$$\begin{aligned}\text{Hence volume of water per acre} &= (250{,}000 \div 2\tfrac{1}{2}) \text{ litres}\\ &= \left(\frac{250{,}000}{2\frac{1}{2}} \times 1\tfrac{3}{4} \times \tfrac{1}{8}\right) \text{ gallons}\\ &= 21{,}875 \text{ gallons.}\end{aligned}$$

EXAMPLES XIV. d.

1. Find in cubic feet the volume of the cuboids whose dimensions are as follows:

(i) Length 4 ft., width $2\frac{1}{2}$ ft., height $1\frac{1}{2}$ ft.
(ii) ,, 9 ft., ,, 4 ft., ,, 21 in.
(iii) ,, $4\frac{1}{2}$ yds., ,, $3\frac{1}{3}$ ft., ,, 16 in.
(iv) ,, 6 ft. 9 in., ,, 5 ft. 4 in., ,, $1\frac{1}{2}$ yd.

2. Find the surface and volume of a cube whose edges measure 4·5 ft.

3. How many cubic yards of masonry are there in a wall 26 yds. 2 ft. long, 20 ft. 3 in. high, and 18 in. thick?

4. Find the cubical content of a beam 13 ft. 6 in. long, if the area of its cross-section is $2\frac{2}{3}$ square feet.

5. How many cubic metres are there in the cuboids which have

(i) Length 3·75 m., breadth 2 m., height 40 cm.?
(ii) Area of base 450 sq. m., thickness 160 cm.?

6. Find the depth of a tank which can contain 2·6 cu. m. of water, the area of the base being 6500 sq. cm.

7. A tank contains 21,000 litres of water; if the length and breadth are 5·6 metres and 2·5 metres respectively, find the depth.

8. Find the weight of an iron bar 48 in. long, 4·5 in. broad, and 2·5 in. thick, given that 1 cu. ft. of iron weighs 480 lbs.

9. How many gallons can be contained in a cubical vessel, each edge of which measures 4 ft., supposing 1 cu. ft. to contain $6\frac{1}{4}$ gallons?

10. A room contains 2156 cu. ft. of air, and its height is 12 ft. 10 in.; find the area of the floor.

11. It costs £1. 8*s.* to cut a trench 16 ft. long and 7 ft. wide at $1\frac{1}{2}$*d.* per cubic foot: find the depth.

12. What must be the depth of a tank whose base is a square on a side of 1 yd., if it holds as much water as a second tank whose dimensions are 4 ft. 6 in., by 2 ft. 3 in., by 1 ft. 4 in.?

13. One ton of lead is rolled into a sheet of uniform thickness 0·42 in. Find approximately in square yards the area of the sheet, having given that 1 cu. ft. of lead weighs 710 lbs.

14. The annual rainfall in a certain place is 28 in. Express this in tons per acre, to the nearest ton. [1 cu. ft. of water weighs 1000 oz.]

15. A pond whose area is half an acre is frozen over with ice 2 in. thick: find, to the nearest ton, the total weight of ice, if 1 cu. ft. of it weighs $57\frac{1}{4}$ lbs.

16. Express in gallons per acre an annual rainfall of 30 in., assuming that 1 cu. ft. contains $6\frac{1}{4}$ gallons.

17. How many hundredweight of sheet-lead, $\frac{1}{8}$ in. thick, will be used to cover a surface of 224 sq. yds.? [See Ex. 13.]

18. If a square foot of metal $\frac{3}{4}$ in. thick weighs 30 lbs., what is the weight in pounds of a plate 6 ft. 6 in. long, 3 ft. 6 in. wide, and $3\frac{1}{2}$ in. thick made from the same metal?

19. A brick (with mortar) occupies a space 9 in. long, $4\frac{1}{2}$ in. broad, and 3 in. high; how many bricks will be required for a wall 30 yds. long, 6 ft. high, and $13\frac{1}{2}$ in. thick?

20. Find the area of the bottom of a box 5 ft. high if the contents (when the box is full) weigh 972 lbs., at the rate of 9 oz. for every 60 cu. in.

21. A block of metal whose dimensions are 2 ft. 9 in., by 1 ft. 8 in., by 1 ft. 4 in. weighs 11 cwt.; what would be the length of a bar of the same metal if the area of its cross-section is 120 sq. in. and its weight is 18 cwt.?

22. In the first fortnight of Dec. 1907 there was a rainfall of 3 inches in the Thames Valley above Teddington. Assuming this area to be 3800 square miles, shew that the rainfall was approximately equivalent to the addition to the normal water of the Thames of three rivers each 670 miles long, 250 feet wide, and 10 feet deep.

246. Suppose a closed box, which measures externally a inches long, b inches wide, and c inches high, is made of wood k inches thick.

(i) Then the internal dimensions are as follows:

$$\text{length} = a - 2k\text{; breadth} = b - 2k\text{; height} = c - 2k.$$

(ii) The number of cubic inches in the *capacity* of the box

$$=(a-2k)(b-2k)(c-2k).$$

(iii) The number of cubic inches of *material* used in the construction of the box $= abc - (a-2k)(b-2k)(c-2k)$.

247. The **specific gravity** of a substance is the ratio of the weights of equal volumes of the given substance and water.

Thus if the specific gravity of lead is 11·4, it follows that the weight of 1 cubic decimetre of lead is 11·4 kilograms.

EXAMPLE 1. *Find the weight of a steel bar* 1·28 *m. long*, 15 *cm. wide, and* 5 *cm. thick, the specific gravity of steel being* 7·7.

$$\begin{aligned}\text{The volume in } \textit{cubic decimetres} &= 12{\cdot}8 \times 1{\cdot}5 \times 0{\cdot}5\\ &= 9{\cdot}6.\\ \therefore \text{ the required weight} &= (9{\cdot}6 \times 7{\cdot}7) \text{ Kg.}\\ &= 73{\cdot}92 \text{ Kg.}\end{aligned}$$

EXAMPLE 2. *A closed wooden chest measures externally* 3 *ft.* 4 *in. long,* 2 *ft.* 4 *in. broad,* 1 *ft.* 5 *in. high; if the thickness of the wood is half an inch; find* (i) *the capacity of the chest,* (ii) *its weight, supposing the specific gravity of the wood to be* 0·972.

The *internal* measurements are respectively 3 ft. 3 in., 2 ft. 3 in., and 1 ft. 4 in.

$$\begin{aligned}\therefore \text{ the } \textit{capacity} \text{ of the chest} &= \left(3\tfrac{1}{4} \times 2\tfrac{1}{4} \times 1\tfrac{1}{3}\right) \text{ cu. ft.} \quad \text{[Art. 246 (ii)]}\\ &= 9\tfrac{3}{4} \text{ cu. ft.}\end{aligned}$$

$$\begin{aligned}\text{The number of } \textit{cubic feet} \text{ of wood} &= \left(3\tfrac{1}{3} \times 2\tfrac{1}{3} \times 1\tfrac{5}{12}\right) - 9\tfrac{3}{4} \quad \text{[Art. 246 (iii)]}\\ &= 11\tfrac{1}{54} - 9\tfrac{3}{4}\\ &= 1\tfrac{29}{108}.\end{aligned}$$

Again since 1 cu. ft. of water weighs 1000 oz., the weight of 1 cu. ft. of the wood weighs 972 oz.

Thus the weight of the box $\left(1\tfrac{29}{108} \times 972\right)$ oz. = 77 lbs. 1 oz.

EXAMPLE 3. *Through a wooden pipe, whose cross-section is a square on a side of* 8 *cm., water flows uniformly at the rate of* 40 *metres a minute. How long will it take to discharge a million litres?*

The volume of water flowing through the pipe per minute is that of a cuboid whose dimensions are 400, 0·8, 0·8 decimetres respectively.

$$\begin{aligned}\text{Hence number of litres per min.} &= 400 \times 0{\cdot}8 \times 0{\cdot}8.\\ &= 256.\\ \therefore \text{ time required for a million litres} &= \tfrac{1000000}{256} \text{ minutes}\\ &= 65 \text{ hrs. } 6\tfrac{1}{4} \text{ min.}\end{aligned}$$

EXAMPLES XIV. e.

[*The following approximate equivalents may be used:*

1 *hectare, or* $(100)^2$ *square metres* $= 2\frac{1}{2}$ *acres.* 1 *kilogram* $= 2\frac{1}{5}$ *pounds.*

1 *cubic foot of water weighs* 1000 *ounces or* $62\frac{1}{2}$ *pounds.*

1 *cubic foot of water contains* $6\frac{1}{4}$ *gallons.*]

1. Find the weight per square metre of sheet zinc 3 mm. thick, the specific gravity of zinc being 7·14.

2. The specific gravity of ebony is 1·2; find the weight of (i) a cube, each edge of which is 2·5 cm., (ii) a ruler, 36 cm. in length, the cross-section being a square on a side of 8 mm.

3. Find the weight in tons of a block of granite (Sp. gr. = 2·56) measuring 4 ft., by $3\frac{1}{2}$ ft., by $2\frac{1}{2}$ ft.

4. What is the weight in pounds of an oak beam, 12 ft. long, 18 in. wide, and 16 in. thick, the specific gravity of oak being 0·85?

5. A level seam of coal has an average thickness of 2 ft. 4 in. If the specific gravity of coal is 1·28, find how many tons the seam will yield per acre.

6. Find in kilograms, and also approximately in pounds the weight of a beam of fir, 4·45 m. long, 24 cm. broad, and 20 cm. thick; given that the specific gravity of fir is 0·55.

7. If 540 Kg. of mercury are poured into a rectangular trough, $2\frac{1}{2}$ m. long by 16 cm. wide, to what depth will the trough be filled? The specific gravity of mercury = 13·5.

8. Water flows through a stone culvert, 3·5 m. wide and 60 cm. deep, at the rate of 2·5 Km. per hour: how many litres will pass under a bridge in $2\frac{1}{2}$ minutes?

9. A tank whose length and breadth are respectively 1·75 m. and 80 cm. receives water from a pipe which discharges 3500 litres per hour. What depth of water will be added in 12 minutes?

10. Find approximately in pounds the weight of a copper bar 1·68 m. in length, 12·5 cm. in width, and 5 cm. in thickness; given that copper weighs 8·8 times as much as its bulk of water.

11. Find, in cubic inches, the capacity of a closed box made of wood $\frac{1}{2}$ in. thick, the external dimensions being 9 in., by 7 in., by 6 in.

12. How many cubic inches of material are required to make a closed box, the sides, lid, and bottom being $\frac{1}{2}$ in. thick, and the external dimensions 13 in., by 11 in., by 9 in.?

13. The sides of an *open* box are $\frac{1}{2}$ in. thick, and the bottom is 1 in. thick; if the external length, breadth, and depth are respectively 14 in., 10 in., and 8 in., find the capacity of the box and the quantity of material used in its construction.

14. A trench, 50 yds. long and 15 ft. wide, has a uniform depth of 6 ft. How long would it take to fill the trench with water by means of a pipe which discharges 125 gallons per minute?

15. Water passes from a reservoir into a canal by a culvert 4 ft. wide and $1\frac{1}{4}$ ft. deep. If the water flows at the rate of 4 miles an hour, find how many gallons pass into the canal in ten minutes.

16. One million litres of water flow on to a skating rink one acre in extent. Find in centimetres the depth to which the rink is flooded.

17. What fraction of a ton of water is required to fill a cistern 3 ft. 8 in. long, 2 ft. 8 in. wide, and 2 ft. deep? Assuming the above cistern to have no lid and to be made of wood 2 in. thick, find the cost of completely covering the wood with paint at 9*d.* per square foot.

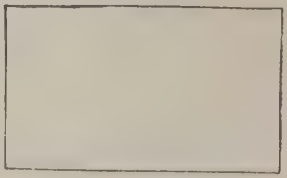
18. What is the capacity of a box which on the outside measures 2 ft. $9\frac{1}{2}$ in. long, 2 ft. $6\frac{3}{4}$ in. broad, and 2 ft. $1\frac{3}{4}$ in. high, the sides and top being $\frac{3}{4}$ in. thick and the bottom 1 in. thick? Find also the number of pounds of water, to the nearest integer, which can be contained in 19 such boxes.

19. A rectangular block of metal, whose dimensions are 1 ft. 6 in., by 1 ft., by 10 in., is thrown into a cistern partly full of water. If the cistern stands on a base 2 ft. 6 in. by 1 ft. 4 in., and the block is completely immersed, how high will the immersion of the metal cause the water to rise?

20. A cistern, measuring internally 5 ft., by 4 ft., by 3 ft. 8 in., has 30 cubic feet of water in it. Porous bricks are placed in the water until the cistern is brim-full, each brick absorbing one-seventeenth of its own volume of water. How many bricks can be put in without the water overflowing, each brick being 9 in., by 3 in., by $2\frac{2}{3}$ in.?

21. An iron plate whose length is 280 cm., breadth 195 cm., and thickness 20 mm. weighs 851·76 Kg.; find the specific gravity of the iron.

22. The adjoining figure represents, on a scale of 1 cm. to 10 yds., a courtyard which is to be paved with blocks of wood 6 in. long, 3 in. broad, and 4 in. deep. How many blocks are used, and how many cartloads of 2·5 tons will they make, if the specific gravity of the wood is 0·8? Also find the cost, to the nearest penny, at 9*s.* 4*d.* per ton.



23. The area of the Trent basin is 4082 square miles, and the annual rainfall 31 inches. The river discharges 220,000 cubic feet per minute. What percentage (to the nearest integer) of the rain that falls is discharged by the river? (C.S.)

24. A town has a population of 400,000 and a water supply of 30 gallons per head per day is required. A reservoir is provided to hold six months' supply [say, July to December inclusive], with a mean depth of 20 feet. What must the area of the reservoir be? Find the area of the catchment basin, if 20 inches of rainfall in the year will yield sufficient water to supply the town. (C.S.)

Give answers to nearest acre and nearest square mile respectively.

25. Lake Tsana has a surface area of 30,000,000 ares, and discharges annually a quantity of water that would raise its level by $1\frac{1}{4}$ metres. It is proposed to erect a barrage at the outlet to control the discharge, with sluice-gates that allow the passage of 170 cubic metres of water per second. If this is done, and the annual discharge is to remain what it now is, during how many days per year (to the nearest integer) may the sluice-gates be open? (C.S.)

26. A river holds 23 grains of solid matter in suspension to every gallon. At a certain point the average velocity of the river is 4 miles an hour, and the area of the cross-section of the river is 1400 sq. ft. Find, to the nearest hundred, how many tons of solid matter in suspension are carried past this point in a year. (C.S.)

27. The diameter of some copper wire is 2 mm., and a cubic decimetre of copper weighs 9·0 Kg. Find, to the nearest metre, the length of wire in a quantity which weighs 7·5 Kg.

[The volume of a length of l metres, of diameter d metres, is $0{\cdot}7854 \times d^2 \times l$ cubic metres.]

CHAPTER XV.

GRAPHS.

248. ONE quantity is often related to another in such a way that if a change is made in the value of one there is a corresponding change in the value of the other. For example, suppose we know the cost of a certain weight of tea; if we double the weight we double the cost, if we treble the weight we treble the cost, and so on. In other words, in this case the cost is always *directly proportional* to the weight. Similarly, when a train is travelling at a uniform speed, the distance travelled is directly proportional to the time.

249. Any expression involving x will have different values if different values are substituted for x. Suppose we wish to find the values of the expression $2x+5$ when x has the series of values 3, 2, 1, 0, -1, -2, -3, the following arrangement will be found convenient.

Let y stand for the expression; that is, suppose $y=2x+5$, and arrange the values as in the following table:

x	3	2	1	0	-1	-2	-3
$2x$	6	4	2	0	-2	-4	-6
$y=2x+5$	11	9	7	5	3	1	-1

Thus corresponding to the values 3, 2, 1, 0, -1, -2, -3 for x we have the values 11, 9, 7, 5, 3, 1, -1, for y, or $2x+5$. Here there is no direct proportion between the values of x and y, but each value of y is dependent on the corresponding value of x.

250. A quantity which may have a series of different values is called a **variable.** In the above table x is a variable, and y (whose value depends on that of x) is also a variable. The relation between two variables thus connected may often be conveniently shewn by means of diagrams which give the values of the variables at a glance.

251. Axes of Reference. Coordinates. On a piece of squared paper select a pair of the thicker horizontal and vertical lines. Let these be marked XOX′, YOY′ as in Fig. 1 below. Then the position of any point P with reference to these lines can be found when we know its distances from each of them. Such lines are known as **axes of reference,** XOX′ being known as the **axis of x,** and YOY′ as the **axis of y.** Their point of intersection O is called the **origin.**

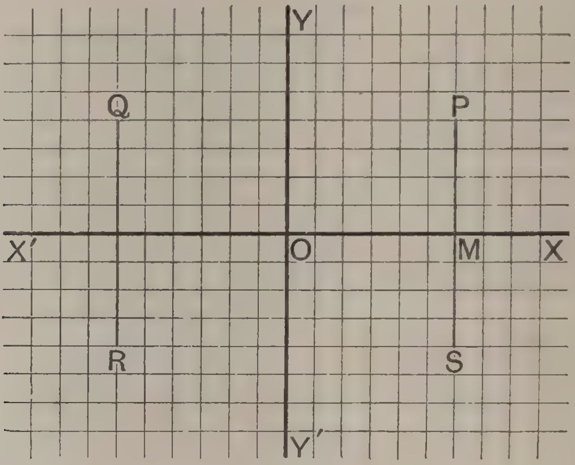


FIG. 1.

Consider the point P in the figure. It will be seen that we can get to P by marking 6 divisions of the paper along OX, that is to the point M, and then taking 4 divisions vertically up from M. Thus if the perpendicular distances of a point from the axes are known the position of the point is fixed. The distances 6 and 4 are known as the **coordinates** of the point P. OM is known as the **abscissa** of P, and PM is known as the **ordinate** of P.

When symbols are used the abscissa is generally denoted by x, and the ordinate by y. A point whose coordinates are x and y is spoken of as "the point (x, y)," the abscissa of the point always being named first. This process of marking the position of a point by means of its coordinates is known as **plotting the point.**

In practice the most convenient paper is that ruled to tenths of an inch, and one or more of the divisions may be taken as the unit of length.

252. The axes of reference divide the plane of the paper into four spaces XOY, YOX′, X′OY′, Y′OX, known respectively as the first, second, third, and fourth **quadrants.**

It is clear that in each quadrant there is a point whose distances from the axes are equal to those of P in the above figure, namely, 6 units and 4 units.

The coordinates of these points are distinguished by the use of the *positive* and *negative* signs, according to the following system: distances measured along the x-axis to the *right* of the origin are *positive*, those measured to the *left* of the origin are *negative*. Distances measured vertically *above* the x-axis (that is, in the first and second quadrants) are *positive*; those which lie *below* the x-axis (that is, in the third and fourth quadrants) are *negative*.

Thus the coordinates of the points Q, R, S, in Fig. 1 are

$$(-6,\ 4),\quad (-6,\ -4)\quad \text{and}\quad (6,\ -4)\ \text{respectively.}$$

The pupil may be reminded that this is a natural extension of the explanation of *opposite signs* given on page 53. (See Art. 49.)

Example 1. *Plot the points*

(i) $(6,\ 8)$; (ii) $(-2,\ 2)$; (iii) $(6,\ 0)$; (iv) $(0,\ 0)$;

and find the distance between the first two.

(i) We first take 6 units to the *right* along OX, and then 8 units at right angles to OX and *above* it. The resulting point P is in the first quadrant.

(ii) Here we may briefly describe the process as follows: Take 2 steps to the *left* then 2 *up*; the resulting point Q is in the second quadrant.

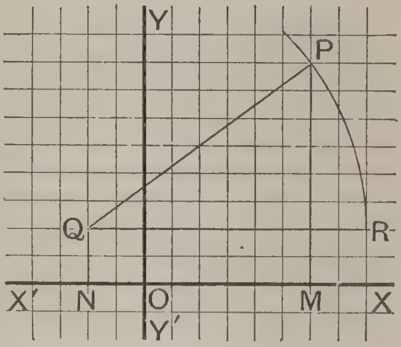


Fig. 2.

(iii) Take 6 steps to the *right*, then *no steps either up or down* from OX. Thus the resulting point M is on the axis of x.

(iv) The point (0, 0) obviously represents the origin O.

To find the distance between Q and P, draw an arc of a circle with centre Q and radius QP. Let this arc cut the horizontal line through Q at R. Then QP = QR.

But QR = 10 divisions, each of which is one-tenth of an inch. Thus QP = 1 inch.

EXAMPLE 2. *A ship sails from harbour; first she sails* 4 *miles due West to a fort, thence* 6 *miles due South, then* 6 *miles due East, and then* 11 *miles due North. Find to the nearest mile her final distance from the fort.*

Here we may conveniently take the origin to denote the position of the harbour, and mark the axes WOE, NOS in order to shew the points of the compass. Let each division of the paper represent one mile; then 4 steps to the left brings us to P which represents the fort. From this point the ship's course is shewn by the dotted lines, and the final position is T. A circle described with centre P and radius PT cuts OE at V. Then PT = PV, which is very nearly 8 divisions from P. Thus the reqd. distance is 8 miles.

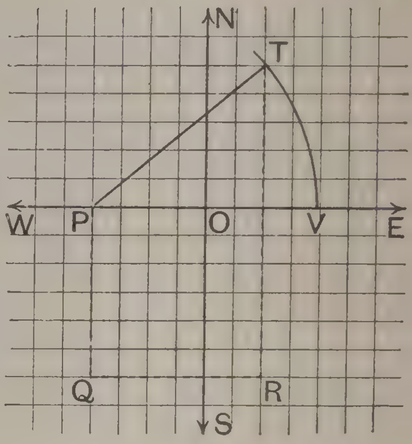


FIG. 3.

EXAMPLES XV. a.

Plot the following pairs of points and draw the line which joins them:

1. (4, 5), (2, 8).
2. (–4, 5), (–2, 8).
3. (3, 0), (0, 6).
4. (–2, 0), (0, –8).
5. (3, –8), (–2, 6).
6. (5, 5), (–2, –2).
7. (–2, 6), (1, –3).
8. (0, 0), (–3, 5).

9. Plot the points (3, 3), (–3, 3), (–3, –3), (3, –3) and find the number of squares contained in the rectangle given by these points.

10. Plot the points (5, 6), (–5, 6), (5, –6), (–5, –6). If each division of the paper is supposed to represent 1 foot, how many square feet are there in the rectangle determined by these points?

11. Plot the following pairs of points, and in each case find the distance between them.

(i) (4, 0), (0, 3);
(ii) (9, 8), (5, 5);
(iii) (15, 0), (0, 8);
(iv) (10, 4), (–5, 12);
(v) (20, 12), (–15, 0);
(vi) (20, 9), (–15, –3).

12. If a man first walks 8 miles West, and then 6 miles North, how far will he be from his starting point?

13. How far will a man be from his starting point after walking East for 12 miles and then South for 5 miles?

14. A ship sails from harbour, first 5 miles W., then 8 miles N., then 14 miles E., and lastly 13 miles S. How far is she now from the harbour, to the nearest mile?

15. Plot the following points and shew experimentally that each set lies in a straight line:

(i) $(9, 7)$, $(0, 0)$, $(-9, -7)$; (ii) $(-9, 7)$, $(0, 0)$, $(9, -7)$.

16. Plot the eight points $(0, 5)$, $(3, 4)$, $(5, 0)$, $(4, -3)$, $(-5, 0)$, $(0, -5)$, $(-4, 3)$, $(-4, -3)$ and shew that they are all at the same distance from the origin.

253. We now return to the expression $2x+5$ discussed in Art. 249. Using the same values of x as before, and putting y to represent the value of the expression, we have the following table of values:

x	3	2	1	0	−1	−2	−3
$y=2x+5$	11	9	7	5	3	1	−1

If we now plot the points given by each pair of values we mark L, M, N, P, Q, R, S in the adjoining figure.

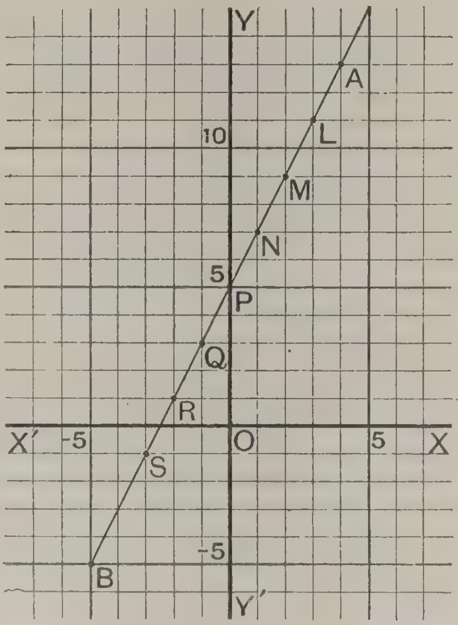


Fig. 4.

It will be seen that they all lie on a *straight line.* This line may be produced in either direction, and is called the **graph** of the expression $2x+5$.

Since y is always equal to $2x+5$, the variations of this expression are seen at a glance by noting the values of the ordinates of the different points.

The advantage of this **graphical** method of illustration is that we can read off from the graph the value of y (that is, of the expression $2x+5$) for *any* value of x.

Thus, from the graph,

when $x=4$, y (or $2x+5$)$=13$, at the point A;
and when $x=-5$, $y=-5$, at the point B.

Note. Any line drawn through a series of plotted points is called a graph. It is not necessarily a *straight* line.

EXAMPLES XV. b.

1. Plot the following series of points:

(i) (5, 0), (5, 2), (5, 5), (5, -1), (5, -4);

(ii) (-4, 8), (-1, 8), (0, 8), (3, 8), (6, 8).

Shew that they lie on two lines respectively parallel to the axis of y, and the axis of x. Read off the coordinates of the point in which they intersect.

2. Plot the values given in the following table:

x	2	-3	4	-5	8
y	2	-3	4	-5	8

shewing that they all lie on a certain line through the origin.

3. If $y=2x+10$, find the values of y when x has the values 0, 1, 3, -2, -5. Draw the graph, and find the coordinates of the points where it cuts the axes.

4. Choosing your own series of values for x, find and tabulate corresponding values for y in the following cases:

(i) $y=x$; (ii) $y=x+5$; (iii) $y=x-5$.

Draw the three graphs using the same pair of axes.

5. Using one pair of axes draw the graphs of

(i) $y=5x$; (ii) $y=5x-4$; (iii) $y=5x+6$.

In this and the preceding example what do you infer about the relative positions of the three lines?

6. Plot the graphs of

(i) $y=2x$; (ii) $y=\frac{1}{2}x$; (iii) $y=3x$; (iv) $y=-3x$.

What do you infer as to the *general character* of the graph of $y=ax$, where a is any numerical quantity?

7. Plot the points given in the following table:

x	0	3	5	4	-5	0	-4	-4
y	5	4	0	-3	0	-5	3	-3

Join these points by a freehand *curved* line. What is the nature of the curve? [Compare Ex. 16, p. 279.]

254. Measurement on Different Scales. In the foregoing examples we have measured abscissae and ordinates on the same scale for the sake of simplicity, but there is no necessity for so doing, and it will often be convenient to measure the variables on different scales so as to get a better diagram.

For example, in drawing the graph of $y=6x+3$,

when x has the values 0, 1, 2, 3, 4,

the corresponding values of y are 3, 9, 15, 21, 27.

Thus some of the ordinates are much larger than the corresponding abscissae and rapidly increase as x increases.

If these points are plotted with x and y measured on the same scale it will be found that with a small unit (such as one-tenth of an inch) the graph is inconveniently placed with regard to the axes. If a larger unit is used the graph requires a diagram of inconvenient size.

[The pupil should prove this for himself by trial.]

The inconvenience may be avoided by measuring the values of y on a considerably smaller scale than those of x. For example, let us take one inch as unit for x, and one-tenth of an inch as unit for y, then the graph of $y=6x+3$ will be found to be as in Fig. 5.

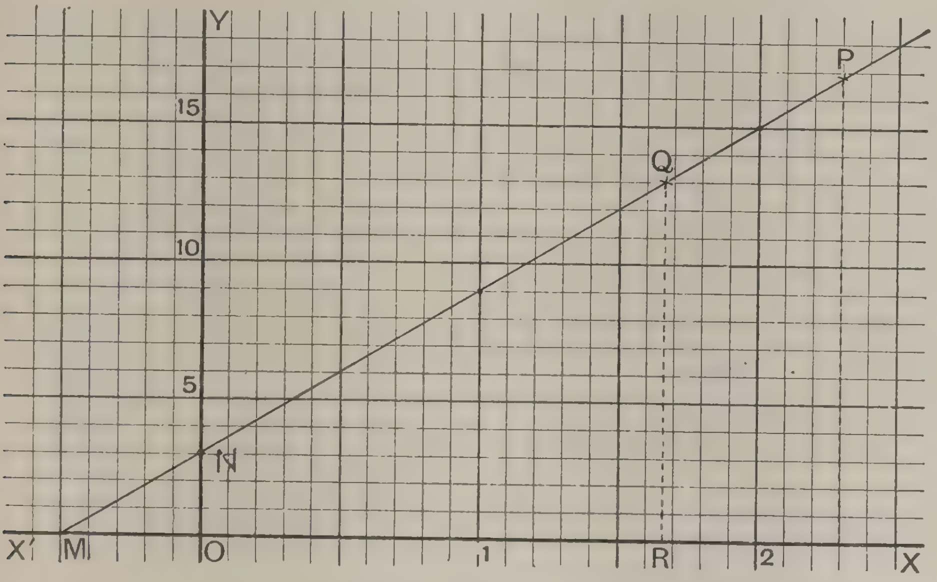


Fig. 5.

Note. Speaking generally, whenever one variable increases much more rapidly than the other, a small unit should be chosen for the rapidly increasing variable and a large one for the other.

255. When a graph has been accurately drawn from plotted points, it can be used to *read off* (without calculation) corresponding values of the variables at intermediate points.

EXAMPLE. *From the graph of the expression* $6x+3$ *find its value when* $x=2{\cdot}3$. *Also find the value of* x *which will make the expression equal to* 13.

Put $y=6x+3$, then the graph is that given in Fig. 5. Now we see that $x=2{\cdot}3$ at the point P, and here $y=17$, nearly.

Again $y=13$ at the point Q; and $x=1{\cdot}66$ very nearly. In reading off this last result we observe that OR is greater than 1·6 and less than 1·7, and we mentally divide the tenth in which R falls into *ten* equal parts (*i.e.* into *hundredths of the unit*) and judge as nearly as possible how many of these hundredths are to be added to 1·6.

EXAMPLES XV. c.

[*In some of the following examples the units are specified; in others the pupil is left to select suitable units for himself. When two or more graphs are involved in the same piece of work they must all be drawn on the same scale. In every case the units employed should be marked on the axes.*]

1. Choosing the values $x=0, 1, 2, 3, 4$, draw the graphs of $y=2x-8$, and $y=-2x+8$. Find the coordinates of the point where they intersect.

[Unit for x, one inch; for y, one-tenth of an inch.]

2. With the same units as in Ex. 1 draw the graphs of $y=9-4x$, and $y=\frac{5}{2}x-4$. Find the coordinates of the point where they intersect.

[Use the values: $x=-2, -1, 0, 1, 2, 3$.]

3. Plot the graphs of

(i) $y=10x+8$; (ii) $y=25-7x$.

In (i) read off the value of y when $x=-\frac{1}{2}$.
... (ii) .... x ...... $y=16$.

4. Using the values $x=-10, -5, 0, 5, 10, 15$, plot the graph of $y=\frac{x}{5}-2$.

Read off the value of x when $y=3$, and the value of y when $x=-15$.

[Use a large unit for y. See Art. 254.]

5. Plot the graph of $y=11x+6$, and find from the graph the value of the expression $11x+6$ when $x=1{\cdot}8$. Also find as nearly as you can the value of x which will make the expression equal to 20.

256. The pupil who has carefully worked the foregoing examples will have inferred that the graph of every equation of the form $y=ax+b$ (where a and b may have any numerical values whatever) is a straight line. Also that the graph of every equation of the form $y=ax$ is *a straight line through the origin.*

Before going further the pupil should verify by trial each of the following statements:

(i) The coordinates of the origin are (0, 0).

(ii) For every point on the axis of x the value of y is 0.

(iii) For every point on the axis of y the value of x is 0.

Note. The points where a graph cuts the axes can always be found by putting $y=0$, $x=0$ successively in the equation. Thus in the equation $y=6x+3$ on page 281,

when $y=0$, $x=-\frac{1}{2}=\text{OM}$ in the figure,

when $x=0$, $y=3=\text{ON}$ ....

The distances OM, ON are known as the *intercepts on the axes.*

257. Equations which contain no higher powers of x and y than the first are called **simple equations** or **equations of the first degree** to distinguish them from *equations of higher degrees* such as $y=ax^2+b$, $y=x^3$. All equations of the first degree can by suitable modifications be reduced to one of the forms $y=ax$, or $y=ax+b$, and since their graphs are straight lines it is convenient sometimes to refer to them as **linear equations.**

Since a *straight* line can always be drawn when *any* two points on it are known, in drawing a *linear* graph only two points need be plotted. The points where the line meets the axes (Art. 256, Note) will always suffice, though they are not always the best to select.

Example. *Draw the graph of* $4x-3y=13$.

$$\text{When } y=0,\ x=\frac{13}{4} \text{ (intercept on the } x\text{-axis)},$$

$$\text{and when } x=0,\ y=-\frac{13}{3} \text{ (intercept on the } y\text{-axis)}.$$

As both of these values involve fractions of the unit, it would be difficult to draw the line with sufficient accuracy. In such a case it is better to find by trial *integral* values of x and y which satisfy the equation.

Thus when $x=1$, $y=-3$, and when $y=1$, $x=4$.

The graph can now be drawn by joining the points $(1, -3)$, $(4, 1)$.

Graphs of Statistics.

258. In all the cases at present considered the graph has been a straight line obtained by first selecting values of x and y which satisfy *an equation of the first degree*, and then drawing a line so as to pass through the plotted points. The method is quite general, and it is easy to see that it may be applied when the variables are connected by an equation *which is not linear*. In such a case it will be found that a line drawn through the plotted points will take the form of some *curve* differing in shape according to the equation which connects the variables. Without discussing such cases in detail we may observe that, whenever two variable quantities depend on each other so that a change in one produces a corresponding change in the other, we can draw a graph to exhibit their variations without knowing any algebraical relation between them, *provided that we are furnished with a sufficient number of corresponding values accurately determined.*

But we frequently have to deal with cases in which a limited number of corresponding values of two variables have been obtained by *observation* or *experiment*. In such cases the data may involve inaccuracies, and consequently the position of the plotted points cannot be absolutely relied on. Moreover we cannot correct irregularities in the graph by selecting other points whose coordinates satisfy a given equation. One method of procedure is to join successive points by *straight* lines. The graph will then be represented by an irregular broken line, sometimes with abrupt changes of direction as we pass from point to point. In cases where no great accuracy of detail is required this simple method is often used to illustrate statistical results. A familiar instance is a Weather Chart giving the height of the barometer at equal intervals of time.

The chief disadvantage of this method is that, although it gives a general idea of the total change that has taken place between the plotted points, it furnishes no accurate information with regard to intermediate points.

Example. *The readings of a thermometer taken at intervals of 2 hours beginning at* 10 a.m. *were* 62·5°, 64°, 69·6°, 69°, 66·5°, 65·7°.

Draw a chart to shew the changes of temperature.

Let the hours be measured on the horizontal axis, taking 5 divisions to represent each interval of 2 hours, beginning at 10 a.m. On the vertical axis let each division represent 1° of temperature, beginning at 60°.

After plotting the points furnished by the data of the question, and joining them by straight lines we obtain the broken line PQRSTV shewn in Fig. 6.

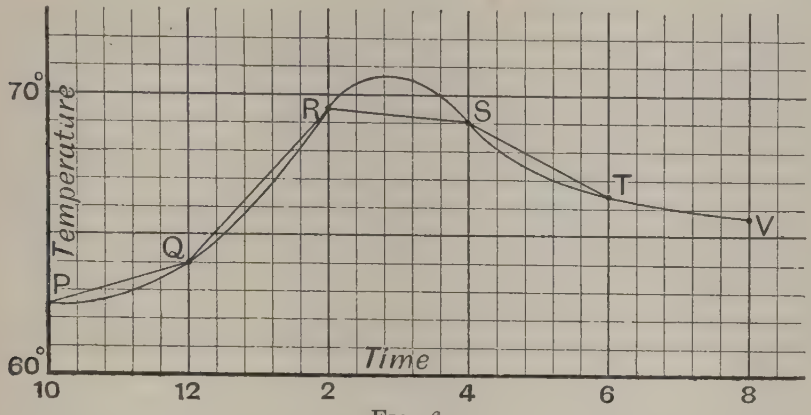


FIG. 6.

But it is contrary to experience to suppose that the abrupt changes of direction at Q and R accurately represent the change of temperature at noon and 2 p.m. respectively. Moreover, it is probable that the maximum temperature occurred at some time between 2 and 4, and not at the time represented at R, the highest of the plotted points. Now if the chart had been obtained by means of a self-registering instrument, the graph (representing change from instant to instant instead of at long intervals) would probably have been somewhat like the continuous waving curve drawn through the points previously registered. From this it would appear that the maximum temperature occurred shortly before 3 p.m., and that TV (which represents a very gradual change) is the only portion of the broken line which records with any degree of accuracy the variation in temperature during two consecutive hours.

259. Although in the last example we were able to indicate the form of the curved line which from the nature of the case *seemed most probable*, it is evident that any number of curves can be drawn through a limited number of plotted points. In such a case the best plan is to draw a curve to lie *as evenly as possible* among the plotted points, passing through some perhaps, and with the rest fairly distributed on either side of the curve. As an aid to drawing an even continuous curve (usually called a *smooth* curve), a thin piece of wood or other flexible material may be bent into the requisite shape, and held in position while the line is drawn. When the plotted points lie approximately on a straight line, the simplest plan is to use a piece of tracing paper on which a straight line has been drawn. When this has been placed in the right position the extremities can be marked on the squared paper, and by joining these points the approximate graph is obtained.

EXAMPLE. *The following table gives statistics of the population of a certain country, where* P *is the number of millions at the beginning of each of the years specified.*

Year	1830	1835	1840	1845	1850	1855	1860
P	20	22	24·5	28	31	31	41

Let t *be the time in years from* 1830. *Plot the values of* P *vertically and those of* t *horizontally and shew the relation between* P *and* t *by a simple curve passing fairly evenly among the plotted points. Find what the population was at the beginning of the years* 1847 *and* 1858.

Take one-tenth of an inch as unit in each case; also it will be convenient if we begin measuring abscissæ at 1830, and ordinates at 20.

The graph is given in Fig. 7; it will be seen that it passes exactly through three of the points and lies evenly among the others.

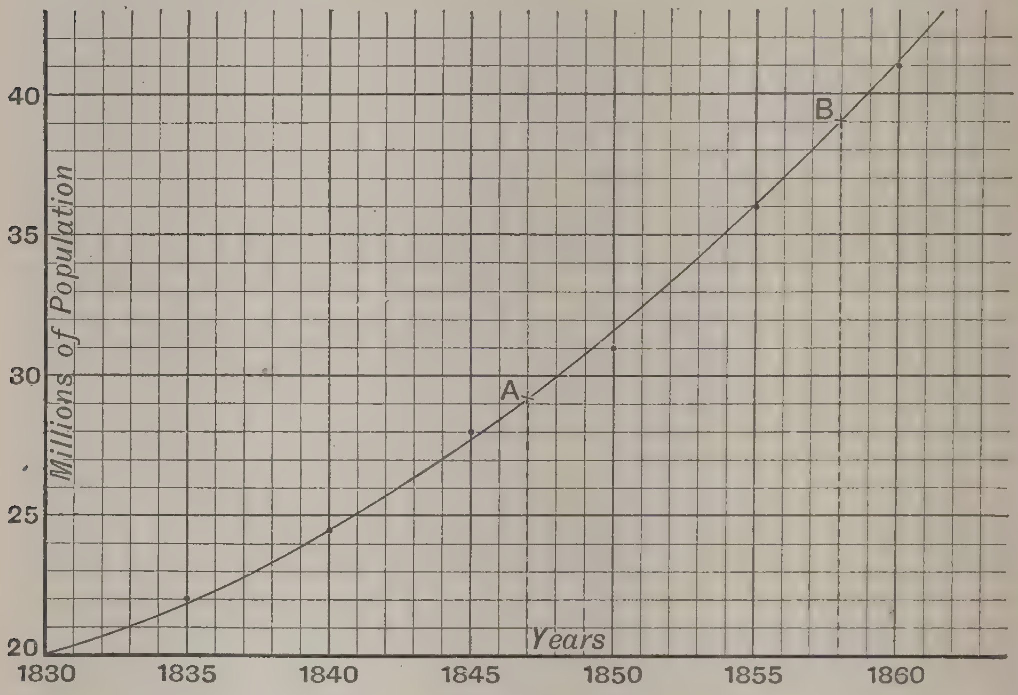


FIG. 7.

The populations in 1847 and 1858, at the points A and B respectively, will be found to be $29\frac{1}{4}$ millions and 39 millions.

260. When the graph is linear it can be produced to any extent within the limits of the paper and so any value of one of the variables being determined the corresponding value of the other can be read off. When large values are in question this method is inconvenient; the following example illustrates the method of procedure in such cases.

EXAMPLE. *Corresponding values of* x *and* y, *some of which are slightly inaccurate, are given in the following table:*

x	1	4	6·8	8	9·5	12	14·4
y	4	8	12·2	13	15·3	20	24·8

Draw the most probable graph. Also find its equation, and thence determine the value of y *when* x = 80, *and the value of* x *when* y = 68.

Let 1 inch be taken to represent 5 units along OX, and 20 units along OY.

After carefully plotting the given points we see that a straight line can be drawn passing through three of them (marked with a cross) and lying evenly among the others. This is the required graph.

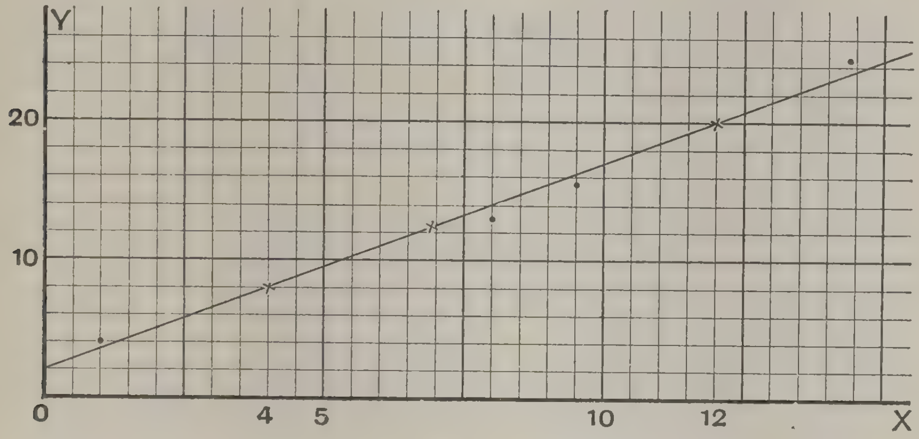


FIG. 8.

Since the graph is a straight line its equation is of the form $y = ax + b$, where a and b are numerical quantities which we have to find. Since the line passes accurately through the points (4, 8), (12, 20), these values of x and y must satisfy the equation.

Thus putting $x = 4$, $y = 8$, we obtain $8 = 4a + b$ ....(1);

again, when $x = 12$, $y = 20$, we have $20 = 12a + b$ ....(2).

By subtraction, $8a = 12$, whence $a = 1·5$.

Substituting this value in (1), we obtain $b = 2$.

Hence the equation of the graph is $y = 1·5x + 2$, and the coordinates of any number of points on the line may now be found by trial.

Thus when $x = 80$, $y = 122$. Again when $y = 68$, $x = \frac{66}{1·5} = 44$.

EXAMPLES XV. d.

[*In Examples* 1–5 *the plotted points may be joined by straight lines. In other cases the graph is to be a straight line or smooth curve lying evenly among the plotted points.*]

1. In a term of 11 weeks a boy's places in his Form were as follows:

8, 6, 11, 10, 9, 6, 6, 4, 2, 1, 1.

Shew these results by means of a graph.

2. The mean heights of the barometer in inches for the first 10 days of January 1904 were as follows:

29·21, 29·12, 29·00, 29·25, 29·37, 29·26, 29·46, 28·83, 28·66, 28·76.

Exhibit these variations by means of a chart.

3. The highest and lowest prices of Consols for the years 1895 to 1904 were as follows:

Year	'95	'96	'97	'98	'99	'00	'01	'02	'03	'04
Highest	$108\frac{1}{8}$	$113\frac{7}{8}$	$113\frac{7}{8}$	$113\frac{3}{8}$	$111\frac{1}{2}$	$103\frac{1}{4}$	$97\frac{7}{8}$	$97\frac{7}{8}$	$93\frac{5}{8}$	$91\frac{1}{4}$
Lowest	$103\frac{1}{2}$	$105\frac{1}{8}$	$110\frac{5}{8}$	$106\frac{3}{4}$	$97\frac{3}{4}$	$96\frac{3}{4}$	91	$92\frac{5}{16}$	$86\frac{7}{8}$	85

Make a chart to shew these variations graphically on the same diagram.

[A convenient scale will be: one inch to £10 vertically, beginning at 85, and 0·5 of an inch to 1 year horizontally.]

4. Make a chart to shew the variations in French Imports and Exports (in millions of pounds), for the years 1896 to 1903 inclusive, from the following data:

Imports	50·1	53·3	51·3	53·0	53·6	51·2	50·6	49·9
Exports	20·6	19·5	20·5	22·2	25·8	23·7	22·2	23·1

5. Illustrate graphically the variations of National Revenue in the years 1898–1906 from the following data:

Year	'98	'99	'00	'01	'02	'03	'04	'05	'06
Revenue in millions of £	95	96	105	115	127	135	125	127	127

6. Plot a graph to shew the variations of population of a certain country from the following statistics, where P is the number of millions at the beginning of each of the years specified.

Year	1830	1835	1840	1850	1860	1865	1870	1880
P	20	22·1	23·5	29·0	34·2	38·2	41·0	49·4

Find what the population was at the beginning of the years 1848 and 1875.

7. The mean temperature on the first day of each month, on an average of 50 years, had the following values:

Jan. 1, 37°;	May 1, 50°;	Sept. 1, 59°;
Feb. 1, 38°;	June 1, 57°;	Oct. 1, 54°;
Mar. 1, 40°;	July 1, 62°;	Nov. 1, 46°;
April 1, 45°;	Aug. 1, 62°;	Dec. 1, 41°.

Represent these variations by means of a smooth curve.

[The difference of length of different months may be neglected.]

8. The following table gives approximately the circumferences of circles corresponding to different radii:

C	15·7	20·1	31·4	44	52·2
r	2·5	3·2	5	7	8·3

Plot the values on squared paper, and from the graph determine the diameter of a circle whose circumference is 12·1 inches and the circumference of a circle whose radius is 2·8 inches.

9. For a given temperature, C degrees on a Centigrade are equal to F degrees on a Fahrenheit thermometer. The following table gives a series of corresponding values of F and C:

C	−10	−5	0	5	10	15	25	40
F	14	23	32	41	50	59	77	104

Draw a graph to shew the Fahrenheit reading corresponding to a given Centigrade temperature, and find the Fahrenheit readings corresponding to 12·5° C and 31° C.

10. Corresponding values of x and y, some of which are slightl inaccurate, are given in the following table:

x	3	6·5	12	14	21	28·6	31·5
y	4	4·8	6·7	7	8·5	11	11·5

Draw the most probable graph. From it find the value of x whe $y=11{\cdot}5$, and the value of y when $x=10$.

11. If W is the weight in ounces required to stretch an elasti string till its length is l inches, plot the following values of W and l:

W	2·5	3·75	6·25	7·5	10	11·25
l	8·5	8·7	9·1	9·3	9·7	9·9

From the graph determine the unstretched length of the string, and the weight the string will support when its length is 1 foot.

12. In an Insurance Society the premium (£P) to insure £100 at different ages is given approximately by the following table:

Age	20	22	25	30	35	40	45	50	55
P	1·8	1·9	2·0	2·3	2·7	3·1	3·6	4·4	5·5

Illustrate the same statistics graphically, and estimate to the nearest shilling the premiums for persons aged 34 and 43.

13. A manufacturer wishes to stock a certain article in many sizes ; at present he has five sizes made at the prices given below:

Length in inches	20	27	33	45	54
Price in shillings	11	14·5	20	35	48·5

Draw a graph to shew suitable prices for intermediate sizes, and find what the prices should be when the lengths are 30 in. and 46 in.

14. The price in pence of a standard Troy ounce of silver on January 1st in each of the ten years 1891–1900 was

45, 40, 36, 29, 30, 31, 28, 27, 27, 28.

Draw a smooth curve shewing its value approximately at any time during these ten years.

15. Plot the points given by the following measured values of x and y, some of which are slightly inaccurate; determine the most probable graph and find its equation. [See Art. 260.]

x	11	20	30	37·2	40	51·3	60	71	80
y	3	7	11·6	15	17	23·2	26·5	33	37

From the graph find the correct value of x when $y=15$, and the correct value of y when $x=71$. From the equation find the value of y when $x=164$.

16. An india-rubber cord was loaded with weights, and a measurement of its length was taken for each load as tabulated. Plot a graph to shew the relation between the length of the cord and the loads.

Load in pounds - - -	10	12	17	21	23	25
Length in centimetres	36·4	37·7	40·5	43·0	44·3	45·4

What was the length of the cord unloaded? Also find the load necessary to produce a length of 41·2 centimetres.

17. The following table gives the population (in thousands) of two towns P and Q at the beginning of each of the years specified:

Year	1835	1845	1855	1865	1875	1880	1890
P	24·4	26	29·5	34	40	43	50
Q	34	36	38·4	41·1	43	44·8	46·7

Plot the graphs on the same diagram, and estimate the population of each town at the beginning of 1870. In what year was the population approximately the same in each?

18. At different ages the mean after-lifetime ("expectation of life") of males, calculated on the death rates of 1871–1880, was given by the following table:

Age - - -	6	10	14	18	22	26	27
Expectation	50·38	47·60	44·26	40·96	37·89	34·96	34·24

Draw a graph to shew the expectation of any male between the ages of 6 and 27, and from it determine the expectation of persons aged 12 and 20.

CHAPTER XVI.

RATIO AND PROPORTION.

261. **Ratio** is the relation that one quantity bears to another *of the same kind*, the comparison being made by considering what multiple, part, or parts the first quantity is of the second. Thus the ratio of one quantity to another is measured by *the fraction which the first is of the second.*

For example, the ratio 21 miles to 14 miles is the abstract fraction $\frac{21}{14}$ or $\frac{3}{2}$. This may be written 21 miles : 14 miles, or 3 : 2.

262. The two quantities compared in a ratio are called its **terms.** The first term is called the **antecedent**; the second the **consequent.**

263. The terms of a ratio must be *of the same kind*; for instance, we cannot compare the magnitude of 21 miles with that of 14 tons. But such a ratio as 1050 lbs. to 2 tons 5 cwt. is possible, since the quantities compared may be expressed in the same denomination, and thus the first quantity may be represented as a fraction of the second.

264. For the purpose of comparison ratios may be reduced to percentages, as explained in Art. 138. [See also Art. 206, Ex. 2.]

EXAMPLE 1. *On a certain day the exchange between London and Paris was* 25·20 *francs to* £1, *and the exchange between London and New York was* 4·86 *dollars to* £1. *What ratio does the value of* 1 *franc bear to that of* 1 *dollar?*

Since $$25{\cdot}20 \text{ francs} = 4{\cdot}86 \text{ dollars},$$

$$\therefore \frac{1 \text{ franc}}{1 \text{ dollar}} = \frac{4{\cdot}86}{25{\cdot}20} = \frac{27}{140}, \text{ or } 27 : 140.$$

EXAMPLE 2. *Understanding the* specific gravity *of a solid as* **the ratio which the weight of any volume of it bears to the weight of an equal volume of water,** *find (correct to two significant figures) the specific gravity of cast iron, having given that* 1 *cubic inch of cast iron weighs* 4·18 *oz., and* 1 *cubic foot of water weighs* 62·43 *lbs.*

Since 1 cubic foot of water weighs 62·43 lbs.,

$$\therefore \text{ 1 cubic inch } \ldots\ldots\ldots\ldots \frac{62{\cdot}43 \times 16}{12 \times 12 \times 12} \text{ oz.}$$

$$= 0{\cdot}578 \text{ oz. (nearly).}$$

$$\text{Now specific gravity of iron} = \frac{\text{weight of 1 cu. in. of iron}}{\text{weight of 1 cu. in. of water}}$$

$$= \frac{4{\cdot}18}{0{\cdot}58} = 7{\cdot}2.$$

EXAMPLES XVI. a.

Express the following ratios in their simplest fractional form :

1. ·65 miles : 91 miles. **2.** 259 tons : 925 tons.

3. £1. 3*s.* : £4. 6*s.* 3*d.* **4.** 255 metres : 1·105 kilometres.

5. $4\frac{2}{5}$ of £5. 12*s.* 6*d.* : £6. 15*s.*

6. Write down as ratios in their simplest form :

(i) 6*d.* in the £ ; (ii) 1*s.* 9*d.* in the £ ;

(iii) 2*s.* $4\frac{1}{2}$*d.* in the £ ; (iv) 5*s.* $3\frac{3}{4}$*d.* in the £.

7. Which is the greatest, and which is the least of the following ratios? Express each as a percentage correct to three significant figures.

(i) $\frac{\text{£25. 2}s.}{\text{£46. 0}s.\ 4d.}$; (ii) $\frac{\text{17 cwt. 2 qrs.}}{\text{1 ton 7 cwt. 2 qrs.}}$; (iii) $\frac{\text{695 metres}}{\text{1·529 kilometres}}$.

8. Having given the following approximate equivalents :

(i) 32 *metres* = 35 *yards ;* (ii) 5 *kilograms* = 11 *pounds ;*

(iii) 4 *litres* = 7 *pints ;* (iv) 2 *hectares* = 5 *acres ;*

write down the approximate ratios of

(i) 1 metre to 1 yard ; (ii) 1 kilogram to 1 pound ;

(iii) 1 litre to 1 pint ; (iv) 1 hectare to 1 acre.

9. The annual rates on two farms are respectively £33. 8*s.* 4*d.* and £55. 2*s.* 9*d.*, the same rate per acre being charged in each case: compare as a ratio the acreage of the two farms.

10. Two steamers perform the same passage in 2 days 1 hour an $3\frac{1}{2}$ days respectively: compare their average speeds per hour.

11. Two trains travel respectively 448 kilometres in 8 hours an 504 kilometres in 12 hours: compare their average speeds.

12. Compare the rates of travelling of a bicyclist who goes 54· kilometres in 2 hrs. 10 min. and a train which travels 95·2 kilometre in 1 hr. 42 min.

13. Two men undertake to drive a distance of 54 miles. Th first performs the journey at an average rate of 8 miles an hour The second, starting half-an-hour later, arrives 15 minutes sooner find the ratio of their speeds.

14. Consider the ratio 5 : 8. Add 10 to each term, and explai why the ratio is thus *increased.* If 10 is added to the antecedent what must be added to the consequent in order that the ratio ma be unchanged?

15. Shew that the ratio 8 : 5 is *diminished* by the addition of an positive quantity a to each term. Find the value of a when the rati $\frac{8+a}{5+a}$ becomes equal to $\frac{7}{5}$.

[The **specific gravity** of a solid substance is the ratio which th weight of any volume of it bears to the weight of an equal volum of *water.*]

16. Given that 1 litre of water weighs 1 kilogram, and that 1 cubic decimetre of gold weighs 19·3 kilograms; write down the specific gravity of gold.

17. Find the specific gravities of the following substances, assuming in each case that 1 cubic foot of water weighs 62·43 lbs.:

(i) *Sheet copper*, having given that 1 cu. ft. of copper weighs 548·1 lbs.

(ii) *Cast lead*, having given that 1 cu. inch of lead weighs 6·53 oz.

(iii) *Zinc*, having given that 1 cu. inch of zinc weighs 4·05 oz.

18. The specific gravity of cast iron is 7·2, and of steel 8·0. Which has the greater volume, 1 lb. of iron or 1 lb. of steel? Two masses of iron and steel have equal weights, find the ratio of their volumes.

[A map and the country it represents are *similar figures*, that is, are of the same shape; and every length measured on the map bears a fixed ratio to the actual distance it represents. This constant ratio is called the **scale** of the map, and when expressed as a fraction with unit numerator is called its **Representative Fraction.** For instance in a map in which 3 inches represents 40 yards, the R.F. $=\frac{3}{40\times 36}=\frac{1}{480}$.]

19. A map is drawn to the scale of 1 inch to 20 miles; what is its Representative Fraction?

On this map Plymouth and Penzance are $3\frac{1}{4}$ inches apart; what is the actual distance?

The actual distance between Exeter and Barnstaple is 35 miles; how far apart will they be on this map?

20. On a map of South Wales a distance of 50 miles is represented by 4·5 inches. What is the scale of the map, that is, how many miles to 1 inch? What is its Representative Fraction?

From Cardiff to Pembroke on the map is 6·6 inches;
from Swansea to Cardigan .... is 3·9 inches;

find the actual distances between these places to the nearest mile.

Simple Proportion.

265. Four quantities are said to be in Proportion when the ratio of the *first* to the *second* is equal to the ratio of the *third* to the *fourth*: that is to say, when the *first* is the **same fraction** of the *second* that the *third* is of the *fourth*.

For example: the numbers 14, 21; 26, 39 are in proportion, since $\frac{14}{21}=\frac{26}{39}$.

This is expressed by saying

14 *is to* 21 *as* 26 *is to* 39;

or $$14 : 21 = 26 : 39.$$

Similarly the numbers a, b; x, y are proportional if

$$\frac{a}{b}=\frac{x}{y};$$

and the relation may be written

$$a : b = x : y.$$

Again the four quantities

1 ton 12 cwt., 2 tons; 5*s*. 4*d*., 6*s*. 8*d*.

are proportionals:

for $$\frac{\text{1 ton 12 cwt.}}{\text{2 tons}}=\frac{\text{32 cwt.}}{\text{40 cwt.}}=\frac{4}{5};$$

and $$\frac{5s.\ 4d.}{6s.\ 8d.}=\frac{64d.}{80d.}=\frac{4}{5};$$

from which we see that 1 ton 12 cwt. is the *same fraction* of 2 tons that 5*s*. 4*d*. is of 6*s*. 8*d*.

This may be written

1 ton 12 cwt. : 2 tons = 5*s*. 4*d*. : 6*s*. 8*d*.

266. A statement expressing the equality of two ratios is called a **proportion**; the four quantities compared are the **terms** of the proportion; the first and last terms are called the **extremes**, and second and third are called the **means.**

NOTE. Since every proportion is the equality of two ratios, it follows that the first and second terms must be of the same kind, and the third and fourth terms must be also of the same kind.

267. When four numbers are in proportion *the product of the extremes is equal to the product of the means.*

For suppose the four numbers a, b ; x, y are proportionals;

that is, suppose $$\frac{a}{b}=\frac{x}{y}.$$

Multiplying both sides by by, we have

$$\frac{a}{b}\times by=\frac{x}{y}\times by,$$

or $$ay=bx.$$

268. When four quantities are in proportion, the last quantity is said to be a **fourth proportional** to the other three.

Thus if $a : b=x : y$, then y is the fourth proportional to a, b, x.

Three quantities a, b, c of the same kind are said to be proportionals, when $a : b=b : c$.

In this case c is said to be a **third proportional** to a and b; and b is said to be a **mean proportional** to a and c; and it will be noticed that

$$b^2=ac.$$

EXAMPLE 1. *What weight has to* 165 *tons the ratio of* 7 *to* 11?

Let x denote the required number of tons;

then $$\frac{x}{165}=\frac{7}{11};$$

$$\therefore\ x=\frac{7}{11}\times 165=105.$$

So that the required weight is 105 tons.

EXAMPLE 2. *What sum of money has to* £2. 6*s*. 8*d*. *the same ratio that* 1 *ton* 4 *cwt. has to* 5 *cwt.*?

Let x denote the required number of pounds;

then $$\frac{x}{2\frac{1}{3}}=\frac{1\text{ ton }4\text{ cwt.}}{5\text{ cwt.}}=\frac{24\text{ cwt.}}{5\text{ cwt.}}=\frac{24}{5};$$

$$\therefore\ x=\frac{24}{5}\times 2\tfrac{1}{3}=11\tfrac{1}{5}.$$

So that the required sum = £11. 4*s*.

Example 3. *Find the 3rd term in the proportion*

$$1\cdot68 : 2\cdot52 = x : 4\cdot29.$$

Here $$\frac{1\cdot68}{2\cdot52} = \frac{x}{4\cdot29};$$

$$\therefore\ x = \frac{1\cdot68 \times 4\cdot29}{2\cdot52} = 2\cdot86.$$

Example 4. *Find the 4th proportional to*

$$9\cdot6\ Kg.,\quad 7\cdot2\ Kg.,\quad 28\cdot8\ metres.$$

Here we have $\dfrac{9\cdot6 \text{ Kg.}}{7\cdot2 \text{ Kg.}} = \dfrac{28\cdot8 \text{ metres}}{x \text{ metres}}$, or $\dfrac{96}{72} = \dfrac{28\cdot8}{x}$.

Hence, as explained in Art. 132,

$$x = 28\cdot8 \times \frac{72}{96} = 21\cdot6.$$

∴ required 4th proportional = 21·6 metres.

EXAMPLES XVI. b.

1. What number has to 65 the same ratio that 9 has to 13?

2. Find the number which is to 6·8 as 8·1 is to 7·2.

3. What sum of money bears to £32 the same ratio that $85\frac{1}{2}$ bears to 114?

4. Find the sum which has to £4. 5*s.* the ratio $65 : 110\frac{1}{2}$.

5. What sum has to £9. 18*s.* the same ratio that 1 ton 1 cwt. has to 2 tons 14 cwt.?

6. To what weight does 8·25 kilograms bear the ratio 55 : 7?

Find the *fourth proportional* to each of the following sets of numbers:

7. 24, 51, 104;

8. $\frac{2}{3}$, $\frac{7}{8}$, $\frac{4}{5}$;

9. 5·6, 46, 0·63;

10. ab^2, a^2b, $\frac{b}{a}$.

Find the *third proportional* to

11. 0·5, 1·5.

12. $5\frac{1}{4}$, 7.

13. 5·6, 0·84.

14. a, b.

Find the *mean proportional* between

15. 7, 63.

16. $\frac{3}{5}$, $\frac{4}{15}$.

17. 1·3, 0·637.

18. a^3b, ab^3.

Find the missing terms in each of the following proportions:

19. £15 : [ ] = 17 : 51.

20. 27 : 39 = [ ] : 364 metres.

21. 3·15 tons : [ ] = 15 : 44.

22. £8 : £27 = 280 miles : [ ].

23. [ ] : £36. 9*s.* = 4 tons : 27 tons.

24. 1 yard : 11 feet = 750 metres : [ ].

25. 12 francs : [ ] = 291 grams : 3·201 Kg.

26. One cubic inch of steel weighs 4·61 oz., and one cubic centimetre of steel weighs 0·28 oz. Express as a decimal (correct to two significant figures) the ratio of 1 cubic centimetre to 1 cubic inch.

27. Find a rough equivalent of 1 ton in kilograms, it being given that the ratio of a kilogram to a pound is nearly 11 : 5.

28. Taking the ratio of a yard to a metre as approximately 200 : 219, find to the nearest metre the equivalent of 2920 yards.

29. The ratio of a litre to a pint being nearly 7 : 4, find roughly the number of gallons in 1 kilolitre.

30. The specific gravities of copper and cast iron are respectively 8·6 and 7·2; find, correct to one decimal figure, the weight of a cubic foot of cast iron, having given that a cubic foot of copper weighs 537·3 lbs.

Simple Proportion by Multiplying-ratios.

269. From the preceding Articles it will be seen that when three terms of a proportion are given, the fourth term may be found by multiplying one of the given quantities by a ratio formed from the remaining two. The method is more fully explained below.

EXAMPLE. *If* 65 *lbs. of a certain tea cost* £9. 15*s.*, *how much should be paid for* 24 *lbs.*?

It will be seen at once that this question may be solved by the Unitary Method, as explained in Chapter VII.; but we may reason more directly from the principles of proportion.

For it is evident that as long as the price per lb. remains the same, *the required cost is the same fraction of* £9. 15*s.* *that* 24 lbs. *is of* 65 lbs.;

that is,
$$\frac{\text{required cost}}{£9.\ 15s.} = \frac{24}{65};$$

$$\therefore \ \text{required cost} = £9.\ 15s. \times \frac{24}{65}$$
$$= £3.\ 12s.$$

That is to say, *noticing that the required cost must be* **less than** *the given cost in the ratio* 24 : 65, *we may at once multiply the given cost by the fraction* $\frac{24}{65}$ (*less* than unity), which we may call the **multiplying ratio.**

270. This arrangement is further illustrated by the following examples which should be carefully compared.

EXAMPLE 1. *A pumping-engine of* 54 *horse-power raises* 306 *gallons per minute from a shaft of a certain depth. What horse-power is needed to raise* 561 *gallons per minute from the same depth?*

Here 306 gallons are raised by 54 horse-power;
561 gallons .... *req^d horse-power.*

[Now if the number of gallons is *increased*, the horse-power must be proportionately *increased.* That is, the required horse-power will be *greater than* 54 in the ratio 561 : 306. Hence the multiplying ratio is the fraction $\frac{561}{306}$, *greater* than unity.]

$$\therefore \text{ required horse-power} = 54 \times \tfrac{561}{306}$$
$$= 99.$$

EXAMPLE 2. *A motor-car runs for* $3\frac{1}{2}$ *hours at an average speed of* 25 *miles an hour. How long would it take to cover the same distance at a speed of* 42 *miles an hour?*

At 25 miles an hour the car takes $3\frac{1}{2}$ hours;
... 42 miles an hour .... *req^d time.*

[Now if the speed is *increased* the time required is proportionally *decreased.* That is, the required time will be *less than* $3\frac{1}{2}$ hours in the ratio 25 : 42. Hence the multiplying ratio is the fraction $\frac{25}{42}$, *less* than unity.]

$$\therefore \text{ the required time} = 3\tfrac{1}{2} \text{ hours} \times \tfrac{25}{42}$$
$$= 2\tfrac{1}{12} \text{ hours} = 2 \text{ hrs. } 5 \text{ min.}$$

271. The reasoning which guided us in forming the multiplying ratios in the last two examples should be carefully noted and compared.

In Example 1, the *greater* the quantity of water raised, the *greater* the necessary horse-power. *Double* the quantity of water would require *double* the horse-power; and *half* the quantity of water would require *half* the horse-power. In fact, if we *multiply* the quantity of water by any number m (whole or fractional) we must also *multiply* the horse-power by the same number m. In this case the required horse-power is said to be **directly proportional to,** or to **vary directly as,** the number of gallons to be raised.

In Example 2, the *greater* the speed, the *less* will be the time required. To *double* the speed would *halve* the time; and to *halve* the speed would *double* the time. In fact, if we *multiply* the speed by any number *m* (whole or fractional) we must *divide* the time by *m*. In this case the time is said to be **inversely proportional to,** or to **vary inversely as,** the speed.

These tests for direct and inverse proportionality will be further discussed and illustrated graphically in a later section. [See p. 307.]

EXAMPLES XVI. c.

1. By the method of multiplying ratios, find

(i) the sum greater than £18 in the ratio 11 : 9;

(ii) the weight greater than $3\frac{3}{4}$ tons in the ratio 12 : 5;

(iii) the sum less than £14. 8*s.* in the ratio 15 : 24;

(iv) the length less than 9 Km. 450 m. in the ratio 17 : 135;

(v) the sum greater than £12. 10*s.* in the ratio of 13 tons : 10 tons.

(vi) the weight less than 3 lbs. 4 oz. in the ratio of $2\frac{1}{2}$ yds. : $4\frac{1}{3}$ yds.

2. A month's wages of 10 men amount to £55; how much would 24 men earn in the same time at the same rate?

3. A vessel sails 74 miles in 12 hours; what distance would be covered in 54 hours at the same speed?

4. On a certain map 7 inches represents 119 miles; what length represents 153 miles?

5. If a supply of rations will last 87 men for 26 days, how long will the same supply last 58 men at the same rate of issue?

6. Steaming at the rate of 18 knots a passage is made in 180 hours; how long would the same passage take at the rate of 10 knots?

7. A tap admitting 20 gallons a minute fills a tank in 21 minutes. How long would a tap admitting 56 gallons a minute take to fill the tank?

8. The railway fare second-class for a journey of 200 miles being £1. 5*s.*, what should be the second-class fare for a journey of 420 miles?

9. On the plan of an estate a distance of 110 yards is represented by $5\frac{1}{2}$ inches. What distance is represented by 3 inches?

10. An Atlantic Liner will make her passage in 5 days, if she maintains a speed of $22\frac{1}{2}$ knots. What must be her average speed if her time is to be reduced to $4\frac{1}{2}$ days?

11. In a coal-mine the winding engine raises the cage in $3\frac{1}{2}$ minutes, when it works at 36 revolutions a minute: if it increases its speed to 42 revolutions, how long will it take to raise the cage?

12. (i) A gang of men has been earning £56 a week. If their rate of pay per hour is increased in the ratio $\frac{21}{20}$, and their hours of work unaltered, what will be their weekly earnings?

(ii) A gang of men is paid at the rate of $7\frac{1}{2}$*d.* per hour each. If their hours of work per week are reduced in the ratio $\frac{10}{11}$, while their total weekly earnings remain the same, what is their new rate of pay?

13. (i) Two motors A and B travel for the same time, A's speed having to B's the ratio 21 : 16. A goes $57\frac{3}{4}$ miles; how far does B go?

(ii) A and B travel a certain distance at the above relative speeds. A takes 3 hrs. 12 min., how long does B take?

(iii) A and B travel a certain distance, A's time having to B's the ratio 16 : 21. A travels 28 miles an hour; what is B's speed?

14. If 9·1 metres of cloth are worth 52 shillings, find the value of 3·5 metres.

15. The weight of an oak beam 5·6 metres in length is 96 kilograms. Find the weight of a like beam whose length is 7·7 metres.

16. Find the value of a kilolitre of wine, of which 7·2 litres can be obtained for 27 shillings.

17. If silk is bought at the rate of 15·5 metres for 130 fr. 20 c., find the cost of 100 metres.

18. If $17\frac{1}{2}$ cubic feet of gravel go to the ton, how many hundredweight go to the cubic yard? [Answer to the nearest unit only.]

19. An express is timed as follows: Paddington, dep. 11.0 a.m.; Bristol, arr. 1.0 p.m., dep. 1.10; Exeter, arr. 2.33, dep. 2.41; Plymouth, arr. 3.48. The whole distance is 246 miles. What is the average running speed per hour? What is the average speed including stoppages?

20. On crushing a sample of quartz weighing 18 tons, a company extracts 23·62 oz. of gold. Estimate the yield per 100 tons. [The answer to be in ounces correct to the 2nd decimal figure.]

21. If 214·5 metres of wire of a certain diameter can be drawn from 13·2 grams of gold, what length of wire of the same size would be given by 8·4 grams?

22. If the diameter of a circle is 15 metres, its circumference is found to be nearly 47·12 metres. Find approximately the circumference of a circle whose diameter is 11 metres. [N.B. *Circumference of circles are directly proportional to their diameters.*]

23. The back and front wheels of a carriage have diameters of 4 feet and $3\frac{1}{2}$ feet respectively. When the former has made 259 revolutions, how many revolutions has the latter made?

24. Two wheels whose diameter are $4\frac{1}{4}$ feet and $5\frac{1}{2}$ feet respectively, are geared together by an endless strap passing over their circumferences. When the smaller wheel has made 220 revolutions, how many will the larger wheel have made?

25. One cubic inch of lead weighs $6\frac{1}{2}$ oz. Find in ounces, correct to the nearest hundredth, the weight of 1 cubic centimetre of lead, having given

$$1 \text{ cu. cm.} : 1 \text{ cu. in.} = 100 : 1639.$$

26. A train runs at an average rate of 40·4 kilometres an hour: how many *miles* are travelled in 48 minutes. [Reckon 1 Km. $=\frac{5}{8}$ mile, and remember that the result will be only approximate.]

27. The Representative Fraction of a map is $\frac{1}{126.720}$; to what scale is it drawn (in miles to the inch)? What length on the map will represent a distance of 7·6 miles? What distance will be represented by 2·55 inches?

28. The distance from Chester to Carlisle is $118\frac{1}{2}$ miles, and is represented on a certain map by 7·5 inches. To what scale is the map drawn (in miles to the inch)? What is its Representative Fraction? The distance between Newcastle and Sheffield is represented by 6·9″; what is the actual distance to the nearest mile?

29. A pipe delivers 255·6 litres of water in 27 minutes. Find in litres, and hence in kilograms, how much water can be obtained in $1\frac{1}{4}$ hour.

30. Find roughly in how many minutes 1 cwt. of water could be drawn from a tap which discharges 420 litres in 1 hr. 6 min. [1 Kg. $=2\frac{1}{5}$ lbs. nearly.]

272. The following examples differ from those of the preceding Exercise only in requiring somewhat more numerical work. When the terms which form the multiplying ratios are compound quantities, they must be brought to the same denomination. In certain cases it is best to reduce to the highest common denomination, in others to the lowest; sometimes to an intermediate denomination.

EXAMPLE 1. *If* 18 *tons* 8 *cwt. of coal cost* £26. 13*s.* 4*d.*, *how much coal could be bought for* £34. 15*s.?*

Here £26. 13*s.* 4*d.* buys 18 tons 8 cwt.

£34. 15*s.* ,, *req*^{d} *weight.*

$$\begin{aligned}\text{Hence required weight} &= 18 \text{ tons } 8 \text{ cwt.} \times \frac{\pounds 34.\ 15s.}{\pounds 26.\ 13s.\ 4d.}\\ &= 368 \text{ cwt.} \times \frac{34\frac{3}{4}}{26\frac{2}{3}}\\ &= 368 \text{ cwt.} \times \frac{139}{4} \times \frac{3}{80}\\ &= 465\tfrac{3}{4} \text{ cwt.}\\ &= 22 \text{ tons } 5\tfrac{3}{4} \text{ cwt.}\end{aligned}$$

EXAMPLE 2. *A bankrupt's debts amount to £5432. 10s., and his available property is valued at £803. 7s.; how much (to the nearest penny) can be claimed on a debt of £40. 12s.?*

DEBT.	PROPERTY.
£5432. 10*s.*	£803. 7*s.*
£40. 12*s.*	*req^d claim.*

Proportional claim on property

$$\begin{aligned} &= \pounds 803.\ 7s. \times \frac{\pounds 40.\ 12s.}{\pounds 5432.\ 10s.}\\ &= \pounds 803{\cdot}35 \times \frac{\pounds 40{\cdot}6}{\pounds 5432{\cdot}5}\\ &= \pounds 8{\cdot}0335 \times \frac{4{\cdot}06}{5{\cdot}4325}\\ &= \pounds 6{\cdot}004, \text{ to } 3^{\text{rd}} \text{ dec. fig.}\\ &= \pounds 6.\ 0s.\ 1d., \text{ to nearest penny.}\end{aligned}$$

```
                8·0335
                  4·06
              ---------
               32·1340
                 ·4820...
              -------
5,·4,3,2,5 ) 32·6160 ( 6·0039
              -------
                  210
                  ---
                   47
```

EXAMPLES XVI. d.

1. The profits of a shop for 28 weeks are £141. 15*s.* 0*d.*; at what rate is this for the year? [Reckon 1 year = 52 weeks.]

2. A coal merchant buys 700 tons of coal for £1404. 1*s.* 8*d.*; what should he give for 48 tons at the same average price?

3. If 23 dozen of wine cost £49. 14*s.* 9*d.*, what would be the cost of 56 dozen of the same kind?

4. The weight of 42 cubic feet of coal is 1 ton 9 cwt. 1 qr.; what is the weight per cubic yard?

5. A coach when actually travelling covers on an average 39 miles in 5 hours; how far would it go between 8.50 a.m. and 5.40 p.m., allowing half-an-hour for changing horses.

6. If $3\frac{1}{4}$ tons of coal cost £4. 11*s.*, what will $2\frac{1}{4}$ tons cost?

7. A man walking at the rate of $3\frac{3}{4}$ miles an hour could walk from his house to the railway station in 1 hr. 40 min. How long would he take to do the same distance at the rate of $3\frac{1}{8}$ miles an hour?

8. How many loads of gravel can be bought for £33. 6*s.* 8*d.*, if 27 loads cost £22. 10s.?

9. A dealer makes two purchases of sheep at the same average price, paying £101. 9*s.* 6*d.* for one lot and £150. 13*s.* 6*d.* for the other. If there are 33 sheep in the first lot, how many are there in the second?

10. How many tons of rice can be bought wholesale for £29 in a market where 45 cwt. cost £16. 6*s.* 3*d.*?

11. If £2. 5*s.* is paid for the carriage of 2 tons 15 cwt. over a distance 75 miles, how far should 6 tons 5 cwt. be carried for the same sum?

12. The rent of 9 acres of land being £10. 10*s.*, how much rent should be paid for 3 roods 32 poles at the same rate?

13. The price per cwt. of a certain commodity is £1. 3*s.* 4*d.*; what weight of it should be supplied for £21. 16*s.* 8*d.*?

14. If the produce of one-sixth of an acre is 1 cwt. 3 qrs. 4 lbs., how much would be yielded at the same rate by 4 ac. 2 r.?

15. A man's hotel bill for 4 weeks 5 days amounted to £28. 17*s.* 6*d.*; how much would he have to pay at the same rate for 6 weeks 4 days?

(*On rates in the £.*)

[A bankrupt's **assets** are such parts of his property (or the value of them) as are available for distribution among his creditors in proportion to the sums severally due to them.]

16. If a bankrupt's debts are £2520, his assets £1890, how much can he pay in the £?

17. A bankrupt owes his creditors £1520, and his assets amount only to £348. 6*s.* 8*d.* How much can he pay in the £?

18. A bankrupt's debts amount to £600 and his assets to £410; what will a creditor receive to whom he owes £125?

19. How much can a man claim for a debt of £141. 1*s.* from a bankrupt whose debts amount to £1792, and whose whole property is only worth £1344?

20. A bankrupt whose liabilities are £1250 pays his creditors 15*s.* in the £. What are his assets, and what is the loss to a creditor to whom he owes £150?

[**Income tax** is charged (with certain exemptions) at the rate of so many pence in every £1 of full or **gross** income; the remaining income is said to be **net.**]

21. If income tax is 11*d.* in the £, shew that

$$\text{(i)} \qquad \text{tax} = \text{gross income} \times \tfrac{11}{240};$$

$$\text{(ii)} \quad \text{net income} = \text{gross income} \times \tfrac{229}{240};$$

$$\text{(iii)} \qquad \text{tax} = \text{net income} \times \tfrac{11}{229}.$$

And write down the corresponding relations when the tax is xd. in the £.

22. If my gross income is £1563. 10*s.*, what is my net income after payment of income tax at 8*d.* in the £?

23. A man's gross income is £610, and on this he has liabilities amounting in all to 3*s.* 11*d.* in the £. What is his net income?

24. What is the gross income of a man who pays £59. 6*s.* 8*d.* as income tax at 10*d.* in the £?

25. After paying income tax at the rate of 9*d.* in the £, my net income is £2117. 10*s.* How much tax have I paid?

[**Rates** are local taxes payable by the occupier of a house or land, and charged as so many pence in every £ on the rent, or on a certain proportion of the rent called the **rateable value** of the property.]

26. The poor rate being 1*s.* 8*d.* in the £, what is the rateable value of a property which pays a rate of £208. 6*s.* 8*d.*?

27. The rateable value of a parish is £5676. 13*s.* 4*d.*; what do the rates amount to at 2*s.* 7½*d.* in the £?

28. A man's income was reduced to £522 after he had paid income tax of 8*d.* in the £. What would have been his net income if the income tax had been 7*d.* in the £?

29. After paying income tax at 10*d.* in the £, and his other rates which exceed the income tax by 1*d.* in the £, a man has £584 left. What is his gross income?

30. In a certain year exemption from income tax was allowed on £120 out of an income not exceeding £600, and the rest was taxed at 9*d.* in the £. I paid £17. 5*s.* 6*d.* as income tax. What was my total income? What was my net income?

31. In the year 1907 *earned* income (that is, income arising from a profession) was taxed at 9*d.* in the £, and *unearned* income (that is, income arising from property) was taxed at 1*s.* in the £. If I paid £70. 9*s.* in all, and my earned income was £1174, what was my unearned income?

(*Miscellaneous Examples for Approximate Work.*)

32. On a certain map 4·7 inches represents a distance of $70\frac{1}{2}$ miles. What length will represent $52\frac{3}{4}$ miles? [Answer to the nearest tenth of an inch.] What is the scale of this map (in miles to an inch)? And what is its representative fraction?

33. A Convalescent Home supports 73 patients at an average weekly cost of £122. 10*s*. Find to the nearest shilling the cost of maintaining 105 patients at the same rate.

34. At an average speed of 71·2 Km. an hour a certain journey is performed in 2 hrs. 16 min. Find to the nearest minute how long the same journey would take at 53·8 Km. an hour.

35. Freight on a consignment of goods for a passage of 600 miles is £28. 19*s*. 6*d*. What should be the freight on the same goods for 423 miles at the same rate per mile? Answer to the nearest penny.

36. By paying an annual premium of £47. 3*s*., I insure my life for £1500. Find to the nearest pound for what sum I might insure for a premium of £38. 15*s*.

37. Find to the nearest gram the weight of 78·6 cu. cm. of silver, it being given that the weight of equal volumes of silver and water are in the ratio 21 : 2.

38. On a gross income of £1253 I pay income tax at the rate of 8*d*. in the £, and then save 3*s*. 6*d*. in every pound that remains. Find to the nearest pound what income remains for current expenditure.

39. A contractor undertook to build a light railway for £5000 in two sections, the first section consisting of 18·740 Km. of line, the second of 6·560 Km. When the first section was complete he found his expenditure had been £3704. Supposing the second section could be built at approximately the same cost per kilometre, estimate his probable gain or loss. [Neglect fractions of a pound.]

40. The weight of a steel wire is $2\frac{3}{4}$ lbs. to the yard; find approximately how many kilograms go to the metre. [Assume 1 metre : 1 yard = 35 : 32, nearly; and 1 Kg. = $2\frac{1}{5}$ lbs. nearly; these data being only near enough to give a result true to one decimal place.]

41. A rainfall of $\frac{1}{2}$ inch gives approximately 11.344 gallons per acre. Express this rainfall approximately in litres per hectare. [1 litre = $1\frac{3}{4}$ pint, and 1 hectare = $2\frac{1}{2}$ acres, nearly.]

42. The weights of equal volumes of gold and water are in the ratio 96 : 5, and a cubic inch of water weighs 0·58 oz. Find the volume of a nugget of gold weighing 8·7 oz. Answer to the nearest hundredth of a cubic inch.

Tests of Proportionality.

Proportion illustrated Graphically.

273. In the foregoing examples no difficulty will have been found in recognising the proportionality of the quantities involved. For instance, when men are all paid at the same rate and for the same time, the *total sum* paid is proportional to the *number of men.* For if we know that a given number of men require a certain sum, then m times the number of men require m times the sum of money, whatever number (whole or fractional) we take for m. [Art. 271.] For a similar reason, when speed is uniform, the *distance* covered is proportional to the *time*: and so on.

274. But two quantities which are so related that when one is increased or diminished, the other is also increased or diminished, are not necessarily proportional.

For instance, when we increase the *side* of a square, we increase the *area*; but the side and area of a square are not proportional; for on *doubling* the side, we multiply the area by **4**; and on *trebling* the side, we multiply the area by **9**.

Again, a body dropped from rest falls 16 feet in the first second; but not 32 feet in the first 2 seconds, nor 48 feet in the first 3 seconds; for the speed is continually increasing. Thus the distance traversed is not proportional to the time.

275. Direct proportion may be thus illustrated graphically.

Consider the series of values of x and y given in the table below, and in each case simplify the ratio $y : x$.

x	4·5	9	12	16·5	19·5
y	3	6	8	11	13

We find
$$\frac{3}{4{\cdot}5}=\frac{6}{9}=\frac{8}{12}=\frac{11}{16{\cdot}5}=\frac{13}{19{\cdot}5}=\mathbf{\frac{2}{3}};$$

so that, for each pair of values given, the ratio $\frac{y}{x}$ is **constant**; since in all cases

$$\frac{y}{x}=\frac{2}{3}; \qquad \text{or} \quad y=\frac{2}{3}\times x.$$

Either of these equations expresses the fact that

y is **directly proportional** to x,

or y **varies directly** as x.

Now plot the points given by the above pairs of values o x and y in the table, and let them be denoted by P, Q, R, S, T as in Fig. 9. It will be found that *all these points lie on a straigh line passing through the origin.*

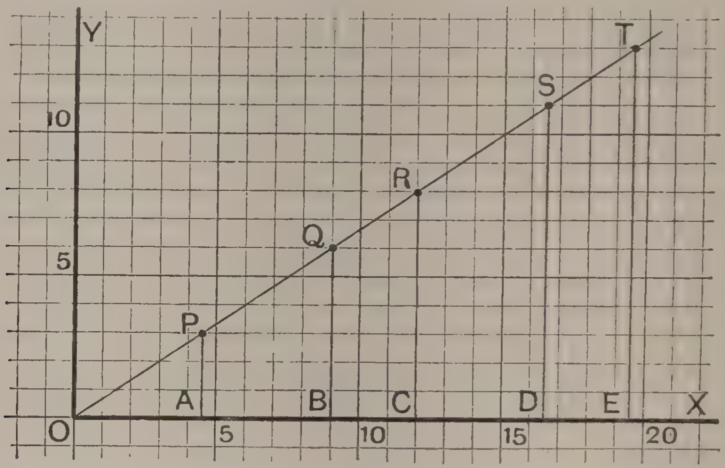


Fig. 9.

Note. The reason for this may be stated as follows: Draw the ordinates PA, QB, RC, ... Then if in the right-angled triangles PAO, QBO, RCO, ..., the vertical sides PA, QB, RC, ... bear a fixed ratio to the horizontal sides OA, OB, OC, ..., then all the triangles are *similar*, that is, have the *same shape*; so that they will all have the same angle at O, and accordingly the points O, P, Q, R, ... will fall in the same straight line. [See *School Geometry*, p. 265, Theorem 64.]

276. From the above reasoning and experiment we infer

(i) that if x and y are directly proportional, then $y=ax$ for every pair of corresponding values, where a is some constant multiplier.

(ii) that the graph of $y=ax$ is a straight line through the origin. [See Art. 256.]

Example. *In a certain mine it was arranged that the rate of wages should rise or fall when the price of coal rose or fell. The following table shews corresponding prices and wages.*

Price per ton at pit's mouth,	10*s*.	12*s*. 6*d*.	14*s*. 6*d*.	16*s*.
Wages per week, - - -	22*s*.	27*s*.	30*s*.	32*s*.

Determine graphically if the wages are proportional to the prices. If not, take the first pair of prices and wages as the standard, and find graphically what wages would be proportional to the remaining prices.

Measure prices horizontally, taking 0·2 in. to represent 1*s.* Measure wages vertically, taking 0·1 in. to represent 1*s.* Plot the points given by corresponding values in the table; call these P, Q, R, S.

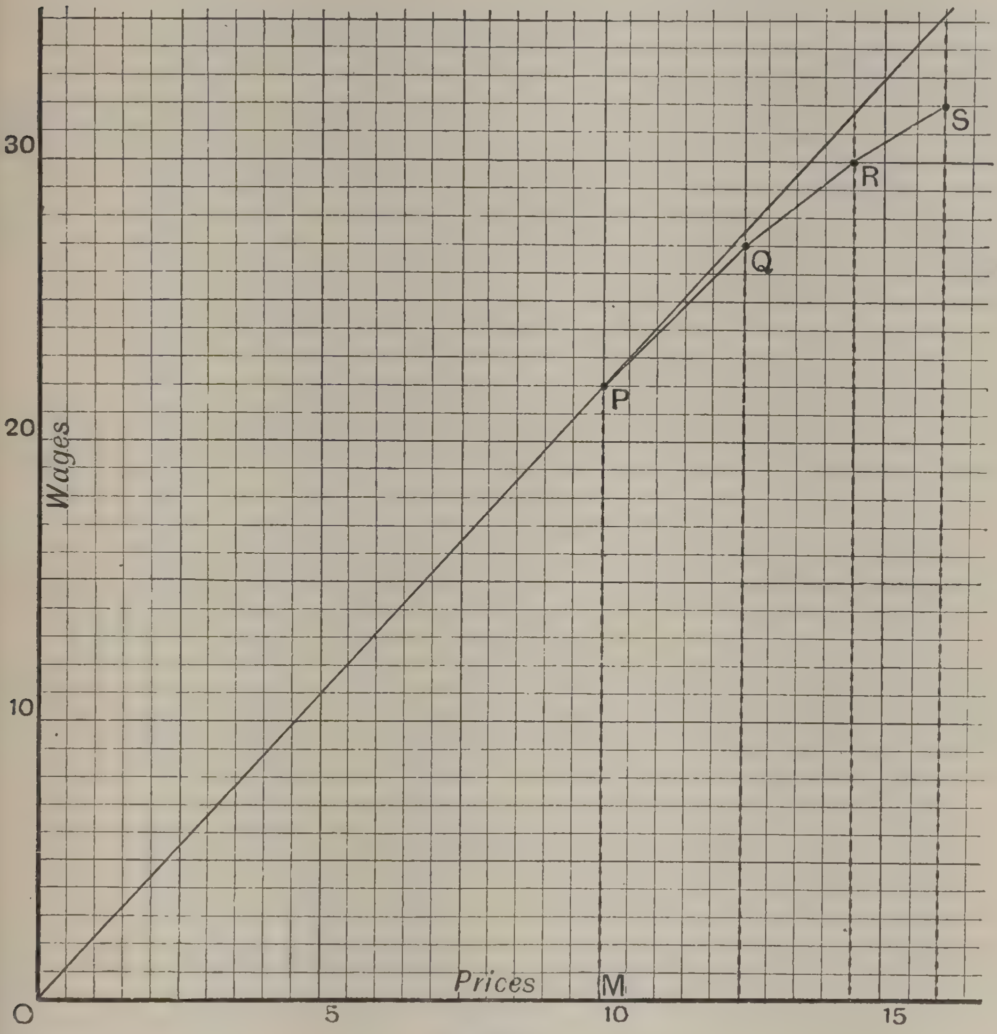


FIG 10.

We see at once that these points are not on a straight line through O. Hence the given wages are not proportional to the prices.

If now we draw and produce the line OP, we obtain a graph for every point of which the ordinate has to its abscissa a constant ratio, namely that of PM to OM, that is, of wages 22*s.* to price 10*s.*

We have now to produce the ordinates of Q, R, S to meet the graph OP, and on measurement the new ordinates are found to be 2·75″, 3·17″, and 3·52″; representing approximately wages of 27*s.* 6*d.*, 31*s.* 8*d.*, and 35*s.* 2*d.*

277. It will now be seen that a graph accurately drawn on a suitable scale may often be used as a 'ready reckoner.'

It is particularly important that the pupil should draw his diagrams on a sufficiently large scale, and that he should be careful in the choice of units. These should always be clearly marked on the axes.

EXAMPLE. *Given that* 5·5 *centimetres are approximately equal to* 2·15 *inches, draw a graph to convert any number of inches into centimetres, or centimetres into inches.*

Suppose y inches are equivalent to x centimetres, then it is clear that the ratio of y to 2·15 = the ratio of x to 5·5;

that is, $$\frac{y}{2{\cdot}15}=\frac{x}{5{\cdot}5}, \qquad \text{or} \quad y=\frac{2{\cdot}15}{5{\cdot}5}x,$$

which is an equation representing a line through the origin, and to draw the graph it is only necessary to mark the position of one other point. Hence if we plot a point P whose coordinates are 5·5 and 2·15, OP is the required graph. See Fig. 11 on the opposite page.

By taking 10 divisions of the paper as unit on each axis the ruled lines will mark tenths of the unit. We can thus read *accurately* to one place of decimals. The second place can be judged by the eye to a fair degree of accuracy as explained in the example of Art. 255.

Thus to find the equivalent of 3 cm., we observe that the ordinate RM is greater than 1·1 and less than 1·2, and we estimate its value as 1·17. Hence 3 cm. = 1·17 in. approximately.

From the graph we may read off the following approximate equivalents:

1 cm. = 0·39 in., 1 in. = 2·56 cm.,
4 cm. = 1·56 in., 4·60 cm. = 1·80 in.

Similarly any other equivalents may be found. The student will find it interesting to check the accuracy of his work by calculating equivalents from the equation of the graph.

NOTE. The size of the page does not admit of a diagram large enough to read the equivalents for large numbers. But if accuracy to only one place of decimals is required, we may use the diagram in the following way. After using the data as already explained to obtain an accurate graph, if we now suppose the unit on each axis to be *one* division of the paper, we can read off results as follows:

15 cm. = 5·9 in., 38 cm. = 14·8 in.,
8 in. = 20·4 cm., 19·5 in. = 50 cm.

Here we read the integers accurately and judge the tenths.

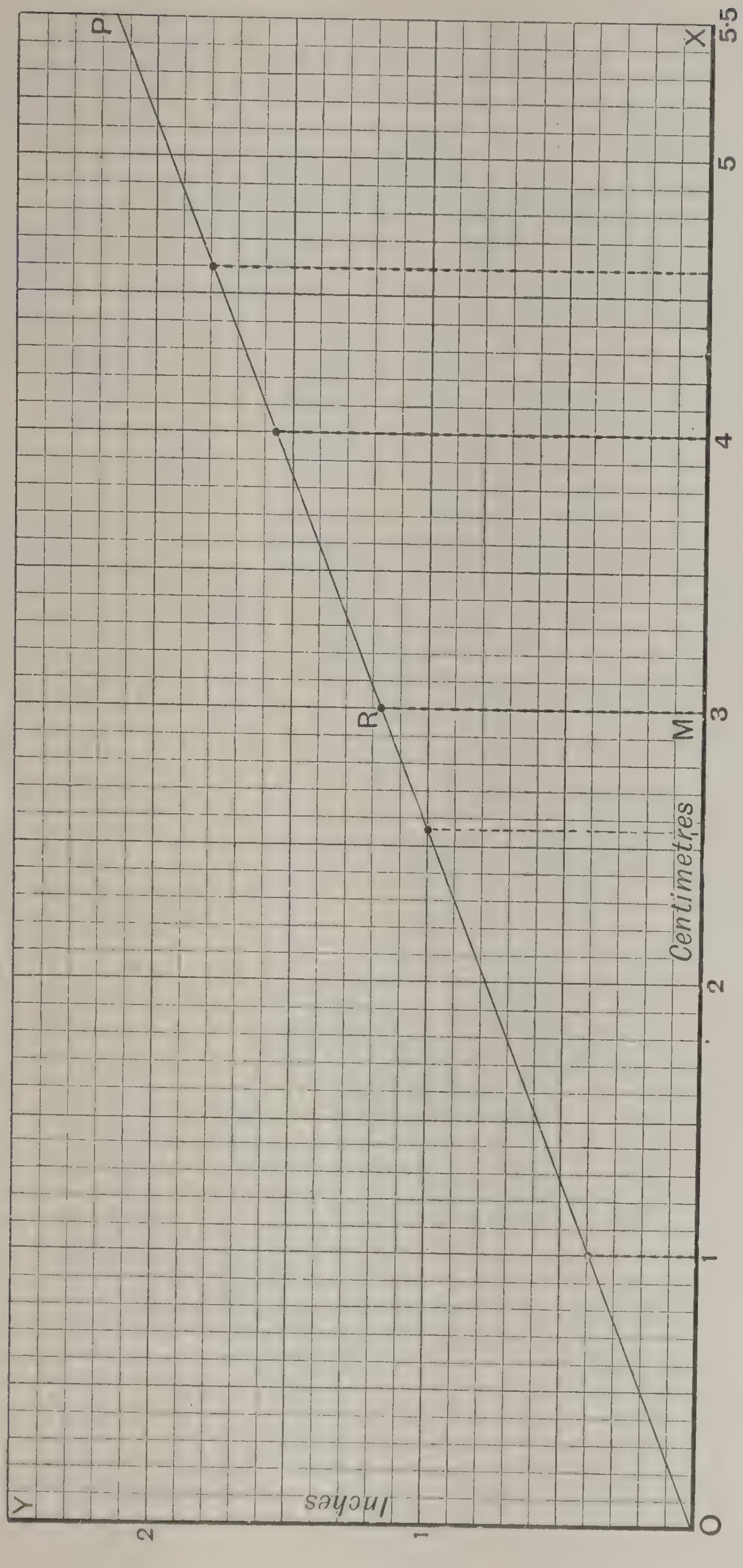


FIG. 11.--Relation between inches and centimetres.

278. Inverse Proportion. Suppose that a distance of 25 miles is to be covered at a uniform speed. Let us find what times the journey would take at 1, $2\frac{1}{2}$, 5, 7, 10, 15, 25 miles per hour respectively.

Here as the speed *increases*, the time *decreases*; and if any speed is multiplied by m, the corresponding time is *divided* by m. That is, the time is inversely proportional to the speed.

Let x denote the speed in miles per hour, and y the corresponding time in hours.

Then, by finding values of y corresponding to the above given values of x, we may complete the following table.

x	1	$2\frac{1}{2}$	5	$7\frac{1}{2}$	10	15	25
y	25	10	5	$3\frac{1}{3}$	$2\frac{1}{2}$	$1\frac{2}{3}$	1

We observe that for each pair of corresponding values

$$x \times y = 25, \qquad \text{so that} \quad y = 25 \times \frac{1}{x}.$$

Thus when y is inversely proportional to x,

$$y \text{ varies as } \frac{1}{x} \text{ (\textit{i.e.} as the reciprocal of } x\text{)}.$$

Now plot points from the above pairs of values of x and y, and draw a freehand curve through them.

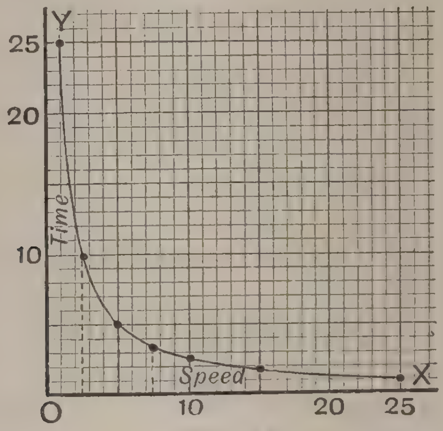


FIG. 12.

This curve is the graph of $y = \frac{25}{x}$, and represents pictorially the variations of two quantities which are inversely proportional.

EXAMPLES XVI. e.

(*Tests of proportionality.*)

1. As a boy grows older he grows taller; does his height vary directly as his age? Give an instance.

2. A boy can run 100 yards in 12 seconds; can he run a mile in 12 sec. $\times \frac{1760}{100}$?

3. A train travels 380 yards in the first minute after starting from a station. Can you say how far it will have travelled in the first 5 minutes, 11 minutes, x minutes?

4. A shell leaves a gun with a muzzle-velocity of 2100 feet per second: will it traverse 6300 feet in 3 seconds, and $(2100 \times m)$ feet in m seconds? Give a reason for your answer.

5. If you had a number of leaden balls of different sizes, would you expect their weights to be proportional to their diameters? If not, would the weights or the diameters increase more rapidly?

6. Suppose that with engines of 1000 horse-power a steamer of a certain size and build can make a speed of 11 knots. Can you suggest a reason why engines of 2000 H.P. would not suffice to drive the same boat 22 knots; nor engines of 3000 H.P. at 33 knots? Is the speed of a steamer proportional to her indicated horse-power?

(*Graphical Examples.*)

7. Assuming that 13 dollars are approximately equal to 54 shillings draw a graph to shew the relation between dollars and shillings for any sum up to £5.

Read off (i) the number of shillings in 7 dollars;
(ii) the number of dollars in £3.

Shew that £4. 7*s.* is very nearly equal to 21 dollars.

8. Given that 20 litres = 4·4 gallons, draw a graph to convert litres to gallons or gallons to litres.

Express (i) $2\frac{1}{2}$ gallons in litres;
(ii) 20·9 litres in gallons.

[Take one gallon to the inch on the axis of x, and 10 litres to the inch on the axis of y.]

9. Given that 6·01 yards = 5·5 metres, draw the graph shewing the equivalent of any number of yards when expressed in metres.

Shew that 22·2 yards = 20·3 metres, approximately.

10. Draw a graph shewing the relation between equal weights in grains and grams, having given that 18·1 grains = 1·17 grams.

Express (i) 3·5 grams in grains.
(ii) 3·09 grains as a decimal of a gram.

11. A train in 2 minutes after leaving a station covers 700 yards; in $3\frac{1}{5}$ minutes 1300 yards; in $4\frac{1}{2}$ minutes 2100 yards; and in $6\frac{1}{2}$ minutes 4400 yards (the times and distances being all measured from the start). Shew by a diagram that the distances are not proportional to the times.

If the train after the second minute had proceeded uniformly at its average speed for the first two minutes, find graphically (to the nearest 100 yards) how far it would have gone in $6\frac{1}{2}$ minutes?

12. Given that the circumferences of circles are directly proportional to their diameters; and that a circle of diameter 10 cm. has a circumference approximately equal to 31·4 cm.; draw a graph shewing the ratio of the circumference to the diameter in any circle.

Use this graph to find as nearly as possible

(i) the circumference of a circle of diameter 16 cm.;

(ii) the diameter of a circle of circumference 40 cm.

13. Draw a graph to serve as a ready reckoner for wages at £20 a year. Read off to the nearest penny the wages for 1 week, 20 days, and 48 days.

How long had a servant worked who received £3. 7*s.* 6*d.* as wages?

[Take 0·1″ to represent 1 day on the x-axis and the same unit to represent 1 shilling on the y-axis. Then since the wages for 73 days amount to £4, the graph is at once obtained by joining the origin to the point (73, 80).]

14. A speed of 60 mi. per hour is equivalent to 88 ft. per second. By means of a graph express in miles per hour the approximate speed of trains which respectively cover 40 ft., 64 ft., 78 ft. per second.

15. A certain Life Assurance Company undertakes to pay £100 to a man's relatives at his death in return for annual premiums whose amounts depend, as in the following table, upon the age at which the premiums begin.

Age -	25	30	35	40	45
Premium	£2. 3*s.*	£2. 9*s.*	£2. 16*s.*	£3. 5*s.*	£3. 16*s.*

Plot these values on a chart, and determine by means of it if the premium is directly proportional to the age.

Taking as your standard "*premium* £2. 16*s. at age* 35," find graphically what would be the proportional premiums at the other ages.

16. At the end of each year a settlement adds to its population $\frac{1}{10}$ of the number of inhabitants at the beginning of the year. At the beginning of 1905 the number was 2000: what was it at the end of 1905, 1906, 1907, 1908? Tabulate and plot the *increases* shewn at the end of each year above the population of Jan. 1st, 1905. Indicate by a chart that these increases are not proportional to the times; and try to account for their not being so.

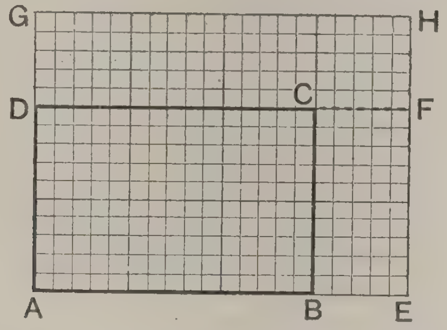
17. If the wages for a day's work of 8 hours are 4*s.* 6*d.*, draw a graph to shew the wages for any fraction of a day, and find (to the nearest penny) what ought to be paid to men who work $2\frac{1}{2}$, $3\frac{1}{2}$, $6\frac{1}{2}$ hours respectively. How many hours' work might be expected for 2*s.* 10*d.*?

18. The highest marks gained in an examination were 136, and these are to be raised so that the maximum is 200. Shew how this may be done by means of a graph, and read off, to the nearest integer, the final marks of candidates who scored 61 and 49 respectively.

Compound Proportion.

279. A question in proportion is said to be *simple* when only one multiplying ratio is to be considered. In *Compound* Proportion the required term is affected by two or more multiplying ratios. Our first example will illustrate the process graphically.

EXAMPLE 1. *A rectangle* ABCD *represents an area of* 150 *sq. ft. If the length is increased in the ratio* 4 : 3, *and the breadth increased in the ratio* 3 : 2, *what area will the new rectangle represent?*



(i) If the length is increased in the ratio $\frac{4}{3}$, that is, from AB to AE (the breadth remaining the same),

we pass from the rect. ABCD to the rect. AEFD;

$$\text{and}\quad \text{rect. AEFD} = \text{rect. ABCD} \times \tfrac{4}{3}.$$

(ii) If now the breadth is increased in the ratio $\frac{3}{2}$, that is, from AD to AG,

we pass from the rect. AEFD to the rect. AEHG;

$$\begin{aligned}\text{and}\quad \text{rect. AEHG} &= \text{rect. AEFD} \times \tfrac{3}{2}.\\ &= \text{rect. ABCD} \times \tfrac{4}{3} \times \tfrac{3}{2}.\end{aligned}$$

Hence the rect. AEHG represents $150 \text{ sq. ft.} \times \frac{4}{3} \times \frac{3}{2}$,

or 300 sq. ft.

EXAMPLE 2. *If 36 men can build a wall 140 yards long in 21 days, how many men would take 18 days to build a similar wall 50 yards in length?*

State the question so that the term corresponding to the answer stands last.

140 yards can be built in 21 days by 36 men;
50 yards .... 18 .... *req^d number.*

(i) If the length of wall is diminished from 140 yards to 50 yards (the time being unaltered), the number of men will be diminished in the ratio $\frac{50}{140}$.

So that the required number of men would be $36 \times \frac{5}{14}$.

(ii) If now the time is diminished from 21 days to 18 days, the number of men will be increased in the ratio $\frac{21}{18}$.

So that finally the required number of men

$$= 36 \times \frac{5}{14} \times \frac{21}{18}$$
$$= 15.$$

280. From these examples we learn that in Compound Proportion the final result is obtained by multiplying the given term (of the same kind as the answer) in succession by the multiplying ratios arising from each pair of corresponding quantities in the statement of the question.

In Example 1, the area of the rectangle is said to vary **jointly** as (that is, as the product of) its length and breadth.

In Example 2, the number of men varies jointly as the amount of work taken *directly*, and the number of days taken *inversely*.

EXAMPLES XVI. f.

1. A certain rectangle has an area of 96 sq. ft.; what would be the area of a rectangle 5 times as long and $\frac{2}{3}$ as wide?

2. A rectangular sheet of lead weighs $2\frac{1}{2}$ cwt.; what would be the weight of another sheet $3\frac{1}{2}$ times as long and $\frac{2}{5}$ as wide (the thickness being the same)?

3. A motor travelling at a uniform speed for a certain time covers 96 miles. Had a run been made at $\frac{3}{4}$ the speed, the time being diminished in the ratio $\frac{5}{8}$, how many miles would have been covered?

4. A gang of men earns £22. 10*s.* in a certain time; how much would be earned at the same rate by $\frac{2}{3}$ of the gang, if their working time were reduced in the ratio $\frac{4}{5}$?

5. A certain block of granite weighs $3\frac{3}{4}$ tons; what is the weight of a second block whose length, breadth, and height are respectively $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{5}$ of the corresponding divisions of the first block?

6. A marine engine working at 2400 horse-power consumes 225 tons of coal per week. How many pounds is this for 1 horse-power for 1 hour?

7. If 6 persons spend £150 in 20 weeks, how long will £105 last 7 persons at the same rate?

8. If 6 persons spend £150 in 20 weeks, what would be the expenditure at the same rate of 9 persons for 24 weeks?

9. If 4 men earn £9 in 12 days, what should be the wages of 6 men for 10 days?

10. How many men will mow 48 acres of grass in 8 days, if 12 men can mow 36 acres in 5 days?

11. How much will 42 men earn in 30 days at the rate of £368 for 69 men for 35 days?

12. If a family of 9 people can live on £120 for 8 months, how many can live for 16 months on £640 at the same rate?

13. If 7 men earn £9. 9*s*. in a certain time, how many will earn £9. 18*s*. at the same rate in $\frac{2}{3}$ of the time?

14. A piece of work is completed by 32 men in 20 days of $7\frac{1}{2}$ hours each; how many men would be required to do it in 8 days of 10 hours each?

15. If 49 men can empty a reservoir in 65 days by pumping 8 hours a day, in how many days of 5 hours each will 196 men do the same work?

16. A wall is built by 17 men in 24 days; in how many days can 18 men do the work, if the working hours per day are reduced in the ratio $\frac{4}{5}$?

17. If when meat is at $8\frac{1}{2}$*d*. per pound, it costs £50 per month to supply a household of 23 persons, what will be the monthly cost of meat at $11\frac{1}{2}$*d*. per pound for a household of 34 persons?

18. If 13 men earn £121. 6*s*. 8*d*. in 8 weeks, in what time will 17 men earn £257. 16*s*. 8*d*. at the same rate?

19. Working for 7 days of 8 hours each, 19 men earn £42. 15*s*. At this rate how much would 49 men earn in 2 days of 10 hours each?

20. How many pages of a magazine will a manuscript occupy which has 120 pages each of 28 lines, with an average of 7 words in a line; if a page of the magazine has 42 lines, with an average of 10 words in a line?

21. An engine of 1 horse-power can lift 33,000 lbs. per minute vertically through 1 foot. What horse-power is required to raise per minute

(i) 3 cwt. of coal up a shaft 550 ft. deep?

(ii) $5\frac{1}{2}$ cwt. of coal up a shaft 100 fathoms deep? [1 fathom = 6 feet.]

(iii) 440 gallons of water up a shaft 100 yards deep? [1 gallon of water weighs 10 lbs.]

And prove that if H denotes the horse-power required to raise W lbs. through h feet per minute,

$$\mathsf{H} = \frac{\mathsf{W} \times h}{33{,}000}.$$

22. A garrison of 1500 men has provisions for 12 weeks. How many men could be maintained for 20 weeks on the same provisions if the daily allowance per man were reduced in the ratio $\frac{9}{10}$?

23. A besieged garrison of 600 men have provisions to last 35 days at a certain rate of issue. How long would the provisions last if the garrison were reinforced by 300 men, and the daily allowance per man reduced in the ratio 5 : 6?

24. If the carriage of 3 tons 14 cwts. for 98 miles cost 18*s*. 6*d*., what will be the cost of carrying 2 tons 9 cwt. a distance of 28 miles at the same rate per ton per mile?

25. If an engine of 40 horse-power is required to pump 220 gallons of water per minute from a depth of 600 feet, what horse-power would be needed to pump 1 ton of water per minute from a depth of 495 feet? [1 gallon of water weighs 10 lbs.]

26. An express train travelling at 55 miles an hour accomplishes a journey in $3\frac{1}{2}$ hours. How long will it take a slow train to travel two-thirds of the distance, its rate being to that of the express as 4 to 9?

27. A locomotive engine, whose driving-wheel is 5 feet in diameter and makes 168 revolutions a minute, travels at 30 miles an hour. What is the speed of an engine whose driving-wheel is 7 ft. in diameter and makes 136 revolutions a minute? [N.B. The circumferences of circles are proportional to their diameters.]

28. If 40 English navvies, each earning 7*s*. a day, do in 30 days a piece of work for which 56 Belgian workmen, each earning 6 francs a day, require 40 days; determine which class of workmen is the more profitable to a contractor, taking the value of a franc at 10*d*. in English money. If a contract carried out by English navvies cost £6000, what would be the cost of the same contract when executed by Belgian workmen?

Proportional Division.

281. The following application of Proportion deserves special attention, namely the process by which a quantity may be divided into parts which bear given ratios to one another.

EXAMPLE 1. *Divide* $2\frac{1}{2}$ *feet into three parts proportional to the numbers* 4, 5, 6.

This means that the 1st part must be divisible into 4 equal subdivisions, of which the 2nd part must contain 5, and the 3rd part 6; so that the whole line must contain 4+5+6, or 15, such subdivisions.

$$\therefore\ 1^{\text{st}}\text{ part} = \tfrac{4}{15}\text{ of }2\tfrac{1}{2}\text{ ft.} = 8\text{ inches};$$

$$2^{\text{nd}}\text{ part} = \tfrac{5}{15}\text{ of }2\tfrac{1}{2}\text{ ft.} = 10\text{ inches};$$

$$3^{\text{rd}}\text{ part} = \tfrac{6}{15}\text{ of }2\tfrac{1}{2}\text{ ft.} = 12\text{ inches}.$$

Or thus:

Let x denote the number of inches in the 1st part;
then $\frac{5}{4}x$ denotes .... 2nd ......
and $\frac{6}{4}x$ denotes .... 3rd ......

$$\text{so that}\quad x + \tfrac{5}{4}x + \tfrac{6}{4}x = 30.$$

From this equation $x = 8$; whence $\frac{5}{4}x = 10$, and $\frac{6}{4}x = 12$.

NOTE. It is clear that the result will not be altered, if the numbers which fix the ratios of the required parts are all multiplied, or all divided, by the same number. Thus division into parts proportional to

(i) 400, 500, 600,
(ii) $\frac{4}{29}$, $\frac{5}{29}$, $\frac{6}{29}$,
(iii) $4m$, $5m$, $6m$,

is in each case performed by dividing into parts proportional to **4, 5, 6.**

EXAMPLE 2. *Divide* £149. 10*s. into three shares proportional to the fractions* $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$.

The given fractions, reduced to a common denominator, are $\frac{6}{12}$, $\frac{8}{12}$, $\frac{9}{12}$; *i.e.* 6 twelfths, 8 twelfths, 9 twelfths; these are evidently proportional to 6, 8, 9.

$$\text{Now}\quad 6+8+9=23;$$

$$\therefore\ 1^{\text{st}}\text{ share} = \tfrac{6}{23}\text{ of }£149\tfrac{1}{2} = £39;$$

$$2^{\text{nd}}\text{ share} = \tfrac{8}{23}\text{ of }£149\tfrac{1}{2} = £52;$$

$$3^{\text{rd}}\text{ share} = \tfrac{9}{23}\text{ of }£149\tfrac{1}{2} = £58.\ 10s.$$

NOTE. The 3rd share may also be found by subtracting the sum of the 1st and 2nd shares from the given sum.

EXAMPLE 3. *Divide 9 kilograms into parts proportional to the decimals* 2·16, 3·52, 4·32.

Here $2{\cdot}16+3{\cdot}52+4{\cdot}32=10$.

$$\therefore\ 1^{\text{st}}\text{ part}=\frac{2{\cdot}16}{10}\text{ of }9\text{ Kg.}=0{\cdot}216\text{ of }9\text{ Kg.}=1{\cdot}944\text{ Kg.};$$

$$2^{\text{nd}}\text{ part}=\frac{3{\cdot}52}{10}\text{ of }9\text{ Kg.}=0{\cdot}352\text{ of }9\text{ Kg.}=4{\cdot}068\text{ Kg.};$$

$$3^{\text{rd}}\text{ part}=\frac{4{\cdot}32}{10}\text{ of }9\text{ Kg.}=0{\cdot}432\text{ of }9\text{ Kg.}=3{\cdot}988\text{ Kg.}$$

282. A quantity is divided into parts **inversely** proportional to given numbers, when the parts are proportional to the **reciprocals** of the given numbers.

For instance, if a quantity is to be divided into parts inversely proportional to 2, 3, 4, the parts must be proportional to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$.

EXAMPLES XVI. g.

Divide

1. £80 into *two* parts in the ratio of 3 : 7.
2. 323 miles ...... *two* parts .... 6 : 11.
3. 450 tons ...... *three* parts proportional to 4, 5, 11.
4. £408 ...... *three* parts .... 7, 8, 9.
5. £20. 5*s*. ...... *three* parts .... 4, 5, 6.
6. 303 yards ...... *three* parts .... 200, 300, 500.
7. 180 Kg. ...... *four* parts .... 5, 9, 10, 12.
8. £2420 ...... *three* parts .... 35, 56, 63.
9. £65 ...... *three* parts .... $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$.
10. 429 tons ...... *three* parts .... $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{24}$.
11. £16. 17*s*. 6*d*. ...... *three* parts .... $\frac{1}{4}$, $\frac{2}{5}$, $\frac{3}{5}$.
12. £81. 10*s*. ...... *four* parts .... $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$.
13. 5 Km. ...... *three* parts .... 3·2, 2·5, 4·3.
14. 1 Kg. ...... *three* parts .... 1·39, 3·24, 5·37.
15. 120 metres ...... *three* parts .... 3·12, 5·20, 6·08, giving answers true to the nearest decimetre.

16. The sides of a triangular field are proportional to 5, 7, 8; and its perimeter is 270 yards. Find the length of each side.

17. An examiner wishes to mark three questions proportionally to 2·0, 3·2, and 4·8. How must he distribute 150 marks between them?

18. Concrete consists of $1\frac{1}{2}$ parts of lime, 4 parts of gravel, and $2\frac{1}{2}$ parts of sand. Out of $6\frac{2}{3}$ tons of concrete how much is lime? What percentage of each ingredient does concrete contain?

19. Divide 289 metres into three lengths inversely proportional to 2, 3, 9.

20. The sides of a triangle are respectively $2\frac{1}{5}$ inches, 3 inches, and $3\frac{1}{4}$ inches. A *similar* triangle has a perimeter of 338 feet; find each side.

21. Three persons *A*, *B*, and *C* join in a commercial venture; *A* contributing £320, *B* £400, and *C* £880. If the total profits amount to £165. 10*s.*, how much should each contributor receive?

[Here the profits are to be divided into shares proportional to the amounts of capital invested; that is, proportional to £320, £400, £880; namely to 4, 5, 11.]

22. Three partners join in a speculation, contributing respectively £500, £300, and £200 to the capital; if the total profits amount to £80, how much should each partner receive?

23. *A*, *B*, and *C* enter into partnership, *A* contributing £750, *B* £1000, and *C* £1250. To what percentage of the annual profits is each partner entitled? How much should *C* receive from a total profit of £1500?

24. A sum of £5100 is to be raised by three towns whose populations are 4250, 5250, and 7500 respectively; if the towns contribute in proportion to their populations, what does each town contribute? And what percentage of the whole sum is furnished by the first town?

25. A bankrupt owes £250, £330, and 400 guineas respectively to three creditors, and his available property is worth only £125. How much should each creditor receive, and how much in the £?

26. A man gave £2 to be divided among his three sons in proportion to their several ages, which were 12 yrs. 9 mos., 9 yrs. 9 mos., and 7 yrs. 6 mos. What was each son's share?

27. A sovereign consists of 22 carat gold, that is, 22 parts by weight of pure gold and 2 parts copper: its weight is 123·274 grains. Find, to the nearest grain, the amount of pure gold in 300 sovereigns.

28. Air is composed of nitrogen, oxygen, and carbonic acid in parts whose weights are nearly proportional to 75·62, 23·04, 1·34. Find, to the nearest gram, the weight of oxygen contained in 3·75 Kg. of air.

29. Given (as in the last example) that nitrogen, oxygen, and carbonic acid enter into air nearly in proportion to 75·62, 23·04, 1·34 by weight; and that 1 litre of air (at mean atmospheric pressure) weighs 1·295 grams: find the weight of oxygen in 1 cubic metre of air to the nearest gram.

30. Steel is composed as follows: iron 98·1 parts by weight, carbon 1·7 parts, other elements 0·2 part. In a steel bar weighing 207·4 Kg. find the weight of carbon. [Answer to three significant figures.]

31. Divide £72. 12*s.* among *A*, *B*, and *C*, so that *A* may get $2\frac{1}{3}$ as much as *B*, and *B* $1\frac{1}{2}$ as much as *C*.

[Suppose *C* gets 1 share; then *B* gets $1\frac{1}{2}$ shares; and *A* gets $2\frac{1}{3}$ of $1\frac{1}{2}$, or $3\frac{1}{2}$ shares. Thus the given sum must be divided proportionally to $3\frac{1}{2}$, $1\frac{1}{2}$, 1; that is, to 7, 3, 2.]

32. Divide £4. 2*s.* 6*d.* among *A*, *B*, and *C*, so that *A*'s share may be $\frac{5}{8}$ of *B*'s, and *B*'s share $1\frac{1}{2}$ of *C*'s.

33. The sum of £12. 10*s.* is to be divided between *A*, *B*, and *C*, so that *A*'s share is to *B*'s in the ratio 5 : 2, and *B*'s share to *C*'s in the ratio 7 : 13. How much should each receive?

34. Divide £146 among *A*, *B*, and *C*, so that as often as *A* gets 4*s.*, *B* may get 5*s.* 4*d.*; and as often as *B* gets 8*s.* 9*d.*, *C* may get 7*s.* 6*d.*

35. Two brass alloys are composed as follows: the 1st contains 3 parts tin, 20 parts copper, 3 parts zinc; the 2nd contains 7 parts tin, 56 parts copper, 2 parts zinc. These alloys are fused together in equal quantities (by weight). In what ratios will the three ingredients enter into the resulting alloy?

(*Division in Compound Proportion.*)

[If it is required to divide a quantity into parts which are *jointly* proportional to *a*, *b*, *c*, and to *x*, *y*, *z*, the required parts must be proportional to $a \times x$, $b \times y$, $c \times z$; namely to *ax*, *by*, *cz*.]

36. *A*, *B*, and *C* join in a business, contributing to the capital as follows: *A* employs £500 for 12 months, *B* £600 for 9 months, *C* £300 for 7 months. How should profits amounting to £384 be divided?

[Here the several claims depend partly on the *amounts* advanced, and partly on the *times* for which these amounts were employed. Consider the employment of £100 for 1 month to be the unit claim.

Then £500 for 12 mos. $= 5 \times 12$, or 60 units;
£600 for 9 mos. $= 6 \times 9$, or 54 units;
£300 for 7 mos. $= 3 \times 7$, or 21 units.

Thus the claims should be as 60 : 54 : 21;
viz. as 20 : 18 : 7.]

37. How should a profit of £160 be divided between two partners, one of whom has contributed £500 for 6 months and the other £850 for 4 months.

38. There are three partners in a business. *A* puts in £600 for 3 months, *B* £400 for 5 months, and *C* £1250 for 2 months; and the profits are £708. 15*s*. How ought they to be divided?

39. *A* begins business with a capital of £4000, and after 4 months takes *B* into partnership with a capital of £300. Two months later *C* joins the firm with a capital of £5000. At the end of the year the profits are found to be £1557. 15*s*.; how much of this sum should each partner receive?

40. *A* and *B* enter into partnership, *A* contributing £800 and *B* £1000. At the end of 6 months they admit *C*, who contributes £600. After the lapse of 3 years they balance the books, discovering a profit of £966. Find the share of each.

41. A certain mixture is formed of two substances combined in *volumes* having the ratio of 4 : 5. The specific gravities of these substances are as 5 : 6. If the weight of the mixture is 15 oz., find the weight of each substance in it. [N.B. The *weight* of a body depends jointly on its *volume* and its *specific gravity*.]

42. The specific gravities of three metals are as 3 : 2 : 5. Volumes of these metals proportional to 7, 4, 2 are mixed to form an amalgam. How much (by weight) of each metal is contained in 78 lbs. of the amalgam?

43. Three partners gain £3650 by a speculation. The first advances one-third of the capital for one-fourth of the time; the second, one-fourth of the capital for one-half the time; the third, the remainder of the capital for the whole time. Find their respective shares of the profit.

[If it is required to divide a given quantity into parts which are *jointly* proportional to a, b, c taken directly, and to x, y, z taken inversely, the required parts must be proportional to

$$a \times \frac{1}{x},\ b \times \frac{1}{y},\ c \times \frac{1}{z};\ \text{that is, to } \frac{a}{x},\ \frac{b}{y},\ \frac{c}{z}.]$$

44. A set of 210 coins consists of three groups, namely of sovereigns, of crowns, and of florins respectively. The values of the groups are as 14 : 4 : 3. What is the number of coins in each group?

45. Three substances whose specific gravities are as 2 : 3 : 4, are mixed together in parts which by *weight* are as 14 : 15 : 16. In what ratios by *volume* are these substances combined? And what volume of each is there in 3·2 litres of the mixture?

Miscellaneous Applications of Proportion and Variation.

283. We have already seen [Art. 275] that if one quantity y *varies directly* as another quantity x, then for every pair of corresponding values of x and y,

$$y = kx, \text{ where } k \text{ is constant :}$$

further, that this is merely another way of saying that y is *directly proportional to* x ; or again, that the ratio $\frac{y}{x}$ is *constant.*

Similarly, if y *varies inversely* as x, that is, if y varies as the *reciprocal* of x,

then $$y = k \times \frac{1}{x}, \quad k \text{ being constant.} \quad [\text{Art. } 278.]$$

In like manner if y varies directly as x^2,

then $$y = kx^2 ;$$

this statement being equivalent to saying that y is *proportional to the square* of x. And so on.

We shall now shew by an example how these principles may be applied.

Example. *Given that the breaking-strain of a steel wire is proportional to the square of its circumference, and that a wire of circumference* 5 *inches can just support a weight of* 40 *tons ; find* (*in tons*) *the breaking-strain of a wire* 3 *inches in circumference.*

Let S denote the breaking-strain (in tons) and C the circumference (in inches) of any steel wire.

Then since S is proportional to C^2, and since $S = 40$ when $C = 5$,

we have $$\frac{S}{40} = \frac{C^2}{5^2} ;$$

so that, when $C = 3$,

$$S = \frac{3^2}{5^2} \times 40 = 14{\cdot}4.$$

That is, the breaking-strain is 14·4 tons.

Or the work may be arranged thus :

Since $$S \text{ varies as } C^2,$$

$$\therefore \; S = kC^2, \text{ where } k \text{ is constant.}$$

Now it is given that $$S = 40, \text{ when } C = 5,$$

$$\therefore \; 40 = k \times 5^2,$$

$$\text{or} \quad k = 1{\cdot}6.$$

Thus $$S = 1{\cdot}6 \times C^2,$$
for all corresponding values of S and C;
so that when $$C = 3,$$
$$S = 1{\cdot}6 \times 3^2 = 14{\cdot}4.$$

284. Similar Figures and Solids. The following important cases of variation frequently occur in Arithmetic and Geometry.

I. In a series of **similar** figures, that is, figures of the same shape, all corresponding dimensions or **lines** similarly drawn in the two figures are **directly proportional.**

For example: If a, b, c denote the sides and p an altitude of a certain triangle; while x, y, z denote the corresponding sides, and q the corresponding altitude of a similar triangle,

then $$\frac{a}{x} = \frac{b}{y} = \frac{c}{z} = \frac{p}{q}.$$

Or again: Since all circles are similar, *the circumferences of circles are proportional to their diameters.*

Hence, if C denotes the circumference and d the diameter of *any* circle,

then the ratio $C : d$ is *constant*, that is to say, $\frac{C}{d}$ has the same value for all circles, so that
$$C = d \times (a\ constant).$$

The constant in this case is a quantity always denoted in mathematics by the symbol π, the numerical value of which is **3·1416**, correct to four decimal figures, or roughly $3\frac{1}{7}$.

II. In a series of similar figures, the **areas** are proportional to the **squares** of corresponding dimensions or lines similarly drawn in the two figures.

For example: If A and L denote respectively the area of a county and the distance between two given points; while a and l denote its area and the distance between the corresponding points as represented on a map,

then $$\frac{A}{a} = \frac{L^2}{l^2}.$$

Or again: *The areas of circles are proportional to the squares of their diameters.*

Hence, if A denotes the area, and d the diameter of any circle,

then $$A = d^2 \times (a\ constant).$$

The constant in this case is $\frac{\pi}{4}$, or 0·7854 to four decimal places.

III. In a series of similar solids, the **volumes** are proportional to the **cubes** of corresponding dimensions or lines similarly drawn in the two figures.

Hence, if the solids are of the same substance, or have the same specific gravity, their **weights** (being proportional to their volumes) will also be proportional to the cubes of corresponding dimensions.

For example: If two rectangular blocks are *similar*, that is, if their lengths, breadths, and heights are proportional, then their volumes are proportional to the cubes of their lengths (or to the cubes of their breadths or heights).

Or again: Since all spheres are similar, *the volumes of spheres are proportional to the cubes of their diameters.*

Hence, if V denotes the volume, and d the diameter of any sphere, then

$$V = d^3 \times (a\ constant).$$

The constant in this case is $\frac{\pi}{6}$, or 0·5236 to four decimal figures.

EXAMPLES XVI. h.

1. In a plan of a certain building-ground drawn to the scale of 1 inch to 15 yards, the distance between two given points appears as 2·4 inches. If the plan is enlarged to the scale of 1 inch to 8 yards, by what length should the given distance be represented?

2. A plan of a rectangular court-yard is drawn to the scale whose representative fraction is $\frac{1}{144}$. The length between two opposite corners in the plan is 3·5 inches: what is the actual distance?

3. In two similar triangles the longest sides are 10 cm. and 25 cm. respectively. The area of the first triangle is 40 sq. cm.: find the area of the second.

4. A circle, whose diameter is 10 cm., has a circumference approximately of 31·4 cm., and an area of 314 sq. cm. In a circle whose diameter is 7 cm. find (i) the circumference; (ii) the area; to the nearest 10th of a centimetre and square centimetre respectively.

5. In the plan of a field a hedge, 63 yards in length, is shewn by a line of 5·6 inches. The area of the field as represented on the plan is found to be 80 sq. in. What is the actual area of the field in square yards?

6. The distance between Exeter and Barnstaple is 35 miles. On a map of Devon this distance is represented by 2·5 in. The area on the map is found to be 13·2 sq. in. Find the actual area of the county to the nearest square mile.

7. Suppose a body falling from rest traverses s feet in the first t seconds. It has been found that s varies as t^2. Given that the distance traversed in the first 3 seconds is 145 feet, find to the nearest foot the distance traversed in (i) the 1st second; (ii) the first 4 seconds. And find the equation connecting s and t.

8. Three cubes of lead have respectively edges measuring 1 in., 2 in., and 3 in. The cube of middle size weighs 3·28 lbs. What do the others weigh?

9. The dimensions of a block of cast iron are 8 cm. long, by 6 cm. wide, by 4 cm. thick; and it weighs 1·4 Kg. The length of a *similar* block of the same metal is 12 cm. Find its breadth, thickness and weight.

10. A solid copper sphere of diameter 2 inches weighs 1·30 lbs. Find in lbs., to two significant digits, the weights of copper spheres whose diameters are 2·5 inches and 7·0 inches.

(*Miscellaneous.*)

11. Three wheels carry teeth whose numbers are proportional to the diameters of the wheels; the diameters being respectively 12 cm., 17 cm., and 20 cm. If the first wheel works into the second, and the second into the third, how many revolutions will the third wheel have made when the first has made 130?

12. *The volume of a given quantity of gas is inversely proportional to the pressure upon it.* At mean atmospheric pressure, namely 14·7 lbs. per square inch, a certain quantity of air occupies 1 cubic foot. Express in cubic inches the volume of the same quantity of air at a pressure of 84 lbs. per square inch.

13. One man works 8 hours a day, and is paid 28 shillings for a week of 6 days; another works 9 hours a day, is paid 25 shillings for a week of 6 days, and takes 5 hours to do work the first man does in 4. Which is it more profitable to employ? (C.S.)

14. For paraffin oil of which $2\frac{1}{2}$ gallons will burn in a stove for 100 hours, I am paying 9*d*. a gallon. Of another kind of oil of the same heating power I find 3 gallons will burn in the same stove for $135\frac{1}{2}$ hours. What would be a reasonable price to pay for it, to the nearest penny per gallon? (C.S.)

15. Two merchants *A* and *B* go into partnership. *A* puts in £2325 and at the end of four months adds £375 to his capital; while *B* withdraws £300 at the end of seven months. If, on a fair division of the profits at the end of the year, *A* and *B* receive equal shares, find how much *B* put in at first.

16. When a circular piece of a certain paper, of diameter $18\frac{1}{2}$ cm., is burnt, the weight of the ash is 0·00605 gram. Find the weight of the ash of a circular sheet of the same thickness and material whose diameter is $29\frac{1}{2}$ cm.

If the weight of the latter is catalogued as 0·01355 gram, find by how much per cent. the calculated result differs from that given in the catalogue. The areas of circles vary as the squares of their diameters. (C.S.)

17. A certain gallon-jar is 5 inches high. Of what height must a jar of the same shape be made to hold a pint? [Vessels of the same shape have volumes proportional to the cubes of their linear dimensions.]

18. A quantity y varies as x^2, and when $x=8$, $y=16$. Find the equation between y and x. Tabulate values of y corresponding to the following values of x:

$$0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,$$

and use the results to shew the relation between y and x by a graph, using 0·4 of an inch as unit on each axis.

What would be the values of y if the above values of x were all *negative?* Plot the points so given, and extend your graph to pass through them. Read off from the graph the values of y, when $x=5{\cdot}5,\ -6{\cdot}5$.

19. It has been proved by experiment that if S represents the breaking-strain (in tons) of a steel wire, and C its circumference (in inches),

then

$$S=C^2\times 1{\cdot}6.$$

From this equation find values of S when

$$C=\tfrac{1}{4}'',\ \tfrac{1}{2}'',\ \tfrac{3}{4}'',\ 1'',\ 1\tfrac{1}{4}'',\ 1\tfrac{1}{2}'',\ 1\tfrac{3}{4}'',\ 2'',$$

and plot a graph shewing the relation of S to C between the given limits.

20. *The pressure of the wind varies as the square of its velocity;* and it has been found that if P denotes the pressure in pounds per square foot, and v the velocity in miles per hour,

then

$$P=0{\cdot}005v^2.$$

Complete the table indicated below, and hence draw a graph comparing the pressure with the velocity of wind.

Velocity in miles per hour	5	10	15	20	25	30	35	40
Pressure in lbs. per sq. ft.								

Read off from the graph

(i) the wind's pressure for 18 miles an hour;

(ii) the wind's velocity for a pressure of 4 lbs. per sq. foot.

MISCELLANEOUS EXAMPLES.

285. EXAMPLE 1. *If 6 men with 7 boys can complete a work in 4 days,* and *4 men with 12 boys can do the same work in* $3\frac{15}{16}$ *days, how long would the work take one man* or *one boy working alone?*

$$\left\{\begin{array}{llll} 6 \text{ men with} & 7 \text{ boys do} & \frac{1}{4} & \text{of the work in } 1 \text{ day,} \\ 4 \;\dots\dots\dots & 12 \;\dots\dots & \frac{16}{63} & \dots\dots\dots\dots\;\; 1 \;\dots\dots \end{array}\right.$$

$$\therefore \left\{\begin{array}{llll} 12 \;\dots\dots\dots & 14 \;\dots\dots & \frac{1}{2} & \dots\dots\dots\dots\;\; 1 \;\dots\dots \\ 12 \;\dots\dots\dots & 36 \;\dots\dots & \frac{16}{21} & \dots\dots\dots\dots\;\; 1 \;\dots\dots \end{array}\right.$$

∴ by subtraction, 22 boys do $\frac{16}{21} - \frac{1}{2}$, or $\frac{11}{42}$ of the work in 1 day,

that is, 1 boy does $\frac{1}{84}$ of the work in 1 day.

Hence one boy would take 84 days working alone, and it easily follows that one man would take 36 days.

EXAMPLE 2. *A man, rowing on a river, finds that after he has rowed down-stream for 2 hours and then drifted with the stream for 1 hour, he is 16 miles from his starting point. The next day he rows up-stream for 6 hours, drifts down-stream for 2 hours, and then finds himself 14 miles from his starting point. Find the rate of the stream, and the rate of rowing* in still water.

Let the boat's rate *in still water* be denoted by x mi. per hr.
and let the rate of the stream .... y ....
Then against the stream the boat goes $(x-y)$ mi. per hr.,
and with .... $(x+y)$ ....

On the first day he rows $2(x+y)$ miles and drifts y miles *further*;

$$\therefore\ 2(x+y)+y=16,$$

$$\text{or}\qquad 2x+3y=16. \qquad\text{(i)}$$

On the second day he rows $6(x-y)$ miles and drifts $2y$ miles *in the opposite direction.*

$$\therefore\ 6(x-y)-2y=14,$$

$$\text{or}\qquad 6x-8y=14. \qquad\text{(ii)}$$

From the equations (i) and (ii) we have to find the values of x and y.

Multiplying (i) by 3, we have

$$6x+9y=48.$$

And from (ii), $\qquad 6x-8y=14.$

By subtraction, $\qquad 17y=34;$

$$\therefore\ y=2.$$

Substituting in (i), $\qquad 2x+6=16;$

$$\therefore\ x=5.$$

Thus the rate of stream is 2 mi. per hour, and the rate of rowing in still water is 5 mi. per hour.

MISCELLANEOUS EXAMPLES IV.

Exercises for Revision.

A.

1. Simplify

$$\text{(i)}\ \frac{3\frac{1}{3}-\frac{5}{9}+\frac{1}{5}}{2\frac{2}{3}+\frac{13}{45}}+\frac{1\frac{1}{5}}{11\frac{1}{5}-2\frac{1}{3}};\qquad \text{(ii)}\ \frac{5\frac{1}{3}-1\frac{1}{12}}{4\frac{2}{3}+6\frac{7}{12}}-\frac{3{\cdot}2-2{\cdot}88}{0{\cdot}97+0{\cdot}83}.$$

2. Find the sum of

$\frac{5}{9}$ of £17. 16*s.* 3*d.*, 5283 farthings, and £21·875.

3. Which goes the faster, a train travelling at 40 miles an hour, o a turbine steamer travelling at 33 knots? A knot is a speed of 608 feet per hour. (C.S.

4. Assuming that the difference between the length of a rod of iro in summer and its length in winter is $\frac{1}{4000}$ of its length in winter, find the total expansion from winter to summer of a bar equal in lengt to the Forth Bridge ($\frac{2}{3}$ of a mile). (C.S.

5. A square courtyard, 35 feet in the side, has to be paved wit square tiles 8 inches in the side. How many tiles will be used, an what space will be left uncovered at the edges, if the tiles cannot b divided? (C.S.

6. Resolve 999999 into its prime factors.

B.

7. Find, to the nearest penny, the value of 20 tons 9 cwt. 3 qrs. o coke at 18*s.* 3*d.* per ton.

8. Simplify $1\frac{4}{7}-\frac{3}{4}\left(\frac{1}{15}+\frac{1}{5}\right)+\frac{3}{104}\div\frac{2\frac{3}{16}}{3-\frac{5}{6}}.$

9. A mixture consists of 15 parts of coffee, purchased at £4. 4*s.* per cwt., and 1 part of chicory, purchased at £1. 16*s.* per cwt. It is sold at 1*s.* 4*d.* per lb.; what profit, to the nearest penny, would b made on the sale of 2 cwt. 24 lbs.?

10. If £1 can be exchanged for 25 fr. 20 c. find to the nearest fran the equivalent of £15. 10*s.*

11. The population of the United Kingdom was 41,609,000 in 1901, and the national income derived from taxes on the sale of intoxicants was £31,428,000. Calculate (to the nearest penny) the average amoun contributed per head in this way. (C.S.)

12. The return railway ticket from a suburb to London costs 2*s.* 2*d.*, and the annual ticket £12. 5*s.* 0*d.* How many days in the year must a man travel to save money by taking an annual ticket instead of payin for each day? (C.S.)

C.

13. How many quarters are there in 13 per cent. of 3 tons 15 cwt.?

14. Calculate the value of $\frac{14 \cdot 52}{83 \cdot 1} + \frac{1 \cdot 575}{13 \cdot 85}$ to five places of decimals.

15. A rectangular field is 330 yards in length and 188 yards in breadth; find the area of the field to the nearest acre; and the money which would be obtained by selling half the field for £17. 4*s*. 6*d*. an acre, and the other half for £21. 15*s*. 6*d*. an acre.

16. Express 2 ac. 2 r. 7·12 p. as the decimal of $3\frac{1}{2}$ acres.

17. Assuming 1 ton = 1016·05 kilograms, express 16 tons 5 cwt. in kilograms correct to the nearest unit.

18. *A* runs 28 yards in 3 secs.; *B* runs 46 yards in 5 secs. If they start to run together, how far will *A* have run when *B* is left 10 yards behind?

D.

19. A cubic foot of water weighs 62·3 lbs., and 41 cubic feet of teak weigh a ton. Is teak heavier or lighter than water, bulk for bulk?

20. What is the value of 0·628125 of a ton of metal at £63. 13*s*. 4*d*. a ton?

21. There are three numbers. The H.C.F. of each pair is 17, and the L.C.M. of all three is 29,172. What is the product of the numbers?

22. Two boxes of eggs, each containing 500, are bought for £2 12*s*. 1*d*.; if the eggs in one box cost 5*d*. per score more than the others, what is the price per score of each lot?

23. If 5 cwts. 3 qrs. 20 lbs. of tea cost £44. 5*s*. 4*d*., what is the value of 2 cwt. 16 lbs.?

24. *A* owes £125 to *B*, *B* owes £90 to *C*, and *C* owes £67 to *A*, and the accounts between them are to be settled by cheques given by *A* to each of the others; for what amounts must the cheques be drawn?

E.

25. A man buys 3 acres 3 roods 15 poles of land at £440 an acre, and builds houses on it costing £3808. 15*s*. in all. What was his total expenditure?

26. If 4 men with 9 boys can do a piece of work in $1\frac{1}{5}$ days, and 3 men with 6 boys can do the same work in $1\frac{5}{7}$ days, how long would one boy or one man working alone take to do the work?

27. Find the value of

$$31{\cdot}213\tfrac{1}{4}+3{\cdot}20\tfrac{7}{8}-20{\cdot}5\tfrac{3}{8}-13{\cdot}632\tfrac{5}{8},$$

and express the result *in thousandths and the fraction of a thousandth.*

28. In 1900–1901 the national expenditure was £183,592,264. Of this the Army cost £91,710,000, the Navy £29,520,000, and the Civi Services £23,500,000. Express each of these as a percentage of the whole, to the nearest integer. [See Ex. 3, p. 210.] (C.S.

29. If a three-mile race is won in 35 min. 33 secs., and the winne beats the second man by 66 yards, shew that the second man's pace i exactly 5 miles an hour.

30. An excursion train, consisting only of first and third-clas carriages, conveys 1575 passengers from London to Portsmouth. The fare for a first-class ticket is 1*s.* 6*d.* more than for a third-class ticket, and 325 of the passengers travel first class. If the total amount received in fares is £300, what is the fare for each class?

F.

31. Find in simplest form the value of

$$\text{(i)}\ \frac{\frac{5}{3}+\frac{3}{4}}{\frac{13}{4}+2\frac{5}{12}}+\frac{1}{3\frac{1}{11}}; \qquad \text{(ii)}\ \frac{3\frac{2}{3}\text{ of }\frac{1}{8}-\frac{5}{12}}{\left(\frac{1}{3}\text{ of }\frac{1}{16}\right)+\left(\frac{3}{16}-\frac{1}{6}\right)}.$$

32. A light was seen at intervals of 13 seconds. It was seen the first time at 1 hr. 54 min. 50 secs. a.m., and the last time at 3 hrs. 17 min. 49 secs. a.m. How many times was the light seen?

33. The wages of 8 men and 4 boys amount to £15, and the wages of 5 men and 3 boys to £9. 15*s.* per week. Find the daily wages of a man and boy.

34. Find the total cost of the following articles: 12 yds. of muslin at $5\frac{1}{2}$*d.* per yd.; 9 yds. of flannel at 1*s.* $11\frac{3}{4}$*d.* per yd.; 3 pairs of gloves at 2*s.* $11\frac{1}{2}$*d.* per pair; 4 pairs of stockings at 2*s.* 3*d.* per pair; 8 handkerchiefs at 16*s.* per dozen.

[Each separate charge should be correct to the nearest farthing.]

35. A landlord's net rental after paying income tax at the rate of 1*s.* in the £ is £7101. 14*s.* 6*d.* Find his gross rental.

36. If 480 ounces Troy of standard gold can be coined into 1869 sovereigns, the proportion of pure gold to alloy in standard gold being 22 to 2, find to the nearest grain the weight of pure gold in a sovereign. [1 oz. Troy = 480 grains.]

G.

37. Find all the prime factors of 11781 and 12376, and hence find their H.C.F.

38. In 1906 there were 6,897,000 bushels of barley produced from 187,000 acres of Irish land. Taking the bushel of barley to weigh 56 lbs., what was the produce in ounces per square yard, correct to one-tenth of an ounce?

39. Assuming that a metre = 39·37 inches, and that £1 is worth 25 fr. 40 c., find the price in francs and centimes per metre equivalent to 3*s.* 11*d.* per yard.

40. Find the value to the nearest penny of

$$£1.\ 7s.\ 1d. \div 1{\cdot}07 + £1.\ 13s.\ 9d. \times 0{\cdot}58 - 1{\cdot}439 \text{ of } 19s.\ 3d.$$

41. Standard gold consists of 11 parts by weight of pure gold and 1 part of copper. If a sovereign contains 113 grains of pure gold, find how many will weigh 1 lb. avoirdupois.

42. A contractor undertakes to dig a canal 4 miles long in 2 years. He employs 1000 men, but finds at the end of a year that only 2980 yards have been completed. How many more men at least must he employ in order to fulfil his contract?

H.

43. Given that 31 lbs. are roughly equivalent to 14 kilograms, draw a graph from which any number of pounds up to 40 may be read off in kilograms, to the same degree of approximation. Express 22 lbs. and 17·5 lbs. in kilograms; and 5 Kg. and 17 Kg. in pounds.

44. A bankrupt owes £315. 5*s.* to *A*, £255. 10*s.* to *B*, and £175 to *C*. His assets are £410. 3*s.* 3*d.*: what should each creditor receive?

45. The members of a musical society have the option of paying either 5*s.* each time they attend a concert; or, an annual subscription of one guinea and 2*s.* 6*d.* each time they attend a concert. How many concerts must the latter class of members attend, during a year, before they gain any advantage over the former class? (C.S.)

46. The number of spectators at a football match was 15,568, the sum of sixpence was charged for admission, and 1245 of the persons admitted paid, in addition, one shilling each for grand-stand tickets. The total receipts were £423. 7*s.* 6*d.* How many persons were admitted without payment? (C.S.)

47. What is the rent of a field containing 71 ac. 1 r. 15 p. at £2. 12*s.* 6*d.* an acre? Give your answer to the nearest penny. (C.S.)

48. A certain field can be reaped by 7 men in a given time, and 5 boys can do as much work as 2 men. Find how many boys would be required in addition to 30 men for the reaping of a field of twice the size in a third part of the time.

I.

49. What is the internal area in square inches of the cross-sectio of a pipe which holds 3 cubic feet in 8 yards of its length? (C.S.)

50. If 1 cwt. of coffee costs £9. 12*s.*, draw a graph to give the price of any number of pounds. Read off the price (to the nearest penny) of 13 lbs., 21 lbs., 23 lbs.

51. If 3 men *or* 7 women can do a piece of work in 32 days, how long will 7 men *and* 5 women require to do a piece of work twice as great?

52. A person who buys tea at 1*s.* 10½*d.* per pound, subsequently finds that the pound weight used was an ounce short in weight. What was the real price of the tea per pound?

53. A person on the platform of a station which is 132 yards long, notices that a train, which takes 3 seconds to pass him, occupies 7½ seconds in passing completely through the station. What is the speed of the train, in miles per hour?

54. A rectangular carpet, 21 ft. 6 in. long and 16 ft. 10½ in. wide, is put down in a room, leaving a space 16 in. wide along each side, and 24 in. wide along each end, unoccupied. If the carpet costs 5*s.* 4*d.* per square yard, and the remaining space is covered with inlaid woodwork, costing 8*d.* per square foot, find the whole cost.

J.

55. Read off as complete decimals

$$4{\cdot}73\tfrac{1}{2},\quad 12{\cdot}3\tfrac{1}{8},\quad 5{\cdot}13\tfrac{1}{4},\quad 7{\cdot}8\tfrac{1}{5},$$

and find their sum.

56. Simplify

$$\text{(i)}\ \frac{\frac{1}{3}\div\left(\frac{3}{4}\times\frac{5}{8}\right)}{\frac{1}{5}\div\frac{1}{4}+2\frac{2}{5}};\qquad \text{(ii)}\ \frac{\frac{2}{7}-\frac{5}{91}-\frac{12}{143}}{12\frac{1}{11}-5\frac{1}{6}\times 2\frac{1}{7}-\frac{17}{56}}.$$

57. Find in square yards and square feet the area of a path 4 ft. wide surrounding a lawn 24 yds. 2 ft. by 22 yds. 1 ft.

58. *A* can do as much work in 2 days as *C* in 3 days, and *B* in 5 days as much work as *C* in 4 days: what time would *B* require to do a work which *A* can finish in 12 weeks?

59. Draw a graph from which the equivalent of a pressure given in pounds per square inch may be read off in kilograms per square centimetre, given that 27 lbs. per sq. in. is approximately equivalent to 1·90 Kg. per sq. cm.

Read off the equivalents of 30 lbs. and 57 lbs. per sq. in.

Express 2·55 Kg. per sq. cm. in lbs. per sq. in.

60. What will be charged for carrying 2½ tons for 46½ miles, if the carriage of 1 cwt. 3 qrs. 21 lbs. for 52¼ miles amounts to 17*s.* 5*d.*?

K.

61. Find the cost of 22 cwt. 3 qrs. 21 lbs. at £12. 16*s.* 8*d.* per cwt.

62. Find the simplest value of

$$\frac{10\frac{2}{3}-(5\frac{2}{5}+4\frac{9}{20})}{10\frac{1}{3}-(2\frac{4}{5}-\frac{4}{15})-7}\times\left(6+\frac{1}{1+\frac{1}{6}}\right).$$

63. Use a contracted method to find the square root, correct to 5 decimal places, of 44100·015241579.

64. A man can row 27 miles up-stream from A to B in 9 hours, and down-stream from B to A in 3 hours. Find (i) the rate of the stream, (ii) how long it would take the man to row from A to B if there was no stream.

65. Assuming that a litre = 0·2201 gallon, find to the nearest litre the quantity of water wasted in an hour by a cistern which leaks at the rate of $1\frac{1}{2}$ pints per minute.

66. A man is timed to skate over a course supposed to be a mile long in 3 min. $25\frac{1}{5}$ sec., but it is found on measurement that the course is 50 yards short. Calculate his time for a true mile at the same rate of speed.

L.

67. A housekeeper takes three half-pints of milk each week-day and one pint on Sunday, and her bill for the week comes to 1*s.* $5\frac{1}{2}$*d.* What is the price of milk per quart?

68. Calculate to the nearest penny the sum to be paid to a creditor whose debt is £183. 13*s.* 10*d.* when his debtor pays 14*s.* $7\frac{1}{4}$*d.* in the £.

69. A proposed railway is to have an average width (including embankments) of 80 ft. How much per mile must the company pay for land at the rate of £165 per acre?

70. On a certain map a road 1320 yds. long is represented by a length of $18\frac{3}{4}$ inches. Determine the scale of the map. What area on the map would represent an area of $\frac{1}{10}$ sq. mile? (C.S.)

71. Shew, without unnecessary work, that the following statements cannot be correct:

(i) $3·984 \times 0·872 = 3·752$,

(ii) $3·984 \div 0·872 = 3·652$,

(iii) $\sqrt{0·324} = 0·18$.

72. During the last 4 months of a year a man's average daily expenditure was 3*s.* $9\frac{1}{2}$*d.* If his average was 4*s.* 5*d.* in September, 3*s.* 4*d.* in October, and 2*s.* 7*d.* in November, what was his average daily expenditure in December, to the nearest penny?

M.

73. A rectangular allotment ground is 24 chains 25 links long by 16 chains 80 links broad. Find, to the nearest penny, its rent at £5. 15*s*. 6*d*. per acre.

74. If 9 men *or* 16 boys can hoe a field of 16 acres in 7 days of 10 hours each, in how many days of 9 hours each will 13 men *and* 15 boys hoe a field of 49 acres?

75. Divide £523. 10*s*. among three persons, *A*, *B*, and *C*, so that *A* may get one-third of what *B* receives, and *C* may receive as much as *A* and *B* together.

76. A railway train, 73 metres in length, is travelling at the rate of 60 kilometres an hour on the up line, and another train, 102 metres in length, is travelling in the opposite direction, on the down line, at the rate of 40 kilometres an hour; find the time occupied by the trains in passing each other.

If the trains were travelling on parallel lines in the same direction, what would be the time occupied by the faster train in passing the slower? (C.S.)

77. A beam, having a square section, is 9 ft. long, and weighs $3\frac{1}{2}$ cwt. A cubic foot of the substance of the beam weighs 32 lbs. What is the thickness of the beam? (C.S.)

78. A basket of 65 oranges is bought for 4*s*. 2*d*. Draw a graph to shew the price for any other number. How many could be bought for 3*s*. 4*d*.? Find the price (to the nearest penny) which must be paid for 36 and for 78 oranges respectively.

N.

79. If a cubic foot of water weighs 62·426 lbs., and a linear foot is equal to 30·48 centimetres, find the number of pounds in a kilogram, correct to the nearest tenth.

80. A block of glass measures, in inches, $5\frac{3}{32}$ by $2\frac{7}{8}$ by $1\frac{5}{16}$. The material weighs 1·6 ounces per cubic inch. Find the weight of the block to the nearest ounce.

Suggest a simple method of testing whether your result contains a serious error. (C.S.)

81. A man rows a certain distance down-stream in 5 hours, and the same distance up-stream in 7 hours. If he can row 8 miles an hour in still water, what is the rate of the stream?

82. A certain length of pathway has to be constructed; it is found that three men can construct one-fifth all but one mile in two days, whilst 18 men can construct one mile more than two-fifths in one day. What is the length of the path? (C.S.)

83. A train overtakes two persons who are walking 2 miles and 4 miles an hour respectively, and completely passes them respectively in 9 seconds and 10 seconds: what is the length of the train and its speed in miles per hour? (C.S.)

84. Justify the following graphical construction for finding approximately 1·414 of any number up to 10. Join the origin to a point P whose coordinates are 10 and 14·14 (or 5 and 7·07), taking 1 inch as unit; then the ordinate of any point on OP is 1·414 times the corresponding abscissa. Read off from the diagram as correctly as possible to two places of decimals, $1·414 \times 2$, $1·414 \times 3·5$, $1·414 \times 8·6$, $\frac{1}{1·414} \times 7·8$.

O.

85. A boy is required to multiply £13. 3*s.* 7*d.* by 105. He multiplies by the factors 3, 5, 7 in order. In this way he gets three products. Explain how many times £13. 3*s.* 7*d.* the sum of these three products will be? Using addition and subtraction only, shew how to find £13. 3*s.* 7*d.* × 93 from the products in the boy's sum.

86. The bottom of a rectangular tank measures 24 ft. by 15 ft. Find the depth to which it will be filled when it contains 9000 gallons of water: Assume that a gallon weighs 10 lbs., and that a cubic foot of water weighs 1000 oz.

87. A garrison of 200 men has provisions sufficient for 24 weeks; at the end of the first week a reinforcement of 80 men arrives, and the rations per day of each man are then reduced from 18 oz. to 15 oz.; for how many days longer can they hold out?

88. A man, by fraudulently using as a pound-weight one short by half-an-ounce, raises the price of tea by $1\frac{1}{4}$*d.* a lb. What is the price of the tea per lb.?

89. The average height of 4 men is 1 m. 72 cm.; the heights of three of them are 1 m. 80 cm., 1 m. 65 cm., and 1 m. 58 cm. respectively: what is the height of the fourth?

If each of these measurements is only correct to the nearest centimetre, between what limits must the height of the fourth man lie?

90. The area of a circular ring is given by the formula

$$(D^2 - d^2) \times 0·7854,$$

where D and d are the external and internal diameters respectively,

(i) Find, to the nearest tenth of a square foot, the area of a path 1·4 ft. wide round a circular flower-bed 8·6 ft. in diameter.

(ii) Find the weight, to the nearest gram, of a circular ring of metal cut from a plate weighing 6·7 grams per square centimetre, the external and internal diameters of the ring being 7·3 cm. and 4·7 cm.

P.

91. After 12 innings a cricketer's average is 24; what must be the average of the remaining 3 innings of the season that his average for the whole season may still be double the number of his innings?

92. Work, as clearly as possible, by the shortest method you can think of, the following:

(i) Find the cost of 87 yards of silk at 7*s.* 7*d.* a yard, together with double that number of yards of ribbon at 1*s.* $2\frac{1}{2}$*d.* a yard.

(ii) Find the cost of 8250 bricks at 10*s.* 6*d.* a thousand.

(iii) Evaluate $49\frac{7}{16}$ of half a mile.

93. A chest whose external length, breadth, and height are 5 ft. 9 in., 4 ft. 3 in., and 3 ft. 3 in. respectively, is made of deal $1\frac{1}{2}$ in. thick. Find the cost of lining the sides and bottom with thin metal at 8*d.* per square foot.

94. On a certain map an area of 12535 sq. mi. 2904 sq. yds. is represented by a rectangle 3·5 in. by 7·4 in. On what scale is the map drawn?

95. If a man rows 21 miles in $3\frac{1}{2}$ hours against a stream the rate of which is 2 miles an hour, how long would he be in rowing 20 miles with the stream?

96. By measuring time along OX (1 inch for 1 hour), and distance along OY (1 inch for 10 miles) shew that a line may be drawn from O through the points (1, 8), (2, 16), (3, 24), ... to indicate distance travelled towards Y in a specified time at 8 miles an hour.

A starts from London at noon at 8 miles an hour; two hours later *B* starts, riding at 10 miles an hour. Find graphically at what time and at what distance from London *B* overtakes *A*. At what times will *A* and *B* be 8 miles apart? If *C* rides after *B*, starting at 3 p.m. at 15 miles an hour, find from the graphs

(i) the distances between *A*, *B*, and *C* at 5 p.m.;

(ii) the time when *C* is 8 miles behind *B*.

CHAPTER XVII.

PERCENTAGES.

286. We have already seen [Chap. VI. Art. 138] that a **percentage** *is a ratio or fraction expressed with* 100 *as its second term*, the first term or numerator of the fraction being the **rate** per cent.

For example: $\frac{7}{100}$ is a percentage, denoting 7 out of one hundred, or 7 per cent.: this is written 7 p.c. or 7 %, 7 being the *rate* per cent.

Thus a percentage of a quantity is a fraction of it expressed in *hundredths*.

287. We have also shewn [Arts. 138, 206, 207]

(i) how to express any fraction or ratio as a percentage;

(ii) how to find a percentage of any given quantity.

It is convenient to repeat these two processes.

EXAMPLE 1. *Express* (i) $\frac{7}{16}$ *as a percentage;*

(ii) $73\frac{1}{3}$ *per cent. as a fraction.*

(i) Let x denote the rate per cent.;

then
$$\frac{x}{100}=\frac{7}{16};$$
$$\therefore\ x=\frac{7}{16}\times 100=43\tfrac{3}{4}.$$

Thus the fraction $\frac{7}{16}$ is equivalent to $43\frac{3}{4}$ per cent.

(ii) $73\frac{1}{3}$ per cent. $=\dfrac{73\frac{1}{3}}{100}=\dfrac{220}{100\times 3}=\dfrac{11}{15}.$

EXAMPLE 2. *Find, to the nearest penny:*

(i) $3\frac{1}{4}$ % of £846; (ii) $5\frac{7}{8}$ % of £786. 11*s*. 4*d*.

(i) $3\frac{1}{4}$ % of £846 $=£846\times\dfrac{3\frac{1}{4}}{100}$
$=£8{\cdot}46\times 3\frac{1}{4}$
$=£27{\cdot}495$
$=£27.\ 9s.\ 11d.$ to the nearest penny.

$$\begin{array}{r|r} & £\\ & 8{\cdot}46\\ & 3\frac{1}{4}\\ \hline & 25{\cdot}38\\ \frac{1}{4} & 2{\cdot}115\\ \hline & 27{\cdot}495\end{array}$$

(ii) $5\frac{7}{8}\%$ of £786. 11*s*. 4*d*. $= £786{\cdot}567 \times \dfrac{5\frac{7}{8}}{100}$

$= £7{\cdot}86567 \times 5\frac{7}{8}$.

$$\begin{array}{r|l} \pounds & \\ 7{\cdot}865 & 6\cancel{7} \\ & 6 \\ \hline 47{\cdot}194 & 0 \\ \textit{less}\ \tfrac{1}{8}\ \ 0{\cdot}983 & 2 \\ \hline 46{\cdot}211 & = \pounds 46.\ 4s.\ 3d. \end{array}$$

Multiply by 6, and deduct $\frac{1}{8}$ of the multiplicand, arranging the work so as to give a result true to the third decimal figure.

NOTE. In a few special cases it may be convenient to work out the result fractionally; but the decimal process here given is generally to be preferred.

288. Commission is a fee paid to an agent for buying or selling property, or for conducting any other business on behalf of another person. Commission is calculated as a *percentage* on the money spent or received by the agent for his employer.

Money paid to effect insurance of property is called a **premium**: it is calculated as a *percentage* on the sum payable in case of loss.

EXAMPLES XVII. a.

1. Find correct to three significant figures the percentages equivalent to the following fractions:

(i) $\frac{2}{7}$; (ii) $\frac{2}{13}$; (iii) $\frac{5}{17}$; (iv) $\frac{11}{21}$; (v) $\frac{8}{15}$.

2. Find the fractions equivalent to

(i) $3\frac{1}{3}\%$; (ii) $7{\cdot}5\%$; (iii) $12\frac{1}{2}\%$; (iv) $33\frac{1}{3}\%$.

How much per cent. is

3. £3. 10*s*. of £25?

4. £17. 5*s*. of £60?

5. 3*s*. 9*d*. of 15*s*.?

6. 22 yards of 1 mile?

Find the values of

7. 4% of £35.

8. 3% of £120.

9. 7% of £225.

10. $4\frac{1}{2}\%$ of £920.

11. $3\frac{1}{2}\%$ of £1175.

12. $5\frac{1}{4}\%$ of 2264 fr.

How much per cent. is

13. 3*s*. $4\frac{1}{2}$*d*. of 16*s*. $10\frac{1}{2}$*d*.?

14. 4 tons 8 cwts. of $27\frac{1}{2}$ tons?

15. 6·60 fr. of 82·50 fr.?

16. 527 grams of 12·4 Kg.

Express decimally, correct to three significant figures :

17. £38 as a percentage of £271.

18. £72. 10*s.* .... £946.

19. £51. 12*s.* .... £85. 14*s.*

20. 3 fr. 26 c. .... 98 fr. 60 c.

21. 7*s.* 9*d.* .... £5. 18*s.* 6*d.*

Working in decimals find to the nearest penny the value of

22. 4 % of £573.

23. 7 % of £1129.

24. $3\frac{1}{2}$ % of £844.

25. $4\frac{1}{4}$ % of £58. 16*s.*

26. 4 % of £150. 13*s.* 6*d.*

27. $5\frac{3}{4}$ % of £804. 8*s.*

28. Find to the nearest penny the commission on

(i) £302. 14*s.* 7*d.* at 3 per cent. ;

(ii) £1572. 11*s.* 10*d.* at $5\frac{1}{2}$ per cent.

29. An agent charges $3\frac{1}{3}$ % commission for collecting rents. If the rents amount to £375, how much commission should be paid ?

30. A yard measure is too long by one quarter of an inch : estimate the error as a percentage of the proper length. [Answer to two significant figures.]

31. A field-force of 7500 men is reduced by sickness and casualties to 6359. Estimate the loss per cent. to three significant figures.

32. In 1890 the population of a town was 4680 ; in 1900 it had risen to 4948. Find the absolute increase, and the increase per cent. reckoned on the population of 1890. [Answer to two significant figures.]

289. The following general methods should be carefully noted.

(i) To a given quantity P add 7 per cent. of itself ;

(ii) From a given quantity P take 7 per cent. of itself ;

each operation being performed in one step.

(i) If to **100** we add 7 p.c. of 100 we get 100 + 7 or **107** ;

∴ the required sum is greater than P in the ratio 107 : 100 ;

that is, required sum $= \mathrm{P} \times \frac{107}{100}$.

(ii) If from **100** we take 7 p.c. of 100 we get 100 − 7 or **93** ;

∴ the required difference is less than P in the ratio 93 : 100 ;

that is, required difference $= \mathrm{P} \times \frac{93}{100}$.

Conversely : (i) What quantity + 7 p.c. of itself gives **Q** ?
(ii) What quantity − 7 p.c. of itself gives **Q** ?

(i) Here the required quantity, *on which the percentage is taken*, corresponds to **100** ; and the given total corresponds to **107**.

∴ the required quantity is less than **Q** in the ratio of 100 : 107 ;

$$\text{that is, required quantity} = \mathbf{Q} \times \tfrac{100}{107}.$$

(ii) Similarly, required quantity $= \mathbf{Q} \times \frac{100}{93}$.

NOTE. In such cases the multipliers $\frac{107}{100}$, $\frac{93}{100}$ are usually best treated as decimals; while the reciprocal multipliers $\frac{100}{107}$, $\frac{100}{93}$ are often best treated as fractions.

EXAMPLE. *Five per cent. of the inhabitants of a town having died of fever, a panic set in, during which* 15 *per cent. of the remaining inhabitants left the town. If the population was then reduced to* 3553, *what was it originally ?*

Out of every 100 persons before the panic 85 are finally left ;

$$\therefore \text{ before the panic, the population} = 3553 \times \tfrac{100}{85}.$$

Again, out of every 100 persons before the fever 95 survived ;

$$\therefore \text{ the original population} = 3553 \times \tfrac{100}{85} \times \tfrac{100}{95}$$
$$= 4400.$$

290. In questions dealing with statistics, where large numbers are involved, approximate methods should be used, as shewn in the following example.

EXAMPLE. *The population of Ireland was returned as* 4,704,750 *at the census of* 1891, *and as* 4,458,775 *in* 1901. *Find, to three significant figures, the percentage decrease.*

The decrease was 245,975 out of a total of 4,704,750.

Let x be the rate per cent. of decrease ;

$$\text{then} \quad \frac{x}{100} = \frac{245{,}975}{4{,}704{,}750}$$
$$= \frac{245{,}975}{4{,}704{,}800}, \text{ nearly.}$$
$$\therefore \; x = \frac{245{,}975}{47{,}048}$$
$$= \frac{24{\cdot}598}{4{\cdot}705}, \text{ nearly,}$$
$$= 5{\cdot}23.$$

Thus the decrease is 5·23 per cent.

Here it is only necessary to retain 4 significant digits in the divisor.

$$\begin{array}{r|r} & 5{\cdot}23 \\ \cline{2-2} 4{\cdot}7_{\backprime}0_{\backprime}5 & 24{\cdot}598 \\ \cline{2-2} & 1\,073 \\ \cline{2-2} & 132 \end{array}$$

EXAMPLES XVII. b.

1. What is a man's income if, after losing $22\frac{1}{2}$ p.c. of it, he has £186 left?

2. The population of a district has decreased by 16 p.c.; if it was originally 15,800, what is it now?

3. The population of a town which is now 62,130 is known to have increased by 9 p.c. since the last census: what was the population at the last census?

4. An army of 145,600 loses $12\frac{1}{2}$ p.c. of its number through disease, and 15 p.c. of the remainder in battle: how many are left?

5. A tradesman makes a reduction of 10 p.c. from the prices marked: what will he receive for an article marked 9*s.* 2*d.*?

6. Goods are being sold at a reduction of 20 p.c. below the marked price: what should be the marked price of an article sold for 11*s.* 8*d.*?

7. A general having lost two-sevenths of his men in battle, and 6 p.c. of the remainder by sickness, has 95,880 men left: how many had he at first?

8. For what sum should goods worth £532 be insured at 5 p.c. so that in case of loss the owner may recover the premium as well as the value of the goods.

[Out of every £100 insured, £5 represents premium. Thus the value of the goods is 95 p.c. of the sum to be insured.]

9. For what sum should a cargo worth £7400 be insured at $7\frac{1}{2}$ p.c. so that in case of loss both cargo and premium may be recovered?

10. A firm adds to its reserve fund in such a way that at the end of each year the fund is increased by 12 p.c. of its amount at the beginning of the year. If the fund now stands at £15,758. 8*s.*, what was it two years ago?

11. Of two routes leading from A to B one is longer than the other by $5\frac{1}{2}$ p.c. of itself. The longer route is 182·460 Km.; what is the length of the shorter to the nearest metre?

12. One year the revenue of a country was £4,780,320; the next year it was £4,957,600. Find the increase per cent. correct to three significant figures.

13. A man embarks his whole capital in four successive ventures; in the first he clears 100 p.c., and in each of the others he loses 20 p.c. Shew that he has gained 2·4 p.c. on his original capital.

14. In a certain year the total revenue was £144,000,000, and the tax on tea produced £5,790,000. What per cent. of the whole revenue, to the nearest whole number, was produced by the tea tax? (C.S.)

15. In Greece in 1893 there were 5,563,100 acres of agricultural land, and of this

Olives	occupied	432,000 acres	yielding	15,000,000 lbs.
Currants	,,	168,000 acres	,,	350,000,000 lbs.
Figs	,,	52,000 acres	,,	60,000,000 lbs.

What was the yield of each per acre, and what percentage of the whole agricultural land was occupied by each? Give the answers correct to the nearest whole number. (C.S.)

16. Our imports for the year 1900 were as shewn below. Find, to the nearest integer, what percentage of the total imports came from each of the sources named:

British possessions, - - - -	£109,530,635
United States of America, - - -	138,789,261
France, - - - - - - -	53,618,656
Other countries, - - - -	221,136,611

(C.S.)

17. The relation between the revenue and expenditure on the Baltimore and Ohio Railway for three years was as follows:

	1868.	1869.	1870.
Earnings, -	$7,558,644	$8,724,915	$8,427,728
Expenses, -	$5,054,448	$5,756,106	$5,453,460

Express the expenses as a percentage of the earnings for each year, and also for the whole period of three years; giving each percentage to the nearest integer. (C.S.)

18. The population of England and Wales was 29,002,525 in 1891, and 32,577,843 in 1901; that of Scotland was 4,025,647 in 1891, and 4,472,103 in 1901. Find which population was increasing at the greater proportional rate during 1891-1901. (C.S.)

19. The following table shews for the year 1903 the estimated population and the number of children attending primary schools in England and Wales, Scotland, and Ireland respectively. Find in each of the three cases the number of children as a percentage of the estimated population to the nearest whole number. (C.S.)

	England and Wales.	Scotland.	Ireland.
Estimated population, -	33.378,000	4,579,000	4,414,000
Number of children attending primary schools, -	5,037,000	669,000	482,000

Profit and Loss.

291. When goods are bought for one sum of money and sold for another there is a **profit** or **loss** according as the selling price is greater or less than the cost price. But in dealing with the results of different business transactions the calculation of *actual* gains or losses does not furnish any useful basis for comparison.

Consider the purchase and sale of two articles as follows:

(i) Cost price £12; selling price £16.

(ii) Cost price £20; selling price £24.

Here the actual profit is £4 in each case, but *relatively to the original outlay*, the profit is $\frac{1}{3}$ in the first case, and $\frac{1}{5}$ in the second. If we express these fractions as *percentages* we see that

in the first case the profit is $\frac{1}{3} \times 100$, or $33\frac{1}{3}$ per cent.,

„ second „ „ $\frac{1}{5} \times 100$, or 20 per cent.

It will be noticed that the profit per cent. has been calculated *with reference to the cost price*, and, unless otherwise stated, profit and loss per cent. are always to be understood in this sense.

In what follows we shall often find it convenient to abbreviate the terms 'cost price' and 'selling price' by the letters C.P. and S.P. respectively.

EXAMPLE 1. *Estimate the gain or loss per cent. when*

(i) *A horse bought for* £30 *is sold for* £28. 10*s*.

(ii) *Articles bought at* £30. 10*s*. *per score are sold at* £22. 2*s*. 6*d*. *per dozen.*

(i) Here the actual loss on £30 is £1. 10*s*.

$$\therefore \text{ loss per cent.} = \frac{1\frac{1}{2}}{30} \times 100 = 5.$$

(ii) The selling price *per score* $= £22{\cdot}125 \times \frac{20}{12}$

$$= £36{\cdot}875.$$

Hence S.P. $= £36{\cdot}875$; C.P. $= £30{\cdot}5$.

$$\therefore \text{ actual gain} = £6{\cdot}375.$$

$$\therefore \text{ gain per cent.} = \frac{6{\cdot}375}{30{\cdot}5} \times 100 = \frac{63{\cdot}75}{3{\cdot}05}$$

$$\begin{array}{r|l} & 20{\cdot}9 \\ 3{\cdot}0{,}5 & \overline{63{\cdot}75} \\ & 275 \end{array}$$

$= 20{\cdot}9$, correct to one decimal fig.

Example 2. *An article which cost £130 was sold at a profit of $7\frac{1}{2}$ per cent.; what was the selling price?*

First Method.

$$\begin{aligned} \text{S.P.} &= \text{C.P.} + 7\tfrac{1}{2}\,\% \text{ of C.P.} \\ &= 107\tfrac{1}{2}\,\% \text{ of C.P.} \\ &= \frac{107\frac{1}{2}}{100} \text{ of } £130 \\ &= £139.\ 15s. \end{aligned}$$

Second Method.

Here we add the actual gain to the cost price. Thus

$$\begin{array}{l} £ \\ 130 \\ \left.\begin{array}{l} \ \ 6{\cdot}5\ \ = 5\,\% \\ \ \ 3{\cdot}25 = 2\tfrac{1}{2}\,\% \end{array}\right\} 7\tfrac{1}{2}\,\% \\ \hline 139{\cdot}75 = £139.\ 15s. \end{array}$$

The second method should only be used in the simpler cases of finding selling price.

Example 3. *By selling a picture for £55. 4s. I lose 8 per cent.; what did the picture cost me?*

Here $\text{C.P.} - 8\,\% \text{ of C.P.} = \text{S.P.};$

that is, $\frac{92}{100} \text{ of C.P.} = \text{S.P.};$

$$\begin{aligned} \therefore\ \text{C.P.} &= £55\tfrac{1}{5} \times \frac{100}{92} \\ &= £\frac{276}{5} \times \frac{100}{92} \\ &= £60. \end{aligned}$$

Note. The percentage loss is 8 p.c. of the *cost price*; hence the *actual* loss cannot be found until the cost price is found. For this reason there is no solution analogous to the second method of Ex. 2.

EXAMPLES XVII. c.

Find the profit or loss per cent. when

1. Goods costing £30 are sold for £36.

2. .... £3. 12s. .... £3. 4s.

3. An article which cost 1s. 8d. is sold for 1s. 6d.

4. .... £2 .... £2. 5s.

5. Pictures costing 5s. 6d. each are sold at £6. 12s. per score: what is the gain or loss per cent.?

Find the selling price in the following cases:

6. Cost price £5; profit 20 p.c.

7. Cost price 6s. 8d.; loss $7\frac{1}{2}$ p.c.

8. Cost price 62 fr. 50 c.; profit 8 p.c.

9. Cost price £11. 5s.; loss 7 p.c.

10. How much per cent. is gained by buying a book-case for 15*s.* 8*d.* and selling it for 19*s.* 7*d.*?

11. If I lose 18*s.* in selling a picture for £3. 2*s.*, what is my loss per cent.?

12. A horse which cost £75 was sold at a loss of 4 per cent.; what did it sell for?

13. At what price must an article which cost 8*s.* 4*d.* be sold so as to gain 14 per cent.?

14. If $12\frac{1}{2}$ per cent. is lost by selling a book for 3*s.* 6*d.*, what was the cost price?

15. Find, correct to one decimal place, the rate per cent. of gain or loss, when

(i) Goods costing £40 are sold for £43. 7*s.* 1*d.*
(ii) .... £25. 15*s.* .... £22. 4*s.* 7*d.*
(iii) .... £77. 4*s.* 8*d.* .... £86. 17*s.* $8\frac{1}{2}$*d.*

16. What is the gain or loss per cent. on groceries retailed at $5\frac{3}{4}$*d.* per lb. and costing £2. 7*s.* 11*d.* per cwt.?

17. A woman bought eggs at 1*s.* 3*d.* per dozen; at what price per hundred must she sell them so as to gain 12 per cent.?

18. If 130 lbs. of sugar are sold for £1. 17*s.* $4\frac{1}{2}$*d.*, what is the gain or loss per cent. if the cost price was 3*d.* per lb.?

19. Knives are bought at 11 for 10*s.* and sold at 10 for 11*s.* What is the gain per cent.?

20. If eggs are bought at 5 for 3*d.* and sold at 25 for 1*s.* 6*d.*, what is the gain or loss per cent.?

21. If 8 p.c. is lost by selling an article for 7*s.* 8*d.*, what did it cost?

22. What was the cost price of an umbrella which was sold for 15*s.* at a gain of 20 p.c.?

23. By selling goods for £17. 4*s.* a profit of $7\frac{1}{2}$ p.c. is made; find the cost price.

24. If 10 p.c. was lost by selling a picture for £65. 5*s.*, what did it cost?

25. A draper sells 150 yards of cloth at 8*s.* 9*d.* per yard, gaining thereby 25 p.c. What did he pay for the whole?

26. If eggs can be sold at 9*d.* per dozen at a profit of 20 p.c., what was the cost price per score?

27. Sugar sold at $2\frac{1}{4}$*d.* per pound produced a gain of $12\frac{1}{2}$ p.c. What was the cost price per ton?

28. A man loses 15 p.c. by selling articles at £20. 8*s.* per gross; what was the cost price of each article? To what sum must he raise the selling price per gross so as to secure a profit of 20 p.c.?

29. A merchant loses $6\frac{1}{4}$ p.c. by selling half a hundredweight of tea for 5 guineas. At what price per pound did he buy it?

292. EXAMPLE 1. *By selling goods for £204 I lose 15 per cent. on my outlay; for what must I sell them so as to gain 10 per cent.?*

Here *given* S.P. = 85 % of C.P.,

required S.P. = 110 % of C.P.;

$$\therefore \text{ required S.P.} = \frac{110}{85} \text{ of given S.P.}$$

$$= £204 \times \frac{110}{85}$$

$$= £264.$$

EXAMPLE 2. *By selling sugar at £21 per ton I gain $12\frac{1}{2}$ per cent.; what percentage should I gain or lose by selling it at 3d. per pound?*

3*d.* per lb. is equivalent to $£\frac{3 \times 2240}{20 \times 12}$, or £28, per ton.

Now
$$2^{\text{nd}} \text{ S.P.} = \frac{28}{21} \text{ of } 1^{\text{st}} \text{ S.P.}$$

$$= \frac{4}{3} \text{ of } 1^{\text{st}} \text{ S.P.}$$

Also
$$1^{\text{st}} \text{ S.P.} = 112\tfrac{1}{2}\,\% \text{ of C.P.};$$

$$\therefore\ 2^{\text{nd}} \text{ S.P.} = \frac{4}{3} \text{ of } 112\tfrac{1}{2}\,\% \text{ of C.P.}$$

$$= 150\,\% \text{ of C.P.};$$

that is, there is a gain of 50 %.

293. When an article of commerce passes through the hands of several dealers, the profit or loss per cent. of each must be calculated on the price at which he bought it.

EXAMPLE. *A sells an article to B at a gain of 10 p.c.; B sells it to C at a gain of $7\frac{1}{2}$ p.c.; C disposes of it at a loss of 2 p.c. If the prime cost to the manufacturer was £6. 7s. 6d., find, to the nearest penny, the price obtained by C.*

A buys for £6·375 and sells at a gain of 10 p.c.;

$$\therefore \text{ the } cost \text{ price to } B \text{ is } £6{\cdot}375 \times \frac{110}{100}.$$

B sells it at a gain of $7\frac{1}{2}$ p.c.;

$$\therefore \text{ the } cost \text{ price to } C \text{ is } £6{\cdot}375 \times \frac{110}{100} \times \frac{107\frac{1}{2}}{100}.$$

C sells it at a loss of 2 p.c.;

∴ the price he obtains for it

$$= \pounds 6{\cdot}375 \times \frac{110}{100} \times \frac{107\frac{1}{2}}{100} \times \frac{98}{100}\,{}^{*}$$

$$= \pounds 6{\cdot}375 \times 1{\cdot}1 \times 1{\cdot}075 \times 0{\cdot}98$$

$= £7{\cdot}388$, to three dec. figs.,

$= £7.\ 7s.\ 9d.$, to the nearest penny.

$$\begin{array}{r}
\pounds \\
6{\cdot}375 \\
1{\cdot}1 \\
\hline
6{\cdot}375 \\
{\cdot}6375 \\
\hline
7{\cdot}0125 \\
1{\cdot}075 \\
\hline
7{\cdot}0125 \\
{\cdot}4908 \\
{\cdot}0351 \\
\hline
7{\cdot}5384 \\
0{\cdot}98 \\
\hline
6{\cdot}7846 \\
{\cdot}6030 \\
\hline
7{\cdot}3876
\end{array}$$

NOTE. At the stage marked *, the pupil must in each case decide for himself whether the reduction will be simpler if treated by fractions.

EXAMPLES XVII. d.

1. By selling goods for £364 I lose 9 p.c.; at what price should I sell to gain 10 p.c.?

2. A merchant sold coal at 19*s.* per ton, and thereby lost 5 p.c.; at what price ought he to have sold it so as to have gained 5 p.c.?

3. If 3 p.c. is gained by selling a piece of cloth for £1. 5*s.* 9*d.*, at what price must it be sold so as to gain 8 p.c.?

4. By selling sugar at $2\frac{1}{2}d.$ per lb. a grocer would lose 2 p.c.; at what price per cwt. could he sell it so as to gain 5 p.c.?

5. A horse was sold for £81 at a loss of 28 p.c.: if the selling price had been £90, what would have been the gain or loss per cent.?

6. What percentage of loss or gain will result from selling cloth at 7*s.* 9*d.* per yard, if 5 p.c. is gained by selling it at 8*s.* 9*d.* per yard?

7. If silk can be sold at 3*s.* 7*d.* a yard at a gain of $7\frac{1}{2}$ p.c., how much per cent. would be lost or gained by selling it at 3*s.* 2*d.* a yard?

8. A tradesman who is selling off states that he will make a reduction of 10 p.c. from the prices marked. At what price must he mark goods for which he wishes to receive 8*s.* 3*d.*?

9. A grocer bought a consignment of 200 eggs, 20 of which were broken in transit; the rest he sold at the average rate of 5 for 9*d.*, thereby clearing 8 p.c. on his outlay: at what price per score did he buy the eggs?

10. If sugar sold at $2\frac{3}{4}d.$ per lb. entails a loss of 12 p.c., what percentage of profit would be made by selling it at £1. 11*s.* 6*d.* per cwt.?

11. By selling oranges at 10*d.* per dozen a woman loses 10 p.c. of her outlay: what would she gain or lose per cent. if she sold them at 10 for a shilling?

12. At a clearance sale an article is reduced in price from 2*s.* to 1*s.* 9*d.* If the first price represents a profit of 28 p.c., how much is gained per cent. by selling at the reduced price?

13. A man bought a cottage and sold it at a loss of 5 p.c. If he had been able to sell it at a gain of 7 p.c. he would have received more than he did by £49. 16*s.* What was the cost price?

[Here *difference of selling prices* $= 12\%$ *of cost price.*]

14. If 20 p.c. more would be gained by selling a piece of land for £46. 10*s.* than by selling it for £39, what did it cost?

15. A carriage was sold at a loss of 20 p.c.; if it had fetched £10 more, there would have been a profit of 10 p.c. What was the cost of the carriage?

16. Find the prime cost of some goods which would fetch 12*s.* 6*d.* more if sold at a profit of $13\frac{1}{2}$ p.c. than if sold at a loss of $6\frac{1}{2}$ p.c.

17. A merchant lost 12 p.c. by selling coal; if he had increased his price by 3*s.* 8*d.* per ton he would have gained 10 p.c. At what price did he sell it?

18. A wine merchant was selling champagne so as to gain 20 p.c. He increased his price by 5*s.* per dozen, and his profit rose to $32\frac{1}{2}$ p.c.: at what price per dozen was he selling the wine at first?

19. It costs A £30 to make a carriage; A sells it to B at a gain of 10 per cent.; B sells it to C at a gain of 5 per cent. What does it cost C?

20. How much should I have to pay for a watch which cost £3. 4*s.* to make, if the maker sold it to a dealer at a profit of 25 per cent., and the dealer sold it to me at a profit of $12\frac{1}{2}$ per cent.?

21. A builder sells a house worth £2000 to an agent at a loss of 16 per cent. If the agent disposes of it at a gain of 25 per cent., what does the purchaser pay?

22. A sells an article to B at a profit of 4 per cent.; B sells it to C at a profit of 5 per cent. If C pays £4. 11*s.* for it, what was the prime cost to A?

[Let £x be A's prime cost; then $x \times \frac{104}{100} \times \frac{105}{100} = 4\frac{11}{20}$;

that is, $x = 4\frac{11}{20} \times \frac{100}{104} \times \frac{100}{105}$. See Note to Example, Art. 293.]

23. A sold some goods to B, making a profit of 10 per cent.; B sold them for £6. 1*s.*, making a profit of 10 per cent. What did A pay for the goods?

24. If the manufacturer makes a profit of 20 p.c., the wholesale dealer a profit of 25 p.c., and the shopkeeper a loss of 5 p.c., what did the shopkeeper get for an article which cost 3*s.* 4*d.* to make?

25. The cost of making a bicycle is £8; the maker adds on a profit of 25 p.c. to this in selling to a dealer, and the dealer sells the machine for £12. 10*s.* Find the dealer's actual profit and his profit per cent.

26. A manufacturer makes an article and sells it to a dealer at a profit of 10 p.c. The dealer sells it to a shopkeeper who obtains for it a price 21 p.c. above the cost of manufacture. What was the dealer's profit per cent.?

27. The total rise in the price of goods passing through three hands is $57\frac{1}{2}$ p.c.; if the first and second secure profits of 20 p.c. and 25 p.c. respectively, what profit per cent. does the third make?

28. *A* makes a wardrobe for £6 and sells it to *B* at a profit of 25 p.c. *B* sells it to *C*, and *C* sells it for £9. 18*s.* making a profit of 10 p.c. What profit per cent. did *B* make?

29. A manufacturer sells to an agent at a profit of 20 p.c.; the agent's wholesale price to a shopkeeper is at a profit of 10 p.c., and the shopkeeper retails his goods at a profit of $12\frac{1}{2}$ p.c. Find

(i) the wholesale price of goods which cost the manufacturer 6*s.* 3*d.*;

(ii) the retail price of an article which cost 25*s.* to make;

(iii) the cost to the manufacturer of goods bought in the shop for £14. 17*s.*;

(iv) the profit per cent. which the manufacturer would make if he sold direct to the customer at the shopkeeper's price.

294. Miscellaneous Examples on Percentages.

Example 1. *One gallon of spirit which contains* 11 *p.c. of water is added to* 3 *gallons containing* 7 *p.c. of water, and to this mixture half a gallon of water is added. Find the percentage of water in the final mixture.*

Here it will be convenient to consider each gallon as consisting of 100 parts.

1^{st} gallon has 100 parts;	89 of spirit, 11 of water.
The 3 gallons have 300 parts;	279 of spirit, 21 of water.
The added $\frac{1}{2}$ gallon has	50 of water.
By addition, 450 parts have	368 of spirit, 82 of water.

$$\text{Thus the required percentage} = \frac{82}{450} \times 100$$
$$= \frac{164}{9}$$
$$= 18\tfrac{2}{9}.$$

Example 2. *In what proportion must tea costing 2s. 9d. per lb. be mixed with tea costing 2s. 2d. per lb. in order that a gain of 20 p.c. may be made by selling the mixture at 3s. per lb.?*

$$\text{The cost price of the mixture} = 3s. \times \frac{100}{120}$$
$$= 2s.\ 6d., \text{ or } 30d.$$

Now suppose that x lbs. of the first kind of tea are taken with y lbs. of the second; then

$$\text{cost price of } (x+y) \text{ lbs. at } 30d. = 30(x+y) \text{ pence.}$$

$$\left.\begin{array}{l}\text{Also the cost price of } x \text{ lbs. at } 33d., \\ \text{together with } \quad y \text{ lbs. at } 26d.\end{array}\right\} = (33x+26y) \text{ pence.}$$

$$\therefore\ 33x+26y = 30x+30y;$$

that is, $$3x = 4y,$$

or $$\frac{x}{y} = \frac{4}{3}.$$

∴ 4 parts of the first must be taken with 3 parts of the second.

EXAMPLES XVII. e.

1. Since the last census the births in a certain district have been 11 p.c., and the deaths 4 p.c. of the whole population. If the population is now 85,600, what was it at the last census? Find also the number of births and deaths that have taken place in the interval.

2. If 8 tons of coal at 25s. per ton are mixed with 20 tons at 20s. per ton, find the gain per cent. if the whole is sold at 24s. per ton.

3. A grocer mixes chicory at 3*d.* per lb. with coffee at 2*s.* per lb. in the proportion of 4 to 17 and sells the mixture at 2*s.* 1*d.* per lb. Find his gain per cent.

4. A rising tradesman saves during each year an amount equal to 20 per cent. of his capital, and adds it to his capital at the end of the year. If on Jan. 1st, 1900, his capital was £500, what was it on Jan. 1st, 1904?

5. A shopman bought eggs at the rate of 7 for a shilling, and sold them at a profit of 40 per cent. How many eggs would a customer get for a shilling?

6. Two partners invest £1250 and £850 respectively in their business, and arrange that 60 per cent. of the profits should be divided equally between them, and the remaining profits treated as interest on the capital. If one partner's share is £30 more than that of the other, what was the whole amount of the profits?

7. A man buys a house for £1150. There is a ground rent of £25 a year, and annual repairs come, on an average, to $1\frac{1}{2}$ per cent. on the purchase price of the house. At what rent must he let the house to clear 8 per cent. per annum on the purchase price? (C.S.)

8. An examination paper is worked by 2500 pupils, of whom one-fifth are girls and the rest boys; 5 p.c. of the boys fail and 40 p.c. of the girls. What percentage of the whole passed?

9. *A* sells goods to *B* at 5 p.c. profit, *B* sells to *C* at 20 p.c. profit, and *C* sells to *D* at 25 p.c. profit. What would be *A*'s profit per cent. supposing that he sold the goods for the price *D* paid for them?

10. If 3 articles are sold for the cost price of 4, what is the profit per cent.?

11. A tradesman's prices are 12 p.c. above cost price: if he allows £1. 6*s.* 3*d.* off a bill of £21, what profit per cent. does he make?

12. A man buys an estate for £36,000. He sells one-third of it at a loss of 20 p.c., and two-fifths of it at a gain of 25 p.c.; at what gain per cent. must he sell the remainder so as to make a profit of 10 p.c. on the whole?

13. A tradesman's takings for a week are £25; what is his net profit if he marks his goods 20% above cost price and pays for rent, labour, etc. 10% of his takings? (C.S.)

14. In what proportion must coffee at 1*s.* 2*d.* per lb. be mixed with coffee at 2*s.* per lb. so that the mixture may be sold at 2*s.* 1*d.* per lb. at a profit of 25 p.c.?

15. A dairyman pays 1*s.* 1*d.* per gallon for his milk; he adds water and sells the mixture at 2*d.* per pint, thereby making 40 per cent. profit. Find the proportion of water to milk received by his customers.

16. In New South Wales, the returns for 1903 shewed that the number of sheep in the country was 26,656,000, and the value of wool produced £8,593,000. In New Zealand similar returns shewed that the number of sheep was 18,955,000 and the value of the wool exported £4,041,000. Assuming that the amount of wool produced per head of sheep was the same in the two States, calculate the percentage of the total wool-production of New Zealand which was exported. (C.S.)

17. In the year 1906 Great Britain took 90 per cent. of the farm products exported by Canada. But it was said that "for every 100 dollars' worth of farm products supplied to the mother country by Canada, other countries furnished her with about 447 dollars' worth."

Taking the value of Canada's farm products exported to Britain as 13·1 millions of pounds sterling, calculate approximately the value of the farm products exported by Canada to *all* countries; also the value of farm products imported by Britain from *all* countries. (C.S.)

18. A bicycle agent allows 25 p.c. discount on his advertised prices, and then makes a profit of 20 p.c. on his outlay. What is the advertised price of a machine on which he gains £3?

19. A man buys milk at $2\frac{1}{2}d.$ per quart, dilutes it with water and sells the mixture at $3d.$ per quart. How much water is added to each quart of milk if his profit is 60 p.c.?

20. A man sells an article at 5 p.c. profit. If he had bought it at 5 p.c. less and sold it for $1s.$ less, he would have gained 10 p.c.; find the cost price.

21. A piano was sold at a gain of 16 p.c. If the selling price had been £2 more the gain would have been 20 p.c.: what did the piano cost?

22. A merchant buys 700 quarters of wheat. He sells 280 of them at $7\frac{1}{2}$ p.c. profit, 320 at 10 p.c. profit, and the rest at $12\frac{1}{2}$ p.c. profit. His whole gain amounts to £100. $19s.$ $7d.$ At what price per quarter did he buy?

23. A grocer sells sugar at $4d.$ per lb. and takes off 5 p.c. for cash payment: find what it costs him per cwt. in order that he may make a profit of 60 p.c.?

24. In what proportion must tobacco at $5s.$ $3d.$ per lb. be mixed with another kind at $6s.$ $6d.$ per lb. so that $33\frac{1}{3}$ p.c. may be gained if the mixture is sold at $7s.$ $10d.$ per lb.

25. A cycle agent buys 30 bicycles, of which 8 are first grade and the rest second grade, for £315. Find at what price he must sell the first grade machines, so that if he sell the second grade machines at three-quarters this price, he may make a profit of 40 p.c. on his outlay.

26. There are two casks of spirit, containing 25 p.c. and 15 p.c. respectively of water. If 2 gallons of the first are added to 3 gallons of the second, what percentage of water will there be in the mixture? If a quart of water is now added, find, to the nearest integer, the percentage of spirit in the final mixture.

27. A tradesman bought some sugar at $3\frac{1}{2}d.$ per kilogram, and sold it at $2d.$ per pound. He assumed that a pound was $\frac{4}{9}$ kilogram, whereas in reality a pound is 453·6 grams. What profit per cent. on his outlay did he think he made, and what profit per cent. did he actually make? (C.S.)

28. A manufacturing firm has a capital of £9000. The gross annual sales are £3750. The cost of materials and manufacture is 35 p.c. of this. Rent, rates, and taxes amount to £387; advertising to £940; office expenses, £136; manager's salary, £200 together with 1 p.c. on the gross sales. Find the rate of interest that can be paid on the capital.

If the advertising is doubled, and in consequence the gross sales increase 40 p.c., what increase will there be in the annual profit? (C.S.)

CHAPTER XVIII.

INTEREST AND DISCOUNT.

295. WHEN money is borrowed in the course of business the lender makes a certain charge for the use of the sum lent. Such a charge is called **Interest**, and the sum lent is called the **Principal.**

Interest is usually reckoned as *a percentage on the principal* for each year until the loan is repaid. The sum paid on each £100 of the loan is called the **rate per cent. per annum.** Unless otherwise stated the interest is always calculated yearly, and the words *per annum* are omitted in quoting the rate per cent.

296. When interest is reckoned on the original principal only, throughout the whole term of the loan, it is called **Simple Interest.** When interest, as it becomes due, is used to increase the principal, the total interest at the expiration of the loan is said to be **Compound.** For the present we shall deal with simple interest only.

Principal together with its interest for a stated time is called the **Amount** for that time.

EXAMPLE. *A sum of £350 is lent for 4 years at 3 per cent. per annum: find the simple interest and the amount.*

Here the interest for *one* year = 3 p.c. of £350

$$= £350 \times \tfrac{3}{100};$$

∴ the interest for *four* years $= £350 \times \tfrac{3}{100} \times 4$

$$= £42.$$

The amount = principal + interest

$$= £350 + £42 = £392.$$

297. Thus it appears that when the *principal, rate per cent.,* and *time* are given, the **simple interest** is found by the following rule:

Multiply the principal by the rate per cent. and by the number of years, and divide the product by 100.

298. If we suppose the principal to be represented by £P, the rate per cent. by £r, the number of years by n, and the resulting interest by £I, the rule gives rise to the formula

$$\mathrm{I}=\frac{\mathrm{P}\times r\times n}{100}.$$

In performing the operations implied in this formula (or in the rule of Art. 297), the pupil must be careful to arrange the details so as to simplify the work as much as possible.

EXAMPLE 1. *Find the interest on* £1866. 13*s*. 4*d*. *for* $3\frac{3}{4}$ *years at* $2\frac{1}{2}$ *per cent.*

$$\text{Required interest} = £1866\tfrac{2}{3}\times 3\tfrac{3}{4}\times\frac{2\frac{1}{2}}{100}$$
$$= £\frac{\overset{7}{\cancel{5600}}}{\cancel{3}}\times\frac{\overset{5}{\cancel{15}}}{\cancel{4}}\times\frac{5}{\cancel{200}}$$
$$= £175.$$

This fractional arrangement should not be used except in the simplest cases, and *never unless the principal can easily be expressed in a fractional form.* It is, however, always worth while to see if a simplification can be obtained by *first multiplying together the number of years and the rate per cent.*

EXAMPLE 2. *Find, to the nearest penny, the interest on* £315. 12*s*. *for* 4 *years* 7 *months at* $2\frac{2}{5}$ *per cent.*

Here *the number of years × rate per cent.* $=4\frac{7}{12}\times 2\frac{2}{5}$

$$=\frac{55}{12}\times\frac{12}{5}=11.$$

Thus we have only to divide the principal by 100 and multiply the result by 11.

$$\text{Hence the required interest} = \frac{£315{\cdot}6}{100}\times 11$$
$$= £3{\cdot}156\times 11$$
$$= £34{\cdot}716$$
$$= £34.\ 14s.\ 4d.\text{, to the nearest penny.}$$

299. It should be noted that a penny is the coin of least value recognised in banking and similar business; it is therefore usually sufficient to obtain a result true to the nearest penny. Hence the best *general method* of finding simple interest is that shewn in the following example in which the principal is decimalised as the first step of the work.

EXAMPLE. *Find, to the nearest penny, the simple interest on £227. 10s. 6d. for 5 years at $3\frac{1}{2}$ per cent.*

Here the principal = £227·525.

$$\begin{array}{r|l} £ & \\ 2{\cdot}275 & 25 \\ & 5 \\ \hline 11{\cdot}376 & 3 \\ & 3\frac{1}{2} \\ \hline 34{\cdot}128 & 9 \\ \frac{1}{2}\quad 5{\cdot}688 & 1 \\ \hline 39{\cdot}817 & \end{array}$$

We divide *first* by 100, then multiply successively by 5 and by $3\frac{1}{2}$, keeping 4 decimal figures throughout, and a final result correct to 3 decimal figures.

Thus required interest = £39·817 = £39. 16s. 4d.

EXAMPLES XVIII. a.

Find the Simple Interest on

1. £200 for 3 yrs. at 6 %. **2.** £450 for 4 yrs. at 5 %.
3. £2000 ,, 6 ,, $3\frac{1}{2}$ %. **4.** £240 ,, 7 ,, 5 %.
5. £540 ,, $2\frac{1}{2}$,, 3 %. **6.** £3375 ,, 4 ,, 3 %.
7. £112. 10s. ,, 4 ,, $3\frac{1}{2}$ %. **8.** £318. 15s. ,, 3 ,, 4 %.
9. £1600 ,, $3\frac{1}{4}$,, $4\frac{1}{2}$ %. **10.** £885. 15s. ,, $2\frac{2}{3}$,, 5 %.

11. £783. 6s. 8d. for 2 years 3 months at $6\frac{2}{3}$ %.

12. £600 for 1 year 5 months at $4\frac{1}{2}$ %.

13. £2526. 5s. for $1\frac{2}{3}$ years at $1\frac{4}{5}$ %.

14. £407. 10s. for 3 years 9 months at $3\frac{1}{5}$ %.

15. £2022. 11s. 3d. for 2 years 8 months at $7\frac{1}{2}$ %.

Find, to the nearest penny, the Simple Interest on

16. £457 for 3 yrs. at $4\frac{1}{2}$ %. **17.** £5127 for $3\frac{1}{2}$ yrs. at 3 %.
18. £641 ,, $3\frac{1}{4}$,, $2\frac{1}{8}$ %. **19.** £3712 ,, $2\frac{1}{2}$,, $3\frac{3}{8}$ %.

20. £761. 11s. 1d. for 6 years 8 months at $4\frac{1}{2}$ %.

21. £408. 2s. 5d. ,, 6 months ,, 4 %.

Find, to the nearest penny, the Amount at Simple Interest on

22. £156. 14s. 6d. for $1\frac{3}{4}$ yrs. at $3\frac{1}{2}$ %.

23. £244. 1s. 8d. ,, 3 yrs. 9 mos. ,, 3 %.

24. £1842. 14s. 5d. ,, 2 yrs. 8 mos. ,, $2\frac{1}{4}$ %.

25. £1147. 2s. 9d. ,, 5 yrs. 4 mos. ,, $2\frac{1}{4}$ %.

26. Find the quarterly interest on £1240 at $3\frac{1}{3}$ %.

27. Find to the nearest cent the half-yearly interest on $548,40 at $4\frac{1}{2}$ % per annum.

28. A sum of £437. 10*s*. was lent at 4 p.c. simple interest; what sum must be paid to cancel the debt at the end of 8 months?

29. A man deposits 6280 fr. in a bank which pays him interest at $2\frac{1}{4}$ p.c. Find what sum he may draw from the bank at the end of half a year.

300. When it is necessary to calculate interest for a number of days, the time must be expressed as a fraction of a year. In estimating the time between the two specified dates it is customary to include one only of the dates named. Thus from April 5th to August 10th would be

$$\overset{\text{April}}{(30-5)} + \overset{\text{May}}{31} + \overset{\text{June}}{30} + \overset{\text{July}}{31} + \overset{\text{Aug.}}{10}, \quad \text{or } 127 \text{ days.}$$

A fraction of a year, with denominator 365, can only be put in a simpler form when the number of days is a multiple of 5 or 73. For example, 146 days is $\frac{2}{5}$ of a year. In other cases it is best to proceed as follows.

Example. *Find, to the nearest penny, the interest on* £1208. 7*s*. 10*d*. *at* 4 *per cent. from April 5th to August 10th.*

The principal = £1208·392, and the number of days is 127.

$$\begin{aligned}
\text{The interest } &= £1208{\cdot}392 \times \frac{4}{100} \times \frac{127}{365} \\
&= £1208{\cdot}392 \times \frac{4}{100} \times \frac{127 \times 2}{730} \\
&= £1208{\cdot}392 \times \frac{1016}{73000} \\
&= £1208{\cdot}392 \times \frac{1{\cdot}016}{73} \\
&= £16{\cdot}818, \text{ to 3 dec. figs.,} \\
&= £16.\ 16s.\ 4d., \text{ to the nearest penny.}
\end{aligned}$$

$$\begin{array}{rr|l}
 & £ & \\
 & 1208{\cdot}392 & \\
 & 1{\cdot}016 & \\
\hline
 & 1208{\cdot}392 & \\
 & 12{\cdot}083 & 9 \\
 & 7{\cdot}250 & 3 \\
\hline
73\,) & 1227{\cdot}726 & (\,16{\cdot}818 \\
 & 497 & \\
 & 597 & \\
 & 132 & \\
 & 596 &
\end{array}$$

Note. In adopting this arrangement we shall always have to divide by 73 (or multiply by $\frac{1}{73}$) as a final step. This may very conveniently be done by the following rule, known as the **third, tenth, and tenth rule**.

To the number of pounds to be divided add one-third of this number, one-tenth of the quotient so obtained, and again one-tenth of the last result; then divide the sum by 100 (*i.e. move the decimal point two places to the left*), *and deduct* $\frac{1}{10,000}$ *of the resulting number.*

Applying this rule to the above example, we have

$$\begin{array}{r r}
 & \pounds \\
 & 1227{\cdot}726 \\
\frac{1}{3} = & 409{\cdot}242 \\
\frac{1}{10} \text{ of } \frac{1}{3} = & 40{\cdot}924 \\
\frac{1}{10} \text{ of } \frac{1}{10} \text{ of } \frac{1}{3} = & 4{\cdot}092 \\
\cline{2-2}
 & 1681{\cdot}984
\end{array}$$

$$\begin{array}{r r}
 & \pounds \\
\text{Moving the decimal point 2 places to the left we get} & 16{\cdot}81984 \\
\text{deduct } \frac{1}{10{,}000} \text{ of this result} & {\cdot}00168 \\
\cline{2-2}
 & 16{\cdot}81816
\end{array}$$

Thus to three places of decimals the result is £16·818, as before.

The *reason for the rule* may be thus explained.

By division, $\frac{1}{73} = {\cdot}01369863\ldots$.

Now $1 + \frac{1}{3} + \left(\frac{1}{10} \text{ of } \frac{1}{3}\right) + \frac{1}{10} \text{ of } \left(\frac{1}{10} \text{ of } \frac{1}{3}\right) = 1 + {\cdot}33\frac{1}{3} + {\cdot}03\frac{1}{3} + {\cdot}00\frac{1}{3}$
$= 1 + {\cdot}37 = 1{\cdot}37.$

$$\begin{array}{r r}
\text{Moving the decimal point 2 places to the left we get} & {\cdot}0137 \\
\text{deduct } \frac{1}{10{,}000} \text{ of this result} & {\cdot}00000137 \\
\cline{2-2}
 & {\cdot}01369863
\end{array}$$

which is the equivalent of $\frac{1}{73}$.

301. Multiplication by aliquot parts may sometimes be used with advantage.

EXAMPLE. *Find the Interest on* £4536. 15*s. for* 219 *days at* $4\frac{11}{16}$ *per cent.*

$$219 \text{ days} = \frac{219}{365} = \frac{3 \times 73}{5 \times 73} = \frac{3}{5} \text{ of 1 year.}$$

And *number of years × rate per cent.* $= \frac{3}{5} \times \frac{75}{16} = 2\frac{13}{16}$.

Thus to find the simple interest, decimalize the principal, divide it by 100, and multiply the result by $2\frac{13}{16}$.

$$\begin{array}{r|r|l}
 & \pounds & \\
 & 45{\cdot}367 & 5 \\
 & & 2\frac{13}{16} \\
\hline
 & 90{\cdot}735 & 0 \\
\frac{1}{2} & 22{\cdot}683 & 7 \\
\frac{1}{4} = \frac{1}{2} \text{ of } \frac{1}{2} & 11{\cdot}341 & 8 \\
\frac{1}{16} = \frac{1}{4} \text{ of } \frac{1}{4} & 2{\cdot}835 & 4 \\
\hline
 & 127{\cdot}596 &
\end{array}
\qquad
\begin{aligned}
\frac{13}{16} &= \frac{8+4+1}{16} \\
&= \frac{1}{2} + \frac{1}{4} + \frac{1}{16}.
\end{aligned}$$

Thus required interest = £127·596 = £127. 11*s.* 11*d.*

EXAMPLES XVIII. b.

Find the Amount at Simple Interest of

1. £750 for 146 days at $3\frac{3}{4}$ per cent.
2. £1835 ,, 292 ,, $1\frac{1}{4}$,,
3. £1687. 10*s.* ,, 219 ,, 6 ,,
4. £252. 1*s.* 8*d.* ,, 73 ,, 3 ,,
5. £1606 ,, 400 ,, $5\frac{1}{2}$,,

Find, to the nearest penny, the Amount at Simple Interest of

6. £820. 4*s.* 2*d.* for 2 yrs. 146 days at $2\frac{1}{2}$ per cent.
7. £426. 10*s.* from March 4th to July 28th at 3 per cent.
8. £219 ,, Jan. 1st ,, July 16th ,, $3\frac{1}{3}$,,
9. £375. 12*s.* ,, May 6th ,, Dec. 11th ,, $4\frac{3}{8}$,,
10. £780. 16*s.* ,, April 7th ,, Aug. 31st ,, $3\frac{7}{16}$,,

[In Examples 11–23 use one of the methods of Art. 300.]
Find, to the nearest penny, the Simple Interest on

11. £316 for 33 days at 5 per cent.
12. £547 ,, 54 ,, $2\frac{1}{2}$,,
13. £7000 ,, 86 ,, 4 ,,
14. £708 ,, 113 ,, $3\frac{1}{2}$,,
15. £965. 14*s.* 7*d.* ,, 98 ,, 4 ,,

Find, to the nearest penny, the Amount on

16. £413 from March 4th to May 8th at 4 per cent.
17. £510 ,, June 4th ,, Aug. 23rd ,, 3 ,,
18. £2450 ,, April 17th ,, July 27th ,, $3\frac{1}{2}$,,
19. £1368. 15*s.* ,, May 17th ,, Dec. 15th ,, $4\frac{3}{4}$,,
20. £402. 10*s.* 9*d.* ,, April 14th ,, July 31st ,, $4\frac{1}{8}$,,

21. A sum of £1250 was borrowed on April 1st and repaid on Aug. 25th of the same year with interest at $3\frac{3}{4}$ p.c. What sum had to be paid?

22. A man deposits £500 at his bank on July 5th and draws the interest at $2\frac{1}{2}$ p.c. on Nov. 18th. At the same time he withdraws £200 of the deposit; the remaining sum with interest he draws on Feb. 20th of the next year. What sums does he receive on these dates respectively?

23. I borrow £350 on May 1st and a further sum of £270 on Aug. 1st on the understanding that I repay the whole with interest at $3\frac{3}{4}$ p.c. at the end of the year. How much do I owe on Dec. 31st?

302. Inverse cases of Simple Interest. In questions on Interest we are concerned with four quantities, viz. *principal*, *rate per cent.*, *time*, and *interest*. When any three of these are given it is possible to find the fourth. Questions in which the interest is among the given quantities, and it is required to find the *principal*, or *rate*, or *time* are called **Inverse.**

CASE I. *When the Principal is required.*

EXAMPLE. *What principal will produce £114. 3s. 9d. as simple interest at $3\frac{1}{2}$ per cent. for $7\frac{1}{4}$ years?*

Let P pounds denote the required principal;

then
$$\frac{\mathsf{P}\times3\frac{1}{2}\times7\frac{1}{4}}{100}=114\frac{3\frac{3}{4}}{20};$$

that is,
$$\mathsf{P}\times\frac{7}{2}\times\frac{29}{4}\times\frac{1}{100}=114\tfrac{3}{16};$$

$$\therefore\ \mathsf{P}=\frac{\overset{9}{\cancel{1827}}}{\underset{2}{\cancel{16}}}\times100\times\frac{2}{\cancel{7}}\times\frac{\cancel{4}}{\cancel{29}}$$
$$=450.$$

Thus the required principal is £450.

Or thus: Interest on P pounds = (Int. on **one** pound) × P.

$$\therefore \text{ number of pounds in req}^{\text{d}}\text{ principal}=\frac{\text{given Int.}}{\text{Int. on £1}};$$

that is,
$$\mathsf{P}=\frac{1827}{16}\div\left(\frac{3\frac{1}{2}\times7\frac{1}{4}}{100}\right)$$
$$=450, \text{ as above.}$$

EXAMPLE 2. *Find, to the nearest pound, what principal will amount to £3521. 8s. at $2\frac{1}{2}$ per cent. in 3 years.*

Let P pounds denote the required principal;

then
$$\mathsf{P}+\frac{\mathsf{P}\times2\frac{1}{2}\times3}{100}=3521\cdot4,$$
$$\mathsf{P}\left(1+\frac{15}{200}\right)=3521\cdot4,$$
$$\mathsf{P}\times1\cdot075=3521\cdot4;$$
$$\therefore\ \mathsf{P}=3275\cdot7.$$

Thus the reqd principal = £3276.

$$\begin{array}{r}
3275\cdot7 \\
1\cdot0\overset{,}{7}\overset{,}{5}\,)\,\overline{3521\cdot4} \\
\underline{2964} \\
\underline{814} \\
\underline{61} \\
7
\end{array}$$

Here we require five figures in the quotient, and the first two are obtained before contracting.

Or thus : Amount of P pounds = (Amt. of **one** pound) × P.

$$\therefore \text{ number of pounds in req}^{\text{d}} \text{ principal} = \frac{\text{given Amt.}}{\text{Amt. of £1}}.$$

Here given Amt. = £3521·4 ; Amt. of £1 = £1·075 ;

$$\therefore\ \mathsf{P} = \frac{3521 \cdot 4}{1 \cdot 075} = 3275 \cdot 7, \text{ as before.}$$

EXAMPLES XVIII. c.

On what Principal will the Simple Interest be

1. £60 in 5 yrs. at 4%? **2.** £90 in 4 yrs. at 5%?
3. £243 ,, 3 ,, 3%? **4.** £37. 10*s.* ,, 5 ,, 3%?

Find the Principal which will produce

5. £15. 4*s.* as interest in 3 years at 4 per cent.
6. £28. 17*s.* 6*d.* ,, ,, $3\frac{1}{2}$,, 6 ,,
7. £44. 6*s.* 8*d.* ,, ,, 4 ,, $2\frac{1}{2}$,,
8. £3. 7*s.* 6*d.* ,, ,, 9 months at 6 per cent.

What Principal will amount to

9. £1417 in 5 yrs. at 6%? **10.** £912 in 4 yrs. at $3\frac{1}{2}$%?
11. £862. 10*s.* ,, 3 ,, 5%? **12.** £899. 10*s.* ,, 6 ,, $4\frac{3}{4}$%?

Find the Principal which will amount to

13. £5430. 7*s.* 6*d.* in 3 months at $2\frac{1}{4}$ per cent.
14. £527. 8*s.* 9*d.* ,, $3\frac{1}{2}$ years ,, $2\frac{1}{2}$,,
15. £204. 16*s.* ,, 146 days ,, 6 ,,
16. £254. 1*s.* 2*d.* ,, $2\frac{1}{2}$ years ,, 3 ,,

17. Find, to the nearest pound, the principal which will give £152 as interest in 5 years at $2\frac{1}{4}$ per cent.

18. On what sum, to the nearest pound, will the interest for $3\frac{1}{2}$ years at 3 per cent. be £24. 18*s.*?

19. Find, to the nearest pound, the sum which must be placed for 8 months at $2\frac{1}{2}$ per cent. to produce as interest £17. 18*s.* 9*d.*

20. Find, to the nearest shilling, the sum which in 1 year 9 months at 3 per cent. will amount to £327. 2*s.* 4*d.*

21. Find, to the nearest pound, what sum a man must deposit at his Bank on his son's 16th birthday, in order that his son on coming of age may be entitled to £2000, simple interest being reckoned at $2\frac{1}{2}$ per cent.

303. CASE II. *When the time is required.*

EXAMPLE 1. *In how many years will* £1250. 12*s.* 6*d.* *amount to* £1375. 13*s.* 9*d.* *at* 4 *per cent.?*

Here the interest = £1375. 13*s.* 9*d.* − £1250. 12*s.* 6*d.*
= £125. 1*s.* 3*d.*

Let the number of years be denoted by n,

then
$$1250\tfrac{5}{8} \times \frac{4}{100} \times n = 125\tfrac{1}{16};$$

$$\therefore\ n = 125\tfrac{1}{16} \div \left(1250\tfrac{5}{8} \times \frac{4}{100}\right)$$

$$= \frac{\cancel{2001}}{\underset{2}{\cancel{16}}} \times \frac{\cancel{8}}{\underset{5}{\cancel{10005}}} \times \overset{5}{\cancel{25}}$$

$$= 2\tfrac{1}{2}.$$

Or thus: Interest for n years = (Int. for **one** year) × n;

$$\therefore\ n = \frac{\text{given Int.}}{\text{Int. for } \textit{one year}},$$

which leads to the same work as before.

EXAMPLE 2. *In how many days will* £2406 *amount to* £2421. 2*s.* *at* $3\frac{1}{2}$ *per cent.?*

Here interest at $3\frac{1}{2}$% for the required fraction of a year
= £2421. 2*s.* − £2406
= £15. 2*s.*
= £15·1.

Interest on £2406 for 1 year $= £2406 \times \dfrac{3\frac{1}{2}}{100}$
= £84·21 ....(i);

$\therefore$ req^{d} fraction of a year $= \dfrac{15{\cdot}1}{84{\cdot}21} = 0{\cdot}179\ldots$ ....(ii).

And 0·179 of a year = 0·179 of 365 days = 65·3 days ......(iii).

This, counting a fraction of a day as a whole, gives 66 days.

The details of work are as follows:

(i)	(ii)	(iii)
£	0·1793	0·179 × 365
24·06	8·4,2,1) 1·5100	= 36·5 × 1·79
$3\frac{1}{2}$	66790	36·5
72·18	7843	1·79
$\frac{1}{2}$ | 12·03	264	36·5
84·21	11	25·5 | 5
		3·2 | 9
		65·3

304. CASE III. *When the rate per cent. is required.*

EXAMPLE. *At what rate per cent. will the simple interest on* £422. 10*s.* *for* 4 *years be* £50. 14*s.?*

Let the required rate per cent. be denoted by r pounds;

then
$$422{\cdot}5 \times \frac{r}{100} \times 4 = 50{\cdot}7\,;$$
$$\therefore\ r \times \frac{422{\cdot}5}{25} = 50{\cdot}7,$$
or
$$r = 50{\cdot}7 \times \frac{25}{422{\cdot}5}$$
$$= 3.$$

Thus the req$^{\text{d}}$ rate per cent. is 3.

Or thus: Interest at r per cent. = (Int. at **one** per cent.) × r.

$$\therefore \text{ required rate per cent.} = \frac{\text{given Int.}}{\text{Int. at } \textit{one per cent.}}$$
$$= 50{\cdot}7 \div \left(422{\cdot}5 \times \tfrac{4}{100}\right),$$

which leads to the same work as before.

EXAMPLES XVIII. c. (*Continued.*)

For what time would the Simple Interest on

22. £750 be £375 at 5 per cent. per annum?
23. £1050 ,, £210 ,, 4 ,, ,,
24. £1860 ,, £558 ,, $3\frac{3}{4}$,, ,,
25. £666. 13*s.* 4*d.* ,, £67. 10*s.* ,, $2\frac{1}{4}$,, ,,
26. £3756 ,, £633. 16*s.* 6*d.* ,, $4\frac{1}{2}$,, ,,

In what time will the Amount on

27. £1500 at 3 per cent. be £1680?
28. £5400 ,, $2\frac{1}{4}$,, £6372?
29. £225 ,, $3\frac{1}{2}$,, £256. 10*s.*?
30. £146. 13*s.* 4*d.* ,, $4\frac{1}{2}$,, £163. 3*s.* 4*d.*?

31. In how many days will £250 produce £3. 10*s.* as simple interest at 5 per cent.?

32. In how many days will the interest on £1572. 17*s.* come to £23. 14*s.* at $5\frac{1}{2}$ per cent.?

33. In how many days will £382 amount to £386. 13*s.* 6*d.* at 4 per cent.?

At what rate per cent. would the Simple Interest on

34. £250 be £37. 10*s*. for 5 years?
35. £375 ,, £90 ,, 4 ,,
36. £318. 15*s*. ,, £44. 12*s*. 6*d*. ,, 4 ,,
37. £168. 13*s*. 4*d*. ,, £18. 19*s*. 6*d*. ,, $2\frac{1}{4}$,,

At what rate per cent. will the Amount of

38. £2500 for $5\frac{1}{2}$ years be £3118. 15*s*.?
39. £485 ,, $2\frac{1}{2}$,, £527. 8*s*. 9*d*.?
40. £320. 15*s*. ,, 5 years 4 months be £363. 10*s*. 4*d*.?

41. If the interest on £650 for 5 months is £12. 3*s*. 9*d*., what is the rate per cent.?

42. The sum of £437. 10*s*. was lent at simple interest, and at the end of 8 months the debt was cancelled by the payment of £449. 3*s*. 4*d*. What was the rate of interest?

43. At what rate per cent. will a sum of money treble itself at simple interest in 25 years?

44. *A* lends *B* a sum of £564, and at the end of $2\frac{1}{2}$ years *B* cancels the debt by a payment of £614: find, correct to two decimal places, what rate per cent. *A* gets for his money.

45. A man lends £3550 on the understanding that he is to receive £100 as interest at the end of 5 months; find, correct to two decimal places, the rate of interest per cent. per annum that the borrower has to pay.

46. What sum, correct to the nearest pound, will amount to £2519. 18*s*. in $4\frac{1}{2}$ years at 5 per cent.?

47. Find, correct to the nearest shilling, what sum must be put out at $2\frac{3}{4}$ per cent. simple interest to give an income of (i) half-a-crown a week; (ii) a penny a day.

48. A man leaves £30. 10*s*. on deposit at 5 per cent., and takes his interest when it has amounted to £1. For how many days was the sum on deposit?

49. A man places £1000 on deposit at 3 per cent., and gives his son the interest on his birthday which falls on June 15th; if the present amounts to £12. 18*s*. 1*d*., on what day was the deposit made?

50. A man makes his wife a birthday present, giving her each year a number of pounds equal to the number of years in her age. If her birthday falls on Aug. 8th, what sum must he place at interest at 4 per cent. on Jan. 1st before she is 36, in order to raise the required sum?

Discount.

305. **Discount** is a sum of money deducted from a debt in consideration of the debt being paid before it is legally due.

Suppose *A* owes *B* a sum of £102, payment being due in six months' time, and that the use of money meantime is valued at 4 per cent. Since at this rate £100 would amount to £102 in 6 months, *A* can equitably discharge his debt to *B* by paying him £100 at once instead of £102 at the end of half-a-year. In this case £100 is called the **Present Value** of £102, and £2 is called the **True Discount** on £102 for six months at 4 per cent. per annum.

In such a case the Present Value of a debt is that principal sum which with its interest for the given time would amount to the sum due, and the true Discount on the *debt* is the same as the Simple Interest on the *Present Value.*

Or briefly :

impt (i) Present Value + Interest on P.V. = Sum due.
(ii) Interest on Present Value = Discount on Sum due.

306. The above reasoning gives the *true* discount, namely the sum which should theoretically be deducted when a debt is paid before it is due. In practice, however, discount is always calculated as *a percentage on the sum due.* In commercial and banking transactions discount always has this latter meaning.

Thus in the case discussed in the last article the practical discount on a debt of £102 due in six months at 4 per cent. would be $\frac{1}{2}$ of 4 p.c. of £102, or £2·04.

NOTE. We shall not give any examples on *true* discount and present value. If the pupil meets with such questions they may all be regarded as illustrations of inverse cases of simple interest. It is only necessary to substitute the words *principal* and *amount* for *present value* and *sum due* respectively, and to remember that the *true discount* on the sum due is the *interest* on the present value.

EXAMPLE 1. *Find to the nearest penny the discount on a sum of* £1208. 7*s.* 10*d. due after* 127 *days at* 4 *per cent.*

Here the required discount is 4 per cent. of £1208. 7*s.* 10*d.* for 127 days. Thus the work is the same as that given in Art. 300.

EXAMPLE 2. *An account after* $2\frac{1}{2}$ *per cent. had been taken off became* £16. 14*s.* 9*d.* *What was the sum due before the discount was deducted?*

The element of time does not enter into a question of this kind. The tradesman simply deducts $2\frac{1}{2}$ % of the sum due to him and accepts £16. 14*s.* 9*d.*

Here £16. 14*s*. 9*d*. = £16·738 to 3 decimal places.

Let the sum due be denoted by x pounds;

then
$$x - x \times \frac{2\frac{1}{2}}{100} = 16{\cdot}738\,;$$
$$\therefore\ x(1 - {\cdot}025) = 16{\cdot}738,$$
that is,
$$x = \frac{16{\cdot}738}{{\cdot}975} = 17{\cdot}167\ldots.$$
$\therefore$ the sum due is £17. 3*s*. 4*d*.

NOTE. In practice a tradesman deducts 1*s*. in the £1 for 5 %, 6*d*. in the £1 for 2½ %, and so on. Moreover, unless the number of shillings comes to 10 or more, usually the pounds only are considered. In the present case the actual discount allowed would probably be 8*s*. 6*d*. (ignoring 3*s*. 4*d*. in the bill), and the sum accepted in payment £16. 14*s*. 10*d*.

EXAMPLES XVIII. d.

(*When not exact the answers are to be given to the nearest penny.*)

Find the Discount on

1.	£2020	due in 3 months at	4 %.
2.	£6331	,, 7 ,,	2½ %.
✓ 3.	£694. 3*s*. 4*d*.	,, 11 ,,	4½ %.
4.	£531. 6*s*.	,, 146 days at	3 %.
✓ 5.	£367. 18*s*. 9*d*.	,, 219 ,,	2½ %.
6.	£890	,, 10 months at	7 %.
✓ 7.	£729. 17*s*. 4*d*.	,, 8 ,,	2½ %.
8.	£531. 4*s*. 4*d*.	,, 11 ,,	3 %.
✓ 9.	£762	,, 176 days at	4 %.
10.	£205. 7*s*.	,, 87 ,,	3 %.

How much cash must be paid to settle the following accounts? [In Examples 11–14 ignore anything less than 10*s*.]

✓11. £25. 13*s*. 8*d*. (Discount 10 %). ✓12. £14. 3*s*. 2*d*. (Discount 5 %).

✓13. £41. 11*s*. (Discount 5 %). ✓14. £38. 4*s*. 8*d*. (Discount 2½ %).

✓15. After 5 % had been deducted from a bill it came to £26. 7*s*. 3*d*. What was the amount before discount was deducted?

✓16. If my tailor allows me 10 % discount for ready money and I pay him £16. 13*s*., what was the amount of my bill before discount was deducted?

307. A **Bill of Exchange** is a written order by which one person demands payment from another of a certain sum by a specified date.

The procedure may be illustrated by the following instance.

A British manufacturer A. B. acting on instructions from a Calcutta merchant L. M., ships to the latter a consignment of goods worth £750. At the same time A. B. sends L. M. a *Bill of Exchange* or demand for payment in the following form:

£750. *Manchester, Oct.* 22*nd*, 1907.

Three months after date pay to me or my order, the sum of seven hundred and fifty pounds, value received.

To L. M. (*Signed*) *A. B.*
Calcutta.

On receiving the goods the Calcutta merchant accepts the liability stated on the Bill by writing across it "accepted" with his signature. The accepted bill is then sent back to A. B., who may now proceed in either of two ways.

(i) He may retain the Bill until it *matures*, that is, until the date when payment is due from L. M., when he will obtain the money through the London Agents of L. M.'s Calcutta Bank. Or (ii) if A. B. should need ready money at some earlier date, he may get the bill *discounted* by his Banker or Bill-broker X. Y.; that is to say, he may sell the bill to X. Y. who will give him in cash its *present value* calculated at the current rate of discount from the *date of discounting* to the *date on which the sum falls due*. Theoretically, the present value should be reckoned by the method of *true* discount, but in practice the Banker deducts *commercial* discount, or simple interest on the sum due.

In this example A. B. is called the **drawer** of the bill, and L. M. the **drawee.** The sum mentioned on the bill and ultimately paid by L. M. to X. Y. is called the **face-value.**

After a bill is nominally due it is customary to allow *three extra days*, called **days of grace**, before the bill is legally due.

308. A **Promissory Note** differs from a Bill of Exchange only in the fact that it is drawn up by the debtor instead of the creditor. Thus in the case already illustrated L. M. might send a **Promissory Note** to A. B., in which he makes a formal promise to pay £750 to A. B. at the date named in the note.

EXAMPLE. *Find, to the nearest penny, the discount on a Bill for* £650 *drawn on April* 15*th*, *at three months, and discounted on May* 19*th* *at* 4 *per cent.*

The bill becomes nominally due on July 15th and legally (allowing for 3 days of grace) on July 18th. The question, therefore, is this:

What discount should be deducted from a debt of £650, due on July 18th, if payment is made on May 19th?

The number of days is $(31-19)+30+18=60$.

$\therefore$ required discount $=4\,\%$ of £650 for 60 days

$$\begin{aligned} &= £650 \times \frac{4}{100} \times \frac{60}{365} \\ &= £6{\cdot}5 \times \frac{48}{73} \\ &= £4{\cdot}274 \\ &= £4.\ 5s.\ 6d. \end{aligned}$$

$$\begin{array}{r} 65 \\ \underline{4{\cdot}8} \\ 260 \\ \underline{52{\cdot}0} \\ 7{,}3\,)\,\underline{312}\ \ (4{\cdot}274 \\ \underline{200} \\ \underline{540} \\ 29 \end{array}$$

Hence the cash obtained on discounting the bill would be

£650 – £4. 5*s*. 6*d*., or £645. 14*s*. 6*d*.

EXAMPLES XVIII. e.

(*Answers to be given to the nearest penny.*)

Find the Discount on the following Bills:

1. £603 drawn on Oct. 4th at 4 months and discounted on Nov. 26th at $2\frac{1}{2}\,\%$.

2. £952. 15*s*. drawn on March 9th at 8 months and discounted on April 7th at $5\,\%$.

3. £381 drawn on April 5th at 7 months and discounted on Aug. 1st at $4\,\%$.

4. £486 drawn on May 18th at 5 months and discounted on July 30th at $3\,\%$.

5. £2763. 5*s*. drawn on April 15th at 6 months and discounted on Aug. 18th at $2\frac{1}{2}\,\%$.

How much cash would be obtained on discounting the following Bills?

6. £317 drawn on Jan. 10th at 9 months, and discounted on May 12th at $2\frac{1}{2}\,\%$.

7. £1934. 4*s*. drawn on Feb. 20th at 12 months, and discounted on May 7th at $2\frac{1}{4}\,\%$.

8. £3675. 11*s*. drawn on May 3rd at 6 months, and discounted on Aug. 15th at $3\frac{1}{2}\,\%$.

9. A bill for £745. 18*s*. 4*d*. drawn on June 8th at 6 months is discounted on Aug. 12th at $3\frac{1}{2}\,\%$. What is the discounted value of the bill?

10. How much cash can I obtain by discounting on Nov. 7th a four-months' bill for £1031. 12*s*. 10*d*., drawn on Aug. 15th, at $6\,\%$?

11. The discount on £505 due in 3 months is £5. 1*s*. What is the discount rate?

12. A man receives a four-months' bill for £270; if he can immediately discount it for £267. 15*s.*, what is the discount rate?

13. If at 5 per cent. £168. 8*s.* 9*d.* is accepted as present payment for a bill of £175, how long has the bill to run?

14. In how many months' time must a debt of £960 be due if it may be cancelled by an immediate payment of £947. 4*s.*, the current rate of discount being 4 %?

15. What rate per cent. does a man get for his money when in discounting a bill due in 10 months he deducts as discount 4 % of the amount of the bill?

16. If a three-months' bill can be discounted at $2\frac{1}{2}$ %, what rate of interest (correct to two decimal places) does the bill-broker get for his money?

Compound Interest.

309. Money is said to be put out at **Compound Interest** when each instalment of interest as it becomes due is added to the principal instead of being paid over to the lender of the principal sum. In this case the principal is continually being increased, and the interest for each period is the interest on the *Amount* at the end of the preceding period.

Thus, if £100 is put out at 5 per cent. compound interest, at the end of one year the amount is £105; this is the principal for the second year, and the interest on it is found to be £5. 5*s.*; thus the principal for the third year is £110. 5*s.*, and so on. It follows that under this system the complete Compound Interest is the sum of the interests for the several periods; this is most conveniently obtained by subtracting the original *principal* from the final *amount*. Unless otherwise stated, interest is supposed to be payable *yearly*.

EXAMPLE 1. *Find, to the nearest penny, the Compound Interest on £550 for 2 years at 4 per cent.*

	£
1st principal	550
1st year's int.	22·00
2nd principal	572·
2nd year's int.	22·88
Amount in 2 years	594·88
Original principal	550
Interest required	44·88

£44·88 = £44. 17*s.* 7*d.*,
to the nearest penny.

Write down the principal; to find the 1st year's interest multiply the principal by $\frac{4}{100}$; that is, multiply by 4 and set down each figure *two places to the right*, the position of the decimal point remaining fixed. Each year's principal is treated in the same way. Lastly, the original principal is subtracted from the final amount.

Example 2. *Find, to the nearest penny, the Amount at Compound Interest of £721. 11s. 7d. for 3 years at 3 per cent.*

	£	
1st principal	721·57$\overset{*}{9}$	17
1st year's int.	21·647	37
2nd principal	743·226	54
2nd year's int.	22·296	80
3rd principal	765·523	34
3rd year's int.	22·965	70
Amt. in 3 years	788·489	

First decimalize the principal correct to *five* places, drawing a vertical line after the 3rd decimal figure. Then multiply the principal by $\frac{3}{100}$, *beginning the multiplication at the first figure to the left of the line*, and writing each figure in the result two places to the right. Add and proceed as before.

Thus reqd amount = £788·489

= £788. 9s. 9d., to the nearest penny.

Since we begin multiplying at the 3rd decimal figure (marked with an asterisk), and place the resulting digit *two* places to the right, we retain *five* decimal figures throughout the work. The figures to the right of the line are only used for carrying purposes in order to secure a result correct to three decimal places, that is to the nearest penny.

EXAMPLES XVIII. f.

(*When not exact the answers are to be given to the nearest penny.*)

Find the Amount at Compound Interest on

1. £200 for 2 yrs. at 5%.
2. £2500 for 2 yrs. at 4%.
3. £3750 ,, 2 ,, 3%.
4. £3500 ,, 3 ,, 5%.
5. £750 ,, 4 ,, 2%.
6. £3300 ,, 4 ,, 5%.

Find the Compound Interest on

7. £225 for 2 yrs. at 4%.
8. £425 for 3 yrs. at 4%.
9. £725 ,, 4 ,, 3%.
10. £3546 ,, 3 ,, 5%.

Find the Amount at Compound Interest on

11. £345. 15s. for 2 years at 4 per cent.
12. £472. 18s. ,, 3 ,, 3 ,,
13. £1243. 14s. 3d. ,, 2 ,, 5 ,,
14. $473,87 ,, 3 ,, 4 ,,

310. When the rate per cent. is not integral the method of aliquot parts may be conveniently used.

Thus for $3\frac{3}{4}\%$, since $3\frac{3}{4}=3+\frac{1}{2}+\frac{1}{4}$, the multiplier at each stage may be used in the form $\frac{3}{100}+\frac{1}{200}+\frac{1}{400}$, and the work is done in three steps, viz. multiplication by 3, division by 2, and by 4, each result being set down two places to the right. The third line may also be obtained in this case by taking half of the preceding line.

Example. *Find the Amount at Compound Interest of £257. 13s., for 2 years 4 months at $3\frac{3}{4}$ per cent.*

	£		
	257·65		
$\frac{3}{100}$……	7·729	5	Int. for 1st year.
$\frac{1}{200}$……	1·288	25	
$\frac{1}{400}$……	·644	12	
	267·311	87	
	8·019	35	Int. for 2nd year.
	1·336	55	
	·668	28	
	277·336	05	
$\frac{1}{100}$……	2·773	36	Int. for $\frac{1}{3}$ of a year.
$\frac{1}{400}$……	·693	34	
	280·803		

Here, after finding the amount for the first two years, we regard the remaining 4 months as a new *period* at *one-third* of the rate per cent. *per annum.* $\frac{1}{3}$ of $3\frac{3}{4}=1\frac{1}{4}$. Thus the multiplier is $\frac{1}{100}+\frac{1}{400}$.

Thus req^{d} amount = £280·803 = £280. 16s. 1d.

Note. The above general method may be modified in special cases. Thus for 5%, we have $\frac{5}{100}=\frac{1}{20}$, and we only have to divide by 2, setting down the result *one* place to the right. Similarly $2\frac{1}{2}\%=\frac{2\frac{1}{2}}{100}=\frac{1}{40}$.

311. Sometimes interest is paid half-yearly or quarterly. For example, banks usually pay interest half-yearly, while on most of the "British Funds" the interest is paid quarterly. In such cases each half-year or quarter is to be considered as a separate period *at the corresponding rate of interest.*

312. If £100 is allowed to accumulate at compound interest at 3% per annum, paid quarterly, after four quarterly periods, each at $\frac{3}{4}\%$, it will be found that the amount is £103·033. Thus the four quarterly payments have the same effect as 3·033% would have if paid annually.

Hence 3·033% is called the **effective annual rate** of 3% per annum, paid quarterly.

EXAMPLES XVIII. f. (*Continued.*)

Find the Amount at Compound Interest on

15. £5000 for 3 yrs. at $2\frac{1}{2}$%. **16.** £4500 for 2 yrs. at $4\frac{1}{4}$%.
17. £4750 ,, 3 ,, $3\frac{1}{2}$%. **18.** £6750 ,, 3 ,, $4\frac{1}{2}$%.
19. £5016. 11*s.* 6*d.* for 2 years at $4\frac{1}{2}$ per cent.
20. £1601. 4*s.* 8*d.* ,, 3 ,, $3\frac{1}{4}$,,

Find the Compound Interest on

21. £2000 for $2\frac{1}{2}$ yrs. at 5%. **22.** £5620 for $2\frac{1}{2}$ yrs. at 4%.
23. £3600 ,, $3\frac{1}{2}$,, $2\frac{1}{2}$%. **24.** £8457·725,, $2\frac{1}{4}$,, $1\frac{1}{4}$%.
25. £504. 13*s.* 9*d.* for 2 yrs. 4 mos. at $4\frac{1}{2}$%.

Find the Amount at Compound Interest, *payable half-yearly*, on

26. £320. 15*s.* in $1\frac{1}{2}$ yrs. at 4%. **27.** £520·8 in 2 yrs. at 5%.
28. £16000 ,, $1\frac{1}{2}$,, $4\frac{1}{2}$%. **29.** £100,000 ,, $2\frac{1}{2}$,, $2\frac{1}{2}$%.

30. If the interest is payable quarterly find what £250 amounts to in 1 year at 5 per cent.

31. Find correct to three decimal places the "effective annual rate" corresponding to

(i) a nominal rate of 10% per annum, payable half-yearly.
(ii) ,, ,, 5% ,, ,, quarterly.
(iii) ,, ,, $2\frac{1}{2}$% ,, ,, quarterly.

313. Let the rate per cent. be denoted by r, and let £R denote the *amount* of £1 in one year; then $R = 1 + \frac{r}{100}$.

The amount of £P at the end of 1st yr. is £PR,
,, ,, £P ,, ,, 2nd ,, £PR × R, or £PR^2,
,, ,, £P ,, ,, 3rd ,, £PR^2 × R, or £PR^3,
and so on.

Thus for n years we have the formula

$$\text{Amount of £P in } n \text{ years} = \pounds PR^n = \pounds P\left(1 + \frac{r}{100}\right)^n.$$

This formula gives another method of finding the amount at compound interest when the principal, time, and rate per cent. are given. When the number of years is large the formula can be very conveniently used with the aid of Logarithms as explained in Chap. XXI.

NOTE. The amount of £P $\doteq$ (Amount of £1) × P.

EXAMPLE. *Find the Amount of £273. 12s. for 3 years at 4 per cent. Compound Interest.*

Here $\mathsf{R}=1+\frac{r}{100}=1+\frac{4}{100}=1{\cdot}04$.

Hence required amount $=£273{\cdot}6\times(1{\cdot}04)^3$

$=£273{\cdot}6\times1{\cdot}124864$

$=£307{\cdot}763$, correct to 3 dec. places

$=£307.\ 15s.\ 3d.$

The work is given below.

$$\begin{array}{r|l}
\multicolumn{2}{l}{£} \\
\multicolumn{2}{l}{273{\cdot}6} \\
\multicolumn{2}{r}{1{\cdot}124864} \\
\hline
273{\cdot}6 & \\
27{\cdot}36 & \\
5{\cdot}472 & \\
1{\cdot}094 & 4 \\
{\cdot}218 & 9 \\
{\cdot}016 & 4 \\
{\cdot}001 & 1 \\
\hline
307{\cdot}763 &
\end{array}$$

$$\begin{array}{ll}
1{\cdot}04 & \\
{\cdot}0416 & =\tfrac{4}{100}\text{ of }1{\cdot}04 \\
\hline
1{\cdot}0816 & =1{\cdot}04\text{ of }1{\cdot}04=(1{\cdot}04)^2 \\
{\cdot}043264 & =\tfrac{4}{100}\text{ of }1{\cdot}0816 \\
\hline
1{\cdot}124864 & =1{\cdot}04\text{ of }(1{\cdot}04)^2=(1{\cdot}04)^3
\end{array}$$

In order to obtain a final result true to 3 places of decimals, it is evident that in this case all six decimal figures in the multiplier 1·124864 must be retained.

NOTE. If the interest had been at the rate of 4 % for the first 2 years, and 5 % for the last year, we should have used the formula

$$\mathsf{A}=\mathsf{P}\times(1{\cdot}04)^2\times1{\cdot}05.$$

Unless the data are very simple such cases are best dealt with by logarithms.

314. All inverse cases of Compound Interest are most conveniently solved by the formula of the last article. The only case we shall here discuss is that in which the amount, rate, and time are given and the principal is required. The other cases cannot be satisfactorily discussed without the aid of logarithms. Some of these will be found in Chap. XXI.

EXAMPLE 1. *What Principal will amount to £307. 15s. 3d. in 3 years at 4 per cent. Compound Interest?*

Since Amount of $£\mathsf{P}=(\text{Amount of }£1)\times\mathsf{P}$

it follows that $\mathsf{P}=\dfrac{\text{given amount}}{\text{amount of }£1}.$

Here given amount $=£307.\ 15s.\ 3d.$

$=£307{\cdot}7625$;

and amount of $£1=£1{\cdot}124864$, as in Art. 313.

$$\therefore\ \mathsf{P}=\frac{£307{\cdot}7625}{£1{\cdot}124864}$$

$$=273{\cdot}6\,;$$

that is, the required principal is £273. 12*s*.

$$\begin{array}{r} 273{\cdot}6 \\ 1{\cdot}124{,}8{,}6{,}4\ )\ 307{\cdot}7625 \\ 82\ 7897 \\ 4\ 0492 \\ 6746 \end{array}$$

Here £273. 12*s*. is the *true* Present Value of £307. 15*s*. 3*d*. due in 3 years at 4 per cent. compound interest. If the term *Present Value* is used in connection with *compound* interest it is always in this sense.

EXAMPLE 2. *Find the difference between the Simple and Compound Interest on* £3500 *for* 2 *years at* 4 *per cent.*

Whether interest is simple or compound, we have

$$\text{Interest on } £\mathsf{P}=(\text{Interest on } £1)\times\mathsf{P}.$$

Hence *the difference between* C.I. *and* S.I. *on* £P

$$=\mathsf{P}\ \textit{times the difference between}\ \text{C.I.}\ \textit{and}\ \text{S.I.}\ \textit{on}\ £1.$$

Now at 4 % for 2 yrs.,

$$\text{Comp. Int. on } £1=£0{\cdot}0816.$$

$$\text{Simple Int. on } £1=£1\times\frac{2\times4}{100}$$

$$=£0{\cdot}08$$

$$\begin{array}{r} £ \\ 1{\cdot}00 \\ {\cdot}04 \\ \hline 1{\cdot}04 \\ {\cdot}0416 \\ \hline 1{\cdot}0816 \end{array}$$

∴ the diff. between C.I. and S.I on £1 $=£0{\cdot}0016$

,, ,, ,, £3500 $=£0{\cdot}0016\times3500$

$=£5{\cdot}6$

$=£5.\ 12s.$

NOTE. It easily follows that the converse question is solved by the formula

$$\mathsf{P}=\frac{\text{difference on } £\mathsf{P}}{\text{difference on } £1}.$$

EXAMPLES XVIII. g.

By the method of Art. 313 find the Amount at Compound Interest of

1. £387 in 2 yrs. at 3 %.

2. £873. 8*s*. in 3 yrs. at 4 %.

3. £953. 15*s*. 6*d*. in 3 yrs. at 5 %.

Find, to the nearest *pound*, the Principal which at Compound Interest will amount to

4. £296 in 2 yrs. at 3 %.

5. £340·95 in 3 yrs. at 3 %.

6. £3244. 18*s*. in 2 yrs. at 4 %.

7. £5343. 2*s*. 3*d*. ,, 3 ,, 4 %.

8. £3820. 3*s*. 3*d*. ,, 3 ,, 5 %.

9. £275. 19*s*. ,, 4 ,, $2\frac{1}{2}$ %.

Find, to the nearest *penny*, the Present Value of

10.	£300	due in	3	yrs. at	4 %	compound	interest.
11.	£850	,,	2	,,	5 %	,,	,,
12.	£2000	,,	3	,,	3 %	,,	,,
13.	£6027. 17*s.* 6*d.*	,,	2	,,	$3\frac{3}{4}$ %	,,	,,
14.	£3925. 5*s.* 4*d.*	,,	$3\frac{1}{2}$	,,	$2\frac{1}{2}$ %	,,	,,

15. Find the amount in 2 years at compound interest of £250 if the rate is 4 per cent. for the first year and 8 per cent. for the second.

16. At 5 per cent. for $2\frac{1}{2}$ years, prove the formula

$$A = P \times (1{\cdot}05)^2 \times 1{\cdot}025.$$

Hence find the Present Value (to the nearest pound) of £22050 due in $2\frac{1}{2}$ years at 5 per cent.

17. What sum would in 3 years amount to £2811. 18*s.* at compound interest if the rate were 3 p.c. for the first year, 4 p.c. for the second year, and 5 p.c. for the third year?

18. Prove that at $3\frac{1}{2}$ p.c. for 2 years the amount at compound interest is approximately 1·07123 times the principal.

19. Prove that at 5 p.c. for 3 years the compound interest is approximately 0·15763 times the principal.

Hence find, to the nearest pound, the principal which in 3 years at 5 p.c. compound interest will be increased by £50.

20. The census is taken every ten years, and it is found that the population of a certain town is greater each census by 12 p.c. of the population at the preceding census. If it was 37632 in 1901, what was it in 1891, and in 1881?

21. The population of a town was 347865. It then increased by 20 per 1000 one year, 25 per 1000 the next, and 30 per 1000 in the third year. What was the population at the end of 3 years, to the nearest hundred?

Find to the nearest penny the difference between the Simple and Compound Interest on

22. £3125 for 3 yrs. at 4 %. **23.** £1000 for 4 yrs. at 4 %.
24. £2400 ,, 3 ,, 5 %. **25.** £2960 ,, 2 ,, $3\frac{3}{4}$ %.

26. £7545. 10*s.* for 2 yrs. at 4 % per annum, interest being paid half-yearly.

27. The difference between the simple and compound interest on a certain sum for 2 years at 5 per cent. is 12*s.* What is the sum?

28. On what sum of money would the compound interest for 3 years at 4 per cent. exceed the simple interest by £15. 4*s.*?

29. The simple interest on a certain sum for 2 years at 3 per cent. is less than the compound interest on the same sum by £3. 7*s.* 6*d.* Find the sum.

30. On what sum of money would the compound interest for 3 years at 5 per cent. exceed the simple interest by £38. 2*s.* 6*d.*?

31. A man buys a farm and buildings valued at £26333. 6*s.* 8*d.* and pays £13000 down. Find what additional sum he ought to pay at the end of two years reckoning compound interest at $3\frac{1}{2}$ per cent. per annum.

32. A man saves £4000 a year, which he invests at the end of the year at 5% compound interest. Find, to the nearest pound, the amount of his savings at the end of the 5th year.

33. A man borrows £200 at 5% compound interest. At the end of each year he pays back £50. How much does he owe at the end of 4 years?

34. In a certain factory the machinery and plant were valued at £2400. If it is supposed to depreciate it each year at $12\frac{1}{2}$% of the value at the beginning of the year, find (to the nearest pound) what its value would be after 4 years.

Graphical Illustrations.

315. If interest is at 4 p.c. per annum we have the following formulæ giving the amount of £1 at the end of n years:

When the interest is simple,

$$A = 1 + \frac{4n}{100} \quad \text{.........................(i).}$$

When the interest is compound,

$$A = (1{\cdot}04)^n \quad \text{.........................(ii).}$$

Since in (i) the quantities A and n are connected by an equation of the first degree, the graph is a straight line [Art. 256]. In this case we can select numerical values for A and n at discretion, and draw the graph. For this purpose two pairs of corresponding values will be sufficient. [Art. 257.]

In (ii) the graph will be some curve the nature of which cannot be recognised from the equation. Moreover, as one of the variables occurs as an index, simultaneous values of A and n cannot be conveniently obtained from the equation.

The comparison between the values, in successive years, of the amount of £1 at simple and compound interest at 4 p.c. per annum can be illustrated graphically as in the following example.

EXAMPLE. *The following details, taken from "Interest Tables," give approximately the amount of* £1 *at* 4 *p.c. for different periods.*

No. of years	0	5	10	15	20	25	30	35
Am^t at S.I.	1	1·20	1·40	1·60	1·80	2·00	2·20	2·40
Am^t at C.I.	1	1·22	1·48	1·80	2·19	2·67	3·24	3·95

Illustrate these data graphically, and use the graphs to find the amount of £100 *in* 28 *years,* (i) *at simple,* (ii) *at compound interest.*

Also find from the graphs in how many years a sum of money will double itself, (i) *at simple,* (ii) *at compound interest.*

In Fig. 13 on the opposite page the years are marked on the horizontal axis, 0·1 of an inch being taken to represent 1 year. On the vertical axis 1 inch is taken to represent £1. On this scale tenths of the unit can be marked and read accurately, and hundredths can be judged with a fair degree of accuracy.

Since when $n=0$, $\mathsf{A}=1$, in each case, both graphs pass through the point (0, 1), and it is convenient to take this point as the origin.

After plotting the points given by the table the graphs will be found as in the diagram. The straight line OP gives the amounts at simple interest of £1 for different periods, and the curved line gives the corresponding amounts at compound interest.

When $n=28$, from the S.I. graph we have $\mathsf{A}=2{\cdot}12$, and from the C.I. graph $\mathsf{A}=3{\cdot}00$.

Thus the amount of £100 in 28 years is £212 at simple interest, and £300 at compound interest. These results will be found correct to the nearest pound.

Again, when $\mathsf{A}=2$, in the graph OP, $n=25$;
.... OQ, $n=17{\cdot}7$, approx.

Thus a sum of money doubles itself in 25 years at simple interest, and in about 17·7 years at compound interest.

NOTE. The simple interest graph being linear can be produced indefinitely beyond P, and thus can be used to read off amounts *for any number of years.* In the case of the compound interest graph the form of the curve is not known beyond the limits of the diagram; thus for purposes of comparison between simple and compound interest the graphs can only be used for periods not exceeding 35 years.

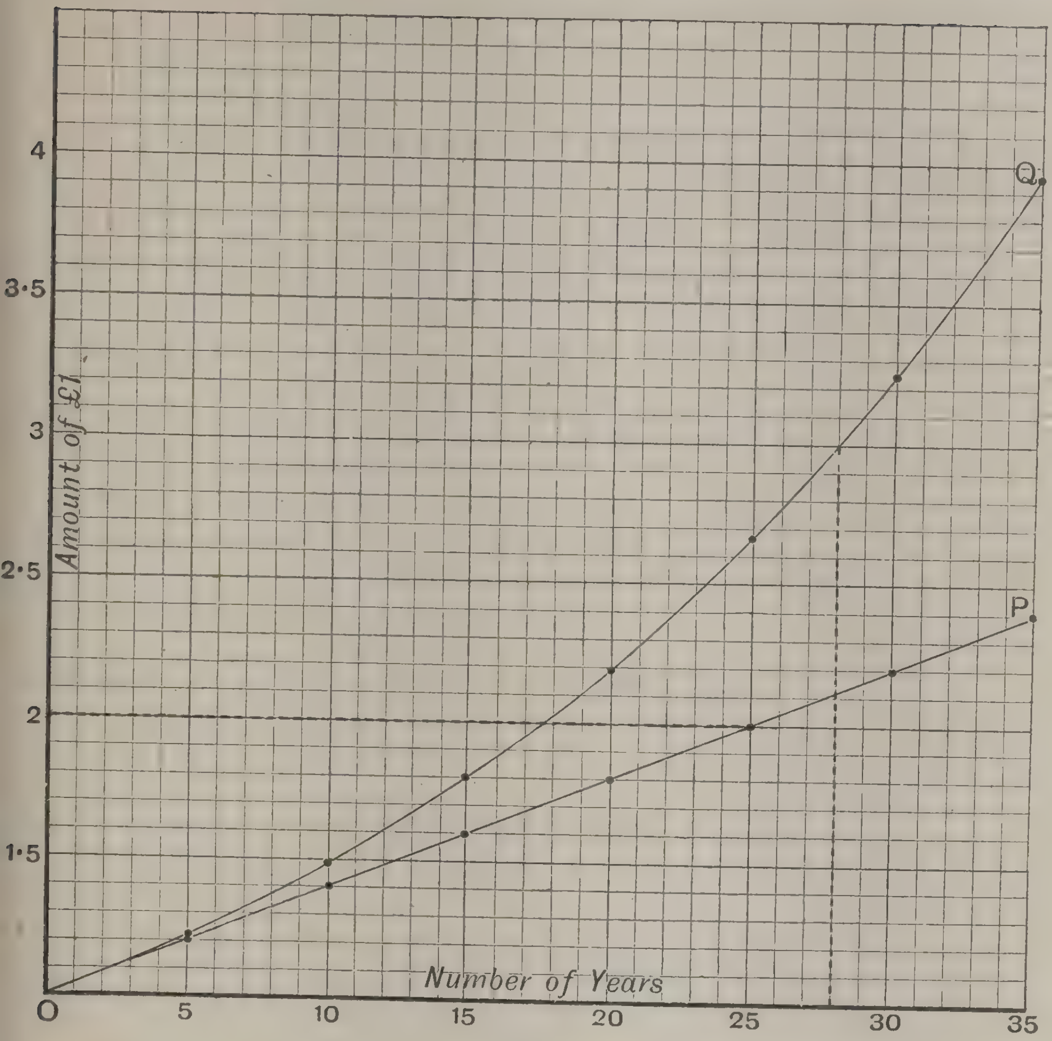


FIG. 13.

316. When a sum of money is at simple interest at a given rate per cent. for any number of years, we have in all cases

$$\text{Amount} = \text{Principal} + \text{Interest},$$

where 'Principal' is constant, and 'Interest' varies with the number of years. Hence the variations of Amount and Time may always be represented by a linear graph in which x is taken to denote the number of years, and y the number of pounds in the corresponding Amount. The equation of the graph will be of the form $y = \mathsf{P} + ax$, where P is the number of pounds in the principal sum, and a is a constant, namely $\mathsf{P} \times \frac{r}{100}$, which has the same value as long as P and r remain the same.

EXAMPLES XVIII. h.

1. From the figure on page 379 find as accurately as possible

(i) the amount of £1 at 4 % compound interest in 34 years;

(ii) the difference between the simple and compound interest a the end of 27 years;

(iii) after how many years the compound interest is approximatel the same as the simple interest at the end of 20 years;

(iv) after how years the difference between the simple and compound interest is approximately £1. 4*s.*;

(v) the present value of £247 due in 23 years at 4 p.c. compoun interest.

2. From the following table draw graphs to shew the amounts a simple and compound interest of £1 at 5 p.c. for different periods.

No. of years	0	5	10	15	20	25	30	35
Amt at S.I.	1	1·25	1·50	1·75	2·00	2·25	2·50	2·75
Amt at C.I.	1	1·28	1·63	2·08	2·65	3·39	4·32	5·52

From the graphs shew that the difference between the simple and compound interest on £10 at the end of 23 years is approximately £9.

Find also

(i) the amount at compound interest of £100 in 26 years;

(ii) in how many years the accumulated interest on £100 wil amount to £400.

3. If £y is the cash payment for a sum of £x after discount a $12\frac{1}{2}$ % has been deducted, find the relation between y and x. Shew that the graph may be conveniently drawn by joining the point (0, 0 with the point (40, 35).

From the graph read off the cash payments for bills of £2. 8*s.* and 12*s* Also find what amounts can be settled by cash payments of 17*s.* 6*d.* and £1. 4*s.* 6*d.*

4. Two sums of money are put out at simple interest at different rates per cent. In the first case the amounts at the end of 6 years and 15 years are £260 and £350 respectively. In the second case the amounts for 5 years and 20 years are £330 and £420. Draw graphs from which the amounts may be read off for any year, and find the year in which the principal with accrued interest will amount to the same in the two cases. Also from the graphs read off the value of each principal.

CHAPTER XIX.

STOCKS AND SHARES.

317. WHEN a Government requires more money than can be obtained by taxation, it invites the public to contribute to a Loan. Such a Loan is raised on special terms, which in their simplest form may be stated thus :

The Government undertakes to pay a fixed rate of interest (say $2\frac{1}{2}$ per cent.) on the money lent, so long as the loan exists. Sometimes the Government names a date for the repayment of the loan : sometimes it names no such date, but reserves the right of repaying the loan at any time convenient to itself.

318. The lender of £100 is said **to hold £100 Stock**; and this stock secures to him the rights named above, viz. (i) income at the stipulated rate, and (ii) repayment of principal if, and when, the Government cancels the debt. But in any case *a lender cannot at will recover his money direct from the Government.* The Stock held is nothing more than *a title to an* **annuity**, or annual income, the amount of the annuity being a fixed percentage on the amount of stock held.

319. Though the original lender cannot at his discretion claim repayment from the Government, he can *sell his stock* to some other person, who, in return for a cash payment, will then become entitled to the income and other rights attaching to the stock.

320. The cash value of stock varies from time to time owing to political and commercial causes ; so that the price at which a holder sells his stock will not necessarily be the same as its nominal value. Thus the holder of £100 *stock* may be able to sell it for no more than £94 *cash* ; or again, in other circumstances he may find buyers willing to give as much as £105 *cash.* In the former case the stock is said **to stand at 94,** in the latter, at **105.**

321. If £100 *stock* will fetch £100 *cash*, the stock is said to be **at par.** Stock standing at 94 is said to be **at 6 discount,** or **6 below par** ; and stock at 105 is at **5 premium,** or **5 above par.**

322. Stock is not necessarily bought or sold in *exact multipl* of £100. For example, a holder of £2000 Government $2\frac{1}{2}$ per cen Stock may sell *any portion* of it (say £1126 stock), thus conveyin to the buyer a part of his income proportional to the amount o stock transferred.

323. The British **Funded** Debt (that is, the debt whic the Government is not obliged to pay off at any fixed time consists chiefly of **Consols**, viz. *Consolidated Annuities*, at th rate of £$2\frac{1}{2}$ on every £100 stock. This Funded Debt is sometime known as **The Funds.**

NOTE. The term *Consolidated* implies that debts contracted a various times and under different conditions have been amalgamate into a single debt, and brought under the same terms. Up to 1888 Consols bore interest at the rate of 3 per cent. In that year th Three per cent. Debt was cancelled and its place taken by a new Debt bearing $2\frac{3}{4}$ per cent. interest till 1903, and afterwards $2\frac{1}{2}$ per cent

324. Money used to establish and work a business is calle **Capital.** Large industrial undertakings such as Railways, Docks Water-works, generally raise the required capital by issuing *stock* in the manner described above; and the purchasers of the stock form an association called a **Joint Stock Company.** In most instances the interest on such stock is not at a fixed rate, but results from periodical divisions of profits. Profits divided amongst stock-holders are called **dividends**; this term is also applied to the *fixed* interest payable on Government and other stocks.

NOTE. Large Companies such as Railways often issue stock in three classes, **Debenture, Preference,** and **Ordinary** Stock. On the first two the dividends are *fixed*, and are respectively the first and second charge upon the profits. The remaining profits, if any, are then applied as dividends on the Ordinary Stock.

In Art. 327 we shall explain the way in which stock is bought and sold, and the commission (called *brokerage*) charged by the agents who transact such business. In the following examples it is for the present supposed that all agents' charges are allowed for in the quoted prices of stock.

EXAMPLE 1. *If £755 Metropolitan* $2\frac{1}{2}$ p.c. *stock is sold at* $77\frac{1}{2}$ *what does the seller obtain?*

Here £100 *stock* is worth £$77\frac{1}{2}$ *cash*;

∴ £755 *stock* ,, £$77\frac{1}{2} \times \frac{755}{100}$ *cash*

= £585. 2*s*. 6*d*.

NOTE. It should be noticed that the given percentage by which the nature of the stock is described does not affect the work. The question might be stated briefly thus:

What is the cash value of £755 stock at $77\frac{1}{2}$?

or, *What is the cost of £755 stock at* $77\frac{1}{2}$?

EXAMPLE 2. *How much Railway* 4 p.c. *stock (correct to the nearest penny) at* $109\frac{3}{4}$ *can be bought for* £1500?

Here £1500 is *cash*, and £100 *stock* costs $109\frac{3}{4}$.

Since £$109\frac{3}{4}$ *cash* buys £100 *stock*;

∴ £1500 *cash* ,, $£100 \times \frac{1500}{109\frac{3}{4}}$ *stock*.

And
$$£100 \times \frac{1500}{109\frac{3}{4}} = £100 \times \frac{6000}{439}$$
$$= £1366{\cdot}743, \text{ correct to 3 decimal places,}$$
$$= £1366.\ 14s.\ 10d., \text{ to the nearest penny.}$$

NOTE. A purchaser who spends £1500 in buying a certain stock is said to **invest** £1500 in that stock. The above question might be stated thus:

A man invests £1500 in stock at $109\frac{3}{4}$; *how much stock does he buy?*

EXAMPLES XIX. a.

What is the cash value of

1. £2800 stock at 94? ✓2. £3750 stock at 102?

✓3. £4120 ,, $92\frac{1}{2}$? 4. £4284 ,, $98\frac{3}{4}$?

✓5. What is the cost of £4715 India 3 p.c. stock at $87\frac{1}{2}$?

✓6. How much cash will be obtained by selling £462. 10s. Railway 4 p.c. stock at 105?

Find the amount of stock which can be bought for

7. £496, price 124. 8. £2527, price 95. 9. £3900, price $97\frac{1}{2}$.

10. What amount of $3\frac{1}{2}$ p.c. Corporation stock at $97\frac{1}{2}$ can be bought for £1560?

✓11. How much 4 p.c. stock can be bought for £621 when the quoted price is $3\frac{1}{2}$ p.c. above par?

12. A man invests £4690 in $2\frac{1}{2}$ p.c. Consols at $83\frac{3}{4}$; how much stock does he buy?

✓13. What will £2880 of $2\frac{3}{4}$ p.c. stock sell for when the price is at a discount of 5 p.c.?

✓14. At what price will £3220 buy £3500 stock?

✓15. I spend £228 in buying £285 stock: what is the price of the stock?

Find to the nearest *pound*

16. The cash value of £855 Consols at $83\frac{1}{2}$.

17. How much stock at $102\frac{1}{2}$ can be bought for £1000.

18. The price of stock when £560 stock can be bought for £480.

Find to the nearest *penny*

19. The proceeds of selling £2912 stock at $94\frac{1}{2}$.

20. The cost of £5782 Preference stock at $125\frac{1}{2}$.

21. The cash value of £3572 stock at $109\frac{1}{2}$.

EXAMPLE 3. *If £4800 India 3 p.c. stock is bought at $87\frac{1}{2}$ and sold at $91\frac{3}{4}$, what profit is made on the transaction?*

Here £100 stock is bought for £$87\frac{1}{2}$ *cash* and sold for £$91\frac{3}{4}$ *cash.*

Hence on £100 *stock* the profit is £$4\frac{1}{4}$;

$$\therefore \quad ,, \quad £4800\ \textit{stock} \quad ,, \quad ,, \quad £4\tfrac{1}{4} \times \frac{4800}{100}, \text{ or } £204.$$

EXAMPLE 4. *A man invests £5500 in 5 p.c. Railway stock at 139, and sells out at $142\frac{1}{2}$; find, to the nearest penny, how much cash he gets by the sale.*

Here the given sum £5500 is *cash* to be invested.

Now £139 *cash* buys stock afterwards sold for £$142\frac{1}{2}$ *cash.*

$$\therefore \quad £5500\ \textit{cash} \quad ,, \quad ,, \quad ,, \quad £142\tfrac{1}{2} \times \frac{5500}{139}\ \textit{cash}.$$

$$\text{And} \qquad £142\tfrac{1}{2} \times \frac{5500}{139} = £5638 \cdot 489\ldots = £5638.\ 9s.\ 9d.$$

NOTE. The pupil should observe that it is not necessary here to find *how much stock was bought*, in order to find *for how much cash it was sold.*

EXAMPLES XIX. a. (*Continued.*)

22. How much is lost by buying £780 stock at 107 and selling it at £102?

23. How much should I gain by buying £650 L. & N.W. Railway stock at $138\frac{1}{4}$ and selling it at $141\frac{1}{2}$?

24. If £750 is invested in stock at $131\frac{1}{4}$ and the stock is sold out at 133, find the proceeds of sale.

25. I invest £455 in 3 p.c. stock at 105; how much cash shall I get by selling out at 102?

26. How much is lost by buying £2500 India $3\frac{1}{2}$ per cents. at $99\frac{3}{4}$, and selling them at $94\frac{1}{4}$?

✓ 27. How much shall I gain by investing £1187. 10*s*. in $4\frac{1}{2}$ p.c. Debentures at $142\frac{1}{2}$ and selling them at 147?

✓ 28. If £3720 stock is bought at $101\frac{1}{2}$ and sold at $107\frac{1}{4}$, find the profit, to the nearest penny.

✓ 29. Find to the nearest penny how much was lost by investing £1642 in 4 p.c. Railway stock at 129 and selling at 125.

325. In the following examples the chief point to notice is the difference between *holding* and *buying* stock. Thus a man *holding* 3 p.c. stock receives an income of £3 *on every* £100 *of stock held*; while a man *buying* 3 p.c. stock at 91 purchases an income of £3 with every £91 *of cash invested.*

Example 1. *A man* holds £5640 *of* $4\frac{1}{2}$ *per cent. stock at* 94 ; *find his income.*

[As we are considering the *income* derived from stock *already in possession*, the *price* at which such stock might be bought or sold does not affect the work.]

On £100 *stock* he gets an *income* of $£4\frac{1}{2}$.

$\therefore$ on £5640 *stock* ,, ,, $£4\frac{1}{2} \times \frac{5640}{100}$, or £253. 16*s*.

Example 2. *A man* invests £5640 *in* $4\frac{1}{2}$ *per cent. stock at* 94 ; *find his income.*

[Here the given sum £5640 is *cash*, with which stock is to be bought. And every £94 *cash* buys £100 *stock*, namely the right to an *income* of $£4\frac{1}{2}$.]

Here £94 *cash* will purchase an *income* of $£4\frac{1}{2}$.

$\therefore$ £5640 *cash* ,, ,, $£4\frac{1}{2} \times \frac{5640}{94}$, or £270.

EXAMPLES XIX. b.

Find the income derived from holding

1. £3400 of 3 % stock. ✓2. £3825 Consols ($2\frac{1}{2}$ %).✓

✓3. £2450 of $5\frac{1}{4}$ % Railway Preference stock.

Find the income derived from investing

4. £1188 in 3 % stock at 81. ✓5. £3840 in 4 % stock at 96.

✓6. What income will be derived from £967. 10*s*. laid out in buying 5 per cent. stock at $107\frac{1}{2}$?

✓ 7. A man invests £11636 in 3 p.c. stock at 104, and £11000 in $4\frac{1}{2}$ p.c. stock at 143. From which source is his income the greater, and by how much?

8. I invest £5000 in 4 p.c. stock at 125; find my half-yearly income after income tax at 1*s.* in the £ has been deducted.

9. When the income tax was at 14*d.* in the £, a man invested £400 in $3\frac{3}{4}$ p.c. stock at 113. What was his net half-yearly dividend?

Find *to the nearest penny*

10. The income derived from holding £786 in a $2\frac{3}{4}$ p.c. stock.

11. The income derived from an investment of £3750 in $4\frac{1}{4}$ p.c. Debenture stock at 140.

12. The income derived from laying out £500 in the purchase of $2\frac{1}{2}$ p.c. Consols at $82\frac{1}{4}$.

EXAMPLE 3. *If the current price of a* 4 *per cent. stock is* $105\frac{1}{2}$, *find*
(i) *how much* stock *a man must* hold to *derive from it an income of* £280;
(ii) *how much* cash *he must* invest *to secure an income of* £280.

(i) £4 *income* is derived from *holding* £100 *stock*;

∴ £280 *income* ,, ,, $£100 \times \frac{280}{4}$, or £7000.

(ii) £4 *income* is derived from *investing* $£105\frac{1}{2}$ *cash*;

∴ £280 *income* ,, ,, $£105\frac{1}{2} \times \frac{280}{4}$, or £7385.

EXAMPLE 4. *What rate per cent. is obtained for money invested in Railway* $4\frac{1}{2}$ p.c. *stock at* $122\frac{1}{2}$?

Here $£122\frac{1}{2}$ *cash* buys an *income* of $£4\frac{1}{2}$;

∴ £100 *cash* ,, *income* of $£4\frac{1}{2} \times \frac{100}{122\frac{1}{2}}$, or £3·67....

That is, income is obtained at the approximate rate of 3·67 per cent. on the money invested.

EXAMPLE 5. *Which of the following stocks yields the better income to an investor:* 3 *per cent. stock at* 87, *or* 4 *per cent. stock at* 115?

Compare the incomes obtained by investing *the same amount of cash* (say £100) in each stock.

Then, as in the last example:

income on investing £100 in 1st stock $= £3 \times \frac{100}{87}$

$= £3{\cdot}45$ (approx.),

income ,, ,, £100 ,, 2nd stock $= £4 \times \frac{100}{115}$

$= £3{\cdot}48$ (approx.).

Hence the 2nd stock pays the better income by 0·03 per cent. of the cash invested.

EXAMPLES XIX. b. (*Continued.*)

13. From how much India 3 p.c. stock would an income of £81 be derived?

14. How much Railway $4\frac{1}{2}$ p.c. stock must be held to secure an income of £297?

15. What sum invested in a $5\frac{1}{2}$ p.c. stock at 110 will produce an income of £60?

16. A man receives £24. 3*s.* from North-Eastern Railway stock which pays a dividend of $5\frac{3}{4}$ p.c. How much stock does he hold?

17. How much must be invested in $5\frac{1}{2}$ p.c. stock at $115\frac{1}{2}$ in order to secure an income of £80. 13*s.* 4*d.*?

18. How much must be invested in $5\frac{1}{2}$ p.c. stock at $141\frac{1}{4}$ so that after paying income tax at 1*s.* 2*d.* in the £1 the net half-yearly dividend may be £17. 12*s.*?

[*Since* 14*d.* *is deducted from every pound*, the net annual *income on* £100 *stock is* £$5\frac{1}{2} \times \frac{226}{240}$.]

19. If my annual income from $2\frac{1}{2}$ p.c. Consols is £35. 7*s.* 3*d.*, find how much stock I hold.

20. Find to the nearest pound what sum must be invested in 3 p.c. stock at 104 to obtain an income of £200.

21. After deducting income tax at 1*s.* in the £ I have an income of £77. 18*s.* a year derived from $4\frac{1}{2}$ p.c. stock which stood at $132\frac{1}{2}$ at the time of purchase. Find to the nearest pound how much I invested.

22. What rate per cent. is obtained by investing in $4\frac{1}{2}$ p.c. stock at 90?

23. What interest shall I get on my capital by investing at 125 in a stock which is paying a dividend of $3\frac{3}{4}$ p.c.?

Find, correct to two places of decimals, the rate per cent. of interest obtained by investing

24. In $5\frac{1}{2}$ p.c. Preference stock at 145.

25. In a $3\frac{1}{2}$ p.c. stock at $78\frac{3}{4}$.

26. In 4 p.c. Railway stock at $119\frac{3}{8}$.

Which of the following stocks yields the better income, and by how much per cent.? [Answer to two decimal figures.]

27. A 3 p.c. stock at 85, or 4 per cents. at 96?

28. A 4 p.c. stock at 126, or a $3\frac{1}{2}$ p.c. stock at 108?

29. A 5 p.c. stock at 140, or 6 per cents. at 174?

30. Which is the more profitable investment, 4 p.c. Preference stock at 108 or $4\frac{1}{2}$ p.c. Debenture stock at 117? By how much would incomes derived from investing £702 in each differ from one another?

31. A man invests a certain sum in 3 p.c. stock at 81, and another invests the same amount in 4 p.c. stock at 112. The difference in their incomes is £2; find the sum invested.

32. *A*'s income is derived from a certain amount of $3\frac{1}{2}$ p.c. stock at $122\frac{1}{2}$; *B*'s income is derived from an equal amount of $2\frac{1}{2}$ p.c. stock at $92\frac{1}{2}$. If *A*'s income exceeds *B*'s by £10, what amount of stock do they each *hold*?

33. At what price must 4 p.c. stock be bought so that an investor may get 5 p.c. for his money?

[*What is the price of* 4 p.c. *stock when* £100 *cash invested in it produces an income of* £5?]

34. At what price must I buy $2\frac{3}{4}$ p.c. stock so as to get $2\frac{1}{2}$ p.c. for my money?

35. An investor in $4\frac{1}{2}$ p.c. stock finds that he is getting 6 p.c. for his money. At what price did he buy?

36. At what price must 3 p.c. stock be bought if an investment of £1600 produces an income of £50?

37. Find the price at which £2400 was invested in $5\frac{1}{2}$ p.c. Preference stock, if the income derived is £120?

38. What is the price of $5\frac{1}{2}$ p.c. stock when a man investing £2353 in it obtains an income of £95. 6*s*. 8*d*.?

39. A man bought American Railway stock paying 6 p.c., and after paying income tax at 10*d*. in the £, he got 5 p.c. for his money. At what price did he buy?

40. The yearly dividend to an investor was £21. 7*s*. 6*d*., and this was 4·45 p.c. for his money; find, to the nearest pound, how much cash he gave for his stock.

41. A fund is being raised to provide a Scholarship of £60 a year. If the necessary income is to be derived from an investment in $2\frac{3}{4}$ p.c. Annuities at 92, find, to the nearest pound, how much must be collected from the subscribers to the fund.

326. In the preceding examples we have dwelt upon the distinction between *holding stock* and *investing money* (or cash); and we have illustrated by examples of a simple kind (i) the buying and selling of stocks, (ii) the income derived from stock held or cash invested, (iii) the rate per cent. (or "yield") to an investor at different prices. These cases cover all the main principles of the subject, and for the sake of simplicity we have supposed that agents' charges and other incidental expenses have been allowed for in the prices of the stocks quoted. We have now to explain how the buying and selling of stocks are effected in actual practice; it will be seen that though modifications in detail are introduced into the working of examples, no new principle is involved.

327. Brokerage. The transfer of stocks from one holder to another is effected through the agency of a Corporation known as a **Stock Exchange,** the members of which are either Stock-Dealers or Stock-Brokers. The former undertake the actual purchase or sale of stocks, while the latter act simply as agents between the Dealers and private clients.

The prices quoted against each stock usually name two prices. Thus the statement "Consols $84\frac{3}{4}$–$85\frac{1}{4}$" means that the Dealers are prepared to buy Consols at the lower price, or to sell them at the higher. The difference (in this case £$\frac{1}{2}$) represents the Dealer's profit on each £100 of stock.

328. The Brokers buy stock from or sell it to the Dealers on behalf of their private clients, from whom they obtain a **brokerage**, or commission, for their services.

Speaking generally, the Broker's charge on all British or Foreign Government stocks is 2*s.* 6*d.*, or £$\frac{1}{8}$, on every £100 stock. On Colonial and Corporation stocks the charge is 5*s.*, or £$\frac{1}{4}$ per £100 stock. In each case the brokerage is calculated as a percentage *on the nominal par value of the stock.*

In this sense, the Broker's charge is usually quoted "brokerage $\frac{1}{8}$" or "brokerage $\frac{1}{4}$," the words "per cent." being omitted.

For example, if a man is buying Consols quoted at $85\frac{1}{2}$, then for every £100 stock bought he pays £$85\frac{1}{2}$ to the Dealer and £$\frac{1}{8}$ to his Broker; that is, he *buys* at $85\frac{1}{2}+\frac{1}{8}$, or $85\frac{5}{8}$.

If he is selling at the same price, for every £100 stock sold he *receives* £$85\frac{1}{2}$ from the Dealer, but *pays* £$\frac{1}{8}$ to his Broker; that is, he *sells* at $85\frac{1}{2}-\frac{1}{8}$, or $85\frac{3}{8}$.

Thus to a *buyer* the brokerage *increases* the price,
while ,, *seller* ,, *diminishes* ,,

329. For Railway and other stocks the charge commonly ranges from 5*s.* to 10*s. per cent.*, and is calculated not on the nominal par value but on *the actual cash value for which the stock is bought and sold.*

In this sense the Broker's charge is quoted as "$\frac{1}{4}$ *per cent.*," or "$\frac{1}{2}$ *per cent.*," the words "per cent." being retained. In this case it has to be calculated as a separate item and not as an addition to or deduction from the price of the stock.

Thus if a Broker received instructions to buy £500 Railway stock at $103\frac{5}{8}$, and the brokerage was $\frac{1}{2}$ per cent., the account to the client (known as a "Contract Note") would be presented somewhat as follows:

April 8th, 1908.

We have this day bought, as per your order, subject to the Rules, Regulations, and Customs of the London Stock Exchange,

		£.	s.	d.
£500 Railway stock at	$103\frac{5}{8}$	518	2	6
Brokerage	$\frac{1}{2}$%	2	11	10
		520	14	4

NOTE. The above Scale of Commission is officially recognised by the London Stock Exchange, but the usage we have explained, though correct in theory, is subject to various modifications in practice. For instance, on Government and Colonial stocks brokerage "$\frac{1}{8}$ or $\frac{1}{4}$ per cent. on the nominal par value" is sometimes charged on each £100 stock or *part of* £100 stock. Again, in addition to brokerage there are other incidental charges, some of which enter into every statement of account from broker to client. As some of these are fixed and others variable they must necessarily occur as separate items in the account. For these reasons brokers find it convenient to put *all* their charges down separately, even in those cases in which the brokerage could theoretically be included in the price of the stock as in Art. 328. The method of that article will, however, be used in some of the examples which follow.

As a rule brokerage alone will be considered: in a few cases in which we shall introduce extra charges, the wording of the examples will make their meaning clear.

EXAMPLE 1. *I invest £1200 in 4 p.c. Colonial stock at $103\frac{1}{4}$, and after receiving a half-year's dividend, I sell out at 106. Find to the nearest penny my total gain, reckoning brokerage $\frac{1}{4}$ in each transaction.*

On *buying*, the price of £100 stock is $103\frac{1}{4} + \frac{1}{4}$, or $103\frac{1}{2}$;

On *selling*, ,, ,, ,, $106 - \frac{1}{4}$, or $105\frac{3}{4}$.

$\therefore$ gain on buying and selling £100 stock $= £2\frac{1}{4}$,

half-year's dividend on £100 stock $= £2$;

$\therefore$ corresponding to every £100 stock the total gain is $£4\frac{1}{4}$.

Hence on an outlay of $£103\frac{1}{2}$ the gain is $£4\frac{1}{4}$;

$\therefore$,, £1200 ,, $£4\frac{1}{4} \times \dfrac{1200}{103\frac{1}{2}}$;

and $£4\frac{1}{4} \times \dfrac{1200}{103\frac{1}{2}} = £49{\cdot}275\ldots = £49.\ 5s.\ 6d.$

Example 2. *A person invests £9075 in 3 p.c. stock at* $90\frac{5}{8}$, *and when it has risen to* $91\frac{1}{8}$ *he sells out and re-invests in* $3\frac{1}{2}$ *p.c. stock at* $97\frac{3}{8}$; *find the change of income, brokerage* $\frac{1}{8}$ *being charged on each transaction.*

(i) Each £100 of the 1st stock costs £$(90\frac{5}{8}+\frac{1}{8})$, or £$90\frac{3}{4}$ *cash.*

Hence £$90\frac{3}{4}$ invested in 1st stock buys an income of £3;

∴ £9075 ,, ,, ,, ,, £$3\times\dfrac{9075}{90\frac{3}{4}}$, or £300.

That is, the first income is £300.

(ii) On *selling*, each £100 of 1st stock produces £$(91\frac{1}{8}-\frac{1}{8})$, or £91 *cash.*

Thus stock bought for £$90\frac{3}{4}$ sold for £91.

∴ ,, ,, £9075 ,, £$91\times\dfrac{9075}{90\frac{3}{4}}$, or £9100.

That is, the first stock realizes £9100.

(iii) The price to a *buyer* of 2nd stock $=97\frac{3}{8}+\frac{1}{8}=97\frac{1}{2}$.

Hence £$97\frac{1}{2}$ invested in 2nd stock buys an income of £$3\frac{1}{2}$;

∴ £9100 ,, ,, ,, ,, £$3\frac{1}{2}\times\dfrac{9100}{97\frac{1}{2}}$, or £$326\frac{2}{3}$.

Thus the gain of income is £26. 13*s.* 4*d.*

EXAMPLES XIX. c.

(*In the following examples we shall use the abbreviations* [B, $\frac{1}{8}$], [B, $\frac{1}{2}$ per cent.] *to indicate* "*brokerage* $\frac{1}{8}$" *and* "*brokerage* $\frac{1}{2}$ per cent." *respectively.*)

1. How much cash would be obtained by selling £6785 stock at $105\frac{3}{4}$? [B, $\frac{1}{4}$.]

2. How much must I pay for £2750 stock at $90\frac{7}{8}$? [B, $\frac{1}{8}$.]

✓ 3. How much stock at $101\frac{1}{4}$ can be bought for £3552. 10*s.*? [B, $\frac{1}{4}$.]

4. The proceeds of the sale of $3\frac{1}{2}$ p.c. stock at $93\frac{1}{4}$ amounted to £6844. 13*s.* 9*d.* Find the amount of stock. [B, $\frac{1}{8}$.]

✓ 5. Find to the nearest penny the cost of £5782 stock at $125\frac{1}{4}$. [B, $\frac{1}{4}$.]

6. If £250 Consols are bought at $91\frac{3}{8}$ and afterwards sold at $90\frac{1}{8}$; how much is lost? [B, $\frac{1}{8}$.]

7. A man invested £3500 in $3\frac{1}{2}$ p.c. stock at $104\frac{3}{4}$; after receiving one half-yearly dividend he sold out at $108\frac{1}{4}$; find the total amount gained. [B, $\frac{1}{4}$.]

8. If I invest £4380 in $2\frac{1}{2}$ p.c. Consols at $91\frac{1}{8}$, what shall I gain by selling out at $93\frac{3}{8}$, after receiving one quarter's dividend? [B, $\frac{1}{8}$.]

9. What income would be derived from investing £4200 in $2\frac{3}{4}$ p.c. stock at $96\frac{1}{8}$? [B, $\frac{1}{8}$.]

10. How much money must be invested in $3\frac{1}{4}$ p.c. stock at $103\frac{3}{4}$ to produce an income of £400? [B, $\frac{1}{4}$.]

11. Find to the nearest pound how much must be invested in 4 p.c. stock at 103 to raise an income of £275. [B, $\frac{1}{8}$.]

12. Find to the nearest pound how much must be invested in 3 p.c. stock at $103\frac{1}{4}$ [B, $\frac{1}{4}$] to produce a net income of £500 a year, after deducting income tax at 14*d.* in the £.

13. At what price must $4\frac{3}{4}$ p.c. stock be bought to yield an income at the rate of 5 p.c.? [B, $\frac{1}{4}$.]

14. Calculate the price of $2\frac{1}{2}$ p.c. Consols when £8670 can be bought for £8019. 15*s.* [B, $\frac{1}{8}$.]

15. A man holding £6000 of 3 p.c. stock sells it at $91\frac{1}{4}$, and with the proceeds buys 4 p.c. stock at $129\frac{3}{4}$. What is his loss of income? [B, $\frac{1}{4}$.]

16. A holder of £5500 of $3\frac{1}{2}$ p.c. stock sells out at $91\frac{1}{4}$ and invests the proceeds in a 4 p.c. stock at 101. If the brokerage is $\frac{1}{8}$ for the first stock and $\frac{1}{4}$ for the second, find the change in his income.

17. A man invests £1426 in 5 p.c. stock at $114\frac{3}{4}$: he sells out at $125\frac{1}{4}$ and invests in 3 p.c. stock at $92\frac{3}{4}$; find the change in his income. [B, $\frac{1}{4}$.]

18. I invest £1365 in 3 p.c. stock at $90\frac{7}{8}$, and sell out £1000 of the stock when the price has risen to $93\frac{5}{8}$, and the remainder when it has fallen to $85\frac{1}{8}$. How much do I lose by the transaction? [B, $\frac{1}{8}$.]

19. I invest £4340 in $3\frac{1}{2}$ p.c. stock at $108\frac{3}{8}$, and sell out when the price has risen to $110\frac{1}{8}$. With the proceeds I buy stock at $120\frac{3}{4}$. If the latter pays dividends at the rate of $5\frac{1}{2}$ p.c. find the increase in my income, brokerage being $\frac{1}{8}$ for the first and $\frac{1}{4}$ for the second stock.

20. A man invests £26180 in 3 p.c. stock at $93\frac{1}{4}$, but shortly after sells out half of it at $92\frac{1}{2}$, and with the proceeds buys 4 p.c. stock at $96\frac{3}{4}$. Find to the nearest penny the difference in his income. [B, $\frac{1}{4}$.]

[*Before working Examples* 21–27 *the pupil should read Art.* 329.]

21. What is the cost (including brokerage $\frac{1}{2}$ per cent.) of £500 Mexican Railway $4\frac{1}{2}$ p.c. Debenture stock at 95?

22. A man instructs his broker to buy £500 Railway stock at $128\frac{1}{2}$, and £500 National Telephone Deferred stock at $111\frac{3}{4}$. What is the total cost? [B, $\frac{1}{2}$ per cent.]

23. I buy £800 Consols at 91 [B, $\frac{1}{8}$], and sell £650 Railway stock at $110\frac{3}{4}$ [B, $\frac{1}{2}$ per cent.]. How much is due from me to my broker?

24. I sell £2375 Railway stock at 125 [B, $\frac{1}{2}$ per cent.], and buy £4000 India $3\frac{1}{2}$ p.c. stock at $97\frac{1}{2}$ [B, $\frac{1}{8}$]. How much do I owe my broker for the whole transaction?

25. If I sell out £3600 of $2\frac{1}{2}$ Consols at $89\frac{3}{8}$ [B, $\frac{1}{8}$] and instruct my broker to buy £2700 of 4 p.c. Railway stock at $119\frac{1}{2}$ [B, $\frac{1}{2}$ per cent.], find how much is due to my broker.

26. What is the broker's account for the purchase of £400 National Telephone Co. Preferred stock at $103\frac{3}{4}$, if the brokerage is $\frac{1}{2}$ per cent., and other charges are as follows: Contract Note Stamp 1*s*., Registration Fee 2*s*. 6*d*., and Transfer Stamp £2. 5*s*.?

27. A client instructs his broker to buy

(i) £300 India 3 % stock at 93 [B, $\frac{1}{8}$];
(ii) £400 Transvaal 3 % stock at $95\frac{1}{2}$ [B, $\frac{1}{4}$];
(iii) £500 Mexican Railway stock at 95 [B, $\frac{1}{2}$ per cent.].

There is a charge of 1*s*. for Contract Note Stamp in each transaction, and in addition on the Railway stock the Transfer Stamp is £2. 10*s*., and Registration Fee 2*s*. 6*d*. What is the total amount due to the broker?

Shares.

330. The Capital of a Company, instead of being raised in the form of Stock, is often divided into **Shares** of definite amount, such as £1, £5, or £10. Thus a Company's Capital, amounting to £500,000, may consist of 100,000 *five-pound* shares, or of 50,000 *ten-pound* shares. Such shares may be bought and sold like stock, except that, while stock may be bought or sold in any amounts whatever, only *whole shares* as a rule may change hands.

331. Sometimes a Company does not at first *call up* the whole of its capital. For example, suppose it is thought that though a capital of £500,000 may ultimately be required, only £300,000 can be usefully employed at first: the company in such a case might issue 100,000 nominal *five-pound* shares, requiring the subscribers to pay at first only £3 cash on each share, and the remaining £2 when called upon. The £300,000 thus raised would be called the **paid-up** capital; or the capital might be said to consist of £100,000 *five-pound* shares, £3 **paid-up**.

332. Dividends are allotted either at *so much per share*, or *as a percentage on the* **paid-up** *capital.* In the following examples, if nothing is said to the contrary, the shares are supposed to be *fully paid-up.*

EXAMPLE 1. *If £3 shares in a Company are sold at 7s. 6d. premium, find* (i) *how many shares can be bought for £540,* (ii) *the cost price of 40 shares,* (iii) *the dividend at 4 per cent. on 60 shares.*

(i) Since each share costs £3. 7*s*. 6*d*., or £$3\frac{3}{8}$;

the req^{d} number of shares $= £540 \div £3\frac{3}{8} = 160$.

(ii) Cost of 40 shares $= £3\frac{3}{8} \times 40 = £135$.

(iii) 60 shares represent a capital of £3 × 60, or £180;

$$\text{and dividend at 4 p.c. on } £180 = £180 \times \frac{4}{100}$$
$$= £7.\ 4s.$$

EXAMPLE 2. *I propose to buy £10 Bank Shares, £7 paid-up, at* $2\frac{3}{8}$ *premium, brokerage* 1*s*. *per share:* (i) *What must I give for 250 shares?* (ii) *Find the half-yearly dividend on this purchase, at 12 per cent. per annum.* (iii) *What rate per cent. per annum should I get for my money?*

(i) A share on which £7 has been paid-up costs, at $2\frac{3}{8}$ premium, £9. 7*s*. 6*d*.; or, with brokerage, £9. 8*s*. 6*d*.:
hence 250 such shares cost £9. 8*s*. 6*d*. × 250,
or, £2356. 5*s*.

(ii) The 250 shares bought represent a *paid-up* capital of £7 × 250; and a half-year's dividend on £7 × 250 at 12 p.c. per annum

$$= £7 \times 250 \times \frac{6}{100} = £105.$$

(iii) Cost of one share, including brokerage, = £9. 8*s*. 6*d*.;

$$\text{annual dividend on one share} \quad = £7 \times \frac{12}{100}.$$

Hence on investing £9·425 I have an income of £7 × ·12.

$$\therefore \quad ,, \quad £100 \quad ,, \quad ,, \quad £7 \times \cdot 12 \times \frac{100}{9 \cdot 425};$$

$$\text{and} \quad £7 \times \cdot 12 \times \frac{100}{9 \cdot 425} = £8 \cdot 91 \ldots.$$

That is, I obtain 8·9 p.c. approximately on my cash investment.

EXAMPLES XIX. d.

Find the total cost of

1. Five hundred £1 mining shares at $3\frac{7}{8}$ per share.

2. Sixty £20 shares at $1\frac{3}{16}$ discount.

all 3. How much must I pay ~~a broker who~~ to buys for me forty-four £1 shares at $2\frac{7}{8}$, brokerage $4\frac{1}{2}d.$ per share, and eighty £5 shares at $1\frac{5}{16}$ premium, brokerage $9d.$ per share?

How much cash will be obtained by selling

4. Fifty £20 shares (£15 paid up) at $2\frac{1}{2}$ premium?

5. Two hundred and fifty £5 shares (£2 paid up) at $\frac{3}{4}$ premium, brokerage $9d.$ per share?

6. How many £20 shares (£15 paid up) at $2\frac{1}{2}$ premium, brokerage $2s.$ per share, can be bought for £352?

7. Find the annual income arising from

(i) Thirty-five £10 shares paying $4\frac{1}{2}$ p.c.

(ii) Forty-seven £5 shares paying $6\frac{1}{2}$ p.c.

(iii) Seventy-five £20 shares (£15 paid) at $7\frac{1}{2}$ p.c., together with a bonus of $6d.$ per share.

8. What is the half-yearly dividend on 20 shares of £10 each at 6 p.c. per annum, less income tax at $10d.$ in the £?

9. How much per cent. for his money does an investor get by buying

(i) £10 shares (£3 paid up) at £5 when the dividend is 10% per annum?

(ii) £20 shares (£9 paid up) paying 8% per annum, when the shares are at $4\frac{1}{2}$ premium?

(iii) £100 shares (£$15\frac{1}{2}$ paid up) at $20\frac{1}{2}$ premium, the half-yearly dividend being $17s.$ per share?

10. If £5 shares in a Company are sold at $2\frac{1}{2}$ premium, find (i) how many shares can be bought for £375, (ii) the cost price of 20 shares, (iii) the dividend at $5\frac{1}{2}$ p.c. on 75 shares.

11. The £20 shares of a Bank have only £6 per share paid up, and their present price is $10\frac{3}{4}$. If brokerage is charged at $1s.\ 3d.$ a share, find (i) how many shares can be bought for £605. $10s.$, (ii) the cost price of 36 shares, (iii) the dividend on 50 shares at $8\frac{3}{4}$ p.c.

12. A man sells out $2\frac{3}{4}$ p.c. stock at $96\frac{1}{4}$ and by investing the proceeds in shares which pay an annual dividend of £4 per share raises his income 5 per cent. What was the price of each share?

all

EXAMPLES XIX. e.

(Miscellaneous Examples on Stocks and Shares.)

[*Unless otherwise stated brokerage and other charges are supposed to be included in the prices quoted.*]

1. I invest £5520 in $2\frac{1}{2}$ p.c. Consols at 92: at what price must I sell out to gain £210 on the transaction?

2. If I invest £1200 in $2\frac{3}{4}$ p.c. Debenture stock at $95\frac{1}{2}$, what is my income to the nearest penny? [B, $\frac{1}{4}$.]

3. Which is the more profitable investment, $4\frac{1}{2}$ p.c. stock at 152, or $2\frac{1}{2}$ p.c. Consols at 85? If £3230 is invested in each, what is the difference in annual income?

4. An income of £176 is derived from the investment of £6200 in a $2\frac{3}{4}$ p.c. stock. At what price was the stock bought? [B, $\frac{1}{8}$.]

5. At what price must I buy 7 p.c. stock so as to get 4 p.c. for my money? [B, $\frac{1}{4}$.]

6. What income is derived from investing £750 in $3\frac{1}{2}$ p.c. stock at $89\frac{3}{4}$? [B, $\frac{1}{4}$.] What would be the loss of capital if the stock were sold out at $87\frac{1}{8}$? [B, $\frac{1}{8}$.]

7. At what price must $4\frac{1}{2}$ p.c. stock be to produce the same income as 5 p.c. stock at 115? Find the half-yearly dividend from an investment of £2346 in either stock.

8. A man invests a fourth of his capital in $2\frac{1}{4}$ p.c. stock at 90 and the remainder in $3\frac{1}{2}$ p.c. stock at 105; find the average rate per cent. on his capital.

9. Find to the nearest penny the cost of £225 Canal stock at $59\frac{7}{8}$, allowing for brokerage at $\frac{1}{2}$ per cent.

10. A sum of £994. 19*s.* had to be raised from the sale of N.S. Wales Inscribed stock at $99\frac{3}{4}$. The brokerage and Contract Stamp amounted to £2. 11*s.* How much stock had to be sold?

11. What sum must be invested in a 5 p.c. stock at $130\frac{1}{4}$ so as to produce an income of £120 a year, after paying income tax at 8*d.* in the £? [B, $\frac{1}{4}$.]

12. My income from 3 p.c. stock after deducting 10*d.* in the £ for income tax is £442. 15*s.* I sell out at $78\frac{7}{8}$ and buy 4 p.c. stock at $102\frac{3}{8}$; find to the nearest penny the alteration in net income, brokerage $\frac{1}{8}$ being charged on each transaction.

13. A man invested £1000, half in 3 p.c. stock at par, subject to an income tax of 1*s.* in the £, and half in $3\frac{1}{4}$ p.c. stock at 104, free of income tax. Which investment was the more advantageous and by how much?

14. What was the total amount of a broker's bill for the purchase of £4376 Great Western Railway stock at 124; brokerage $\frac{1}{2}$ per cent. upon the money value; Contract Stamp 1*s.*; Registration Fee 2*s.* 6*d.*; Transfer Stamp £27. 5*s.*?

15. A man invests half his capital in 3 p.c. stock at $97\frac{1}{2}$, and the other half in 4 p.c. stock at 125; his total income from both sources is £510. How much did he invest?

16. The difference between the incomes derived from investing a certain sum in 6 p.c. stock at 126 and in 9 p.c. stock at 210 is £22. 10*s.* What is the amount invested?

17. How much must a man bequeath, subject to a legacy duty of 10 p.c., so as to provide a sufficient investment in $4\frac{1}{2}$ p.c. stock at $103\frac{1}{2}$ to bring in a net income of £60 a year, after deducting income tax at 10*d.* in the £.

18. I am a holder of 3 p.c. stock which yields me an income of £300 a year. I sell half of it at 92, and invest the proceeds in an American railway at 46. What dividend per cent. per annum ought the latter stock to pay so that the transaction may increase my income by £50?

19. A man sells £4560 of $2\frac{1}{2}$ per cent. Consols at 85, and with the proceeds purchases 4 per cent. stock at 114. After deducting income tax (at the same rate in both cases), he finds his income better by £20. 18*s.* 0*d.* What is the income tax per pound?

20. If fifty £10 shares in a company paying a dividend of 8 p.c. are sold for £18 each, and the proceeds invested in £5 shares in another company at £3. 10*s.* each, find what difference in income results if the second company pays a dividend of $3\frac{1}{2}$ p.c.

21. One company pays $5\frac{1}{2}$ p.c. on shares of £100 each; another pays at the rate of $3\frac{1}{2}$ p.c. on shares of £10 each; if the price of the former be £115. 10*s.*, and of the latter £7. 15*s.*, compare the rates of interest which the shares return to a purchaser.

22. A man sells out £3840 of 8 p.c. mining stock at $187\frac{1}{2}$, and re-invests the proceeds in $4\frac{1}{2}$ p.c. Railway stock at 108, his net income being thus diminished by £6. 18*s.* Find the rate of income tax in the £.

23. A person invests £8000 in $2\frac{3}{4}$ p.c. stock at 97 and pays income tax at 1*s.* in the £. On the stock rising to 99 he sells out and invests the proceeds in $2\frac{1}{4}$ p.c. stock at 73, free of income tax. Find to the nearest penny the net change in his income.

24. A firm of Solicitors instructed their brokers to purchase sufficient India 3 p.c. stock to enable them to pay a sovereign per week from the dividends to an old servant. India stock was then at $97\frac{3}{4}$, brokerage $\frac{1}{8}$, income tax 1s. in the £. There was a charge of 1s. for Contract Note Stamp. What (to the nearest penny) was the sum paid to the brokers?

25. A man held one hundred £10 shares which stood at 150 p.c. premium, and he decided to sell and re-invest in the purchase of certain £1 shares which stood at $16\frac{2}{3}$ p.c. discount, paying all charges connected therewith from a separate fund. How many shares did he buy?

26. A lady has £1000 to invest: she tells her broker to purchase £500 Canada $3\frac{1}{2}$ p.c. stock at 104, and to invest the balance in New Zealand 3 p.c. stock at $97\frac{1}{2}$. Find to the nearest penny how much N.Z. stock he must buy. [B, $\frac{1}{4}$.]

27. A Banker had £50,000 to invest: he bought £20,000 Jamaica $3\frac{1}{2}$ p.c. stock at $101\frac{1}{8}$, £10,000 Cardiff Corporation 3 p.c. stock at 97, and sufficient Bristol 3 p.c. stock at $97\frac{1}{4}$ to complete the amount. Find to the nearest pound how much Bristol stock he bought. Also find, to two places of decimals, the average income per cent. on the whole investment after deducting 1s. in the £ for income tax.

28. A speculator buys £4000 stock at 65, and his bank lends him money to the extent of 40 % of the actual value, charging 4 % for the loan. He holds for a year, takes a dividend of $2\frac{1}{4}$, and sells out at 70. Find his actual profit. Also find, to one decimal place, the profit per cent. on his capital outlay.

MISCELLANEOUS EXAMPLES.

333. EXAMPLE 1. *By investing £1272, partly in 3 p.c. stock at 96, and partly in 4 p.c. stock at 120, a man secures an annual income of £41; how much does he invest in each kind of stock?*

Suppose he invests £x at 3 %, and £y at 4 %; then

$$x\times\frac{3}{96}+y\times\frac{4}{120}=41,$$

or,
$$x\times\frac{1}{32}+y\times\frac{1}{30}=41, \text{ ....(i).}$$

Also
$$x+y=1272. \text{ ....(ii).}$$

Multiply (i) by $2\times16\times15$, the L.C.M. of 32 and 30; then

$$15x+16y=19680;$$

and from (ii)
$$15x+15y=19080.$$

By subtraction, $y=600$; hence from (ii), $x=672$.

Thus he invests £672 at 3 %, and £600 at 4 %.

EXAMPLE 2. *One alloy of metals contains 90 per cent. of copper and 10 per cent. of tin. Another alloy contains 93 per cent. of copper and 4 per cent. of tin. Find in what proportions they must be mixed that the mixture may contain 9 per cent. of tin, and what percentage of copper it will contain.* (C.S.)

Suppose x parts of the first metal have to be mixed with y parts of the second; then the mixture contains $x+y$ parts with 9 % of tin.

Also 10 % of x and 4 % of y are tin.

$$\therefore\ \frac{10}{100}x+\frac{4}{100}y=\frac{9}{100}(x+y),$$

that is,
$$10x+4y=9x+9y;$$

$$\therefore\ x=5y,$$

so that
$$\frac{x}{y}=\frac{5}{1}.$$

Hence 5 parts of the first metal must be taken with 1 part of the second.

Now suppose 500 parts of the first are mixed with 100 parts of the second.

Then
$$\left.\begin{array}{l}\text{450 parts of the first}\\ \text{93 .... second}\end{array}\right\}\text{ are copper.}$$

Hence in 600 parts there are 543 parts of copper; so that the percentage of copper is $\frac{543}{6}$, or 90·5.

MISCELLANEOUS EXAMPLES V.

Exercises for Revision.

A.

1. Find, to the nearest kilogram, the weight of 1 ton, assuming that 1 Kg. = 2·2046 lbs.

2. Find, correct to the nearest penny, the value of 10 tons 13 cwt. 74 lbs. at £1. 3*s*. 4*d*. per ton.

3. The profits of a business are £4875. One of the partners sells 0·24 of his share of the profits for £780. What fraction of the profits belong to him?

4. The length of a room is 23 ft. 6 in., and it costs £5. 17*s*. 6*d*. to paint the ceiling at 2*s*. 3*d*. per square yard. What is its breadth?

5. A tradesman sells goods for £80 at a loss of 4 per cent.; what must he sell them for to gain 15 per cent.?

6. A man receives £8. 8*s*. 9*d*. as his dividend on some $3\frac{3}{4}$ p.c. stock. How much of the stock does he hold?

B.

7. It costs £339. 7*s*. to fence in a square enclosure at £2. 11*s*. 5*d*. per 20 yds.; what is its acreage?

8. A man buys 300 eggs at 15 for 1*s*. and sells them at 10*d*. per dozen. Find (i) his actual, (ii) his percentage gain or loss.

9. When 52 lbs. of coffee are worth as much as 12 lbs. of tea, 22 lbs. of tea are worth as much as 572 lbs. of sugar, a cask of sugar costs 2 guineas, and 1 cwt. of coffee costs 8 guineas, what is the weight of a cask of sugar?

10. A piece of wire, $3\frac{1}{2}$ metres long, with a sectional area of 2 square mm., is heated, and then drawn to a length of 18 metres. Find, to the nearest hundredth of a square mm., the area of a cross-section of the wire thus drawn. (C.S.)

11. If 1 cwt. of sugar costs £1. 6*s*. 8*d*., draw a graph to find the price of any number of pounds. Find the cost of 26 lbs. How many pounds can be bought for 4*s*. 10*d*.?

12. Calculate (to a penny) the compound interest on £1756 for 3 years at $2\frac{1}{4}$ per cent. per annum.

C.

13. Express as a decimal the ratio of £1. 7*s*. $7\frac{1}{2}$*d*. to £8. 2*s*. 6*d*. Hence state what percentage the first sum is of the second.

14. Find the square root of 9·0707 correct to 6 places of decimals.

15. If a money-lender charges $1\frac{1}{2}d.$ a week for a loan of £1, what rate per cent. of interest per annum does he obtain?

16. A rectangular carpet 16 ft. 3 in. long by 12 ft. 2 in. broad lies on the floor of a room 21 ft. 8 in. long and 18 ft. 3 in. broad. What fraction of the area of the room is covered by the carpet?

17. Three men together fill a pit in an hour and a half, throwing in respectively 8, 10, and 12 equal shovelfuls of earth per minute. In what time could each man have filled up the pit by himself?

18. A tradesman gains 35 per cent. by selling an article for 6*s.* 9*d.* What would he have gained per cent. if he had sold it for 8*s.* 3*d.*?

D.

19. How many bricks $9'' \times 4\frac{1}{2}'' \times 3''$ can be carried by a truck whose load is 5 tons? The bricks in question weigh 145 pounds per cubic foot. Give your result to two significant figures. (C.S.)

20. *A* walks at the rate of 4 miles an hour, *B* at the rate of $4\frac{1}{2}$ miles an hour: how many yards start can *B* give *A* in walking over a course of 1350 yards?

21. A tradesman marks articles 30 per cent. above their cost price, but takes off 5 per cent. for cash: find the cost of a vase for which I pay down £8. 4*s.* 8*d.*

22. Assuming 1 lb. $=0{\cdot}4536$ Kg., find the price, to the nearest franc, of 2 cwt. 26 lbs. of coffee at 4 fr. 75 c. per kilogram.

✓ **23.** If £9000 of a 4 per cent. stock is sold at $112\frac{1}{2}$, and with the proceeds a $5\frac{1}{2}$ per cent. stock is bought at $168\frac{3}{4}$, what is the resulting change of dividend?

24. How must a sum of £1130 be divided into two parts so that by receiving 7 per cent. on one part and 2 per cent. on the other a man may receive 5 per cent. on the whole sum?

E.

25. Add together $2\frac{2}{9}$, $3\frac{11}{12}$, $4\frac{3}{8}$, $2\frac{5}{6}$ and divide the result by the difference between $3\frac{7}{18}$ and $2\frac{5}{16}$.

26. Express in its simplest form the ratio of £17. 19*s.* 4*d.* to £33. 7*s.* 4*d.*

27. Find the cost, to the nearest shilling, of making a road 47 mi. 3 fur. 115 yds. long at £38. 12*s.* per mile.

28. A man buys 50 bicycles for £500, but 20 of them are soiled. He determines to sell each soiled one at three-fourths the price at which he sells a new one. What must this price be in order that he may make a profit of 35 per cent. on his outlay?

29. Two men and a boy can do a piece of work in 5 days, whilst a man and two boys can do it in 6 days. If a man is paid at the rate of 28*s*. a week, what should be the wages of a boy?

30. The populations of the upper and lower parts of a town were equal, and after the former had fallen 20 per cent., and the latter risen 15 per cent., the total number of inhabitants was 39,390. What was the original population of each part?

F.

31. What will be the gain per cent. if eggs bought at the rate of six for 5*d*. are sold at the rate of five for 6*d*.?

32. A man measures the length and breadth of a rectangular field by pacing and takes the length of his pace as 30 inches. He makes the area 2 acres 3 roods. His pace is really 32 inches. Find the true area. (C.S.)

33. A man sells out £10,000 of $2\frac{3}{4}$ per cent. stock at par, and with part of the proceeds purchases enough 4 per cent. stock at 140 to yield him the same income as before. What amount of cash does he retain in hand?

34. Find to the nearest penny the amount at simple interest of £389. 6*s*. 8*d*. from May 17th to July 29th at $3\frac{1}{2}$ per cent.

35. A man has to glaze 36 picture frames, each of which requires a piece of glass 17 inches by 9 inches. He cuts these from sheets of glass measuring 18 inches by 30 inches. How many such sheets must he use? If he buys the sheets at the rate of twopence per square foot and sells the glass actually put into the frames at threepence per square foot, what profit per cent. does he make?

36. A man, his wife, and son, worked in a garden; the man for 2 days, his wife for $3\frac{1}{2}$ days, the boy for 4 days. Their daily wages were: man's to woman's in the ratio of 7 to 4, and man's to boy's in the ratio of 7 to 3. Their total earnings amounted to £1. What were the daily wages of each?

G.

37. Find the price of a steel casting weighing 103 tons 5 cwt. 3 qrs. at £5. 8*s*. 6*d*. a ton.

If the steel weighs 488 lbs. per cubic foot, what is the volume of the casting? (C.S.)

38. A closed box whose outside measurements are 26, 19, and 18 inches is made of wood which is $\frac{1}{2}$ inch thick. If a cubic foot of wood weigh 40 lbs., find the weight of the box.

39. The wages of a man for a day's work are 5*s.* 6*d.*, and those of a woman 2*s.* 9*d.* A man can do one and a half times as much work as a woman can do in the same time. Certain work has to be done which would occupy 50 men for 80 days; find the difference in the cost if 20 men and 30 women were employed to do the work instead of 50 men. (C.S.)

40. What sum will amount to £308. 14*s.* if allowed to accumulate for 3 years at compound interest at the rate of 5 % per annum?

41. At what price must a man mark an article which cost £1. 18*s.* so that he may take off discount at 5 %, and still make a profit of 5 %?

42. A man invests £2310 partly in 3 p.c. stock at 81 and partly in $4\frac{1}{2}$ p.c. stock at 135, and derives an annual income of £80. How much does he invest in each kind of stock?

H.

43. Find, correct to four places of decimals, the cube of 1·2345.

44. Simplify $$\frac{4\frac{1}{3}-1\frac{1}{12}}{6\frac{2}{3}+4\frac{7}{12}}-\frac{2{\cdot}1-1{\cdot}94}{{\cdot}93+{\cdot}87}.$$

45. If 100 yds. of copper wire weigh 14·08 lbs., find to the nearest gram the weight of 1 metre, having given that

1 metre = 1·09 yds., and 1 gram = 0·0022 lb.

46. A milk-vendor receives two adulterated supplies which contain respectively 10 per cent. and 3 per cent. of water. How much must be taken of each kind so as to form 84 gallons of a mixture which contains 95 per cent. of milk?

47. A clock which shews correct time on Monday, at noon, gains 2 minutes 15 seconds per day. When it shews 5 o'clock on Wednesday evening, what is the correct time, to the nearest second?

48. Two partners invest £1350 and £750 respectively in their business, and arrange that 40 per cent. of the profits should be divided equally between them, and the remaining profits treated as interest on the capital. If one partner's share is £45 more than that of the other, what was the whole amount of the profits?

I.

49. A tailor sends in a bill for £9. 10*s.*, but he will accept £9. 0*s.* 6*d.* as payment in full if the bill is paid at once. What rate of discount does he allow?

50. When the rateable value of the county of London was £41,647,310 it was necessary to collect a rate of 2*s.* 11*d.* in the pound to pay various expenses. How much were these expenses to the nearest thousand pounds?

If the rateable value increases by 5 per cent. and these expenses increase by 10 per cent., find, to the nearest halfpenny, what rate in the pound must then be collected to pay these expenses? (C.S.)

51. 216 eggs are bought at 12 for 10*d.* and are sold at 10 for a shilling; some eggs are broken, but a profit of 20 per cent. is made: how many eggs are broken?

52. A sum of money is to be paid down, such that with compound interest at 4 per cent. it will amount to £800 at the end of 4 years. The interest is to be added annually. Find, to the nearest pound, how much must be paid. (C.S.)

53. It costs £28 to carpet the whole floor of a room 32 ft. long and 25 ft. wide; if a space 1 ft. 3 in. be left uncovered all round, what will be the saving in cost?

54. Three labourers, *A*, *B*, *C*, are employed in filling carts with sand. When all work together they can fill a cart in 8 min.; *A* and *C* together can do so in 12 min., and *A* and *B* in 13 min. 20 sec. If each man work for the same time, in what proportions should a wage of 10*s.* be divided between them?

J.

55. Shew how each of the following calculations can be done mentally, giving results:

(i) 480 articles at 13*s.* $3\frac{1}{2}d.$ each.

(ii) $24 \times 25 \times 26$.

(iii) The income from £1200 invested in the $2\frac{3}{4}$ per cents. at 110.

(iv) Which investment gives the larger income: Railway Stocks at 250, paying $7\frac{1}{2}$ per cent. dividend, or Bank stock at 400, paying $11\frac{1}{4}$ per cent. dividend?

56. If 3 men *or* 8 boys can do a piece of work in 17 days, in what time will 7 men *and* 4 boys do a piece of work twice as great?

57. In what time will the simple interest on a sum of money become equal to the principal at 4 per cent.?

58. A cistern is constructed to hold 100 gallons, and the base of the cistern is a square yard. What is the depth of the cistern, to the nearest inch? A cubic foot is 6·23 gallons. (C.S.)

✓ **59.** I invest equal sums in a 4 per cent. stock and in a 3 per cent. stock, and get 5 per cent. for my money; the 4 per cents. are at 90: what is the price of the 3 per cents.?

60. If *A* runs a mile in 7·5 minutes, and *B* runs at the rate of 7·5 miles an hour, how far behind will the loser be in a race which the faster wins in 6 minutes?

K.

61. A man in India wishes to send his wife in England £100 a year in monthly instalments. How many rupees worth 1*s.* 4*d.* each must he transmit monthly? (C.S.)

62. Find the weight (to the nearest kilogram) of an iron rod of square section, 10 metres long and 2·3 centimetres broad. A cubic centimetre of iron weighs 7·207 grams. (C.S.)

63. Find (as intelligently as you can) the *total* amount of the following bill, showing all your methods of calculating. You may combine items. 135 eggs at 2*s*. 3*d*. a score, 135 eggs at 1*s*. 9*d*. a score, 110 lb. of butter at 1*s*. $7\frac{1}{2}$*d*. per lb., 106 cakes at 11*s*. 3*d*. a dozen, 220 lb. of lard at $8\frac{1}{4}$*d*. per lb., 318 lb. of biscuits at 4*s*. 3*d*. per dozen lb.

Take off a discount of 10*d*. in the £ from the total of the bill. (C.S.)

64. I hold £720 stock, partly in 3 per cents. and partly in 5 per cents., thereby obtaining the same income as if I held the whole in a stock paying $3\frac{1}{3}$ per cent. How much of each kind of stock do I hold?

65. Two jars containing respectively 3 and 5 gallons are filled with mixtures of alcohol and water. In the smaller jar 25 per cent. of the mixture is alcohol, in the larger 25 per cent. of the mixture is water. The jars are emptied into a 9-gallon cask, which is filled up with water. Find the percentage of alcohol in the cask. (C.S.)

66. A workman is able to save $12\frac{1}{2}$ per cent. of his wages, but if his wages were raised 2*s*. a week and his expenses were increased by 10 per cent., his annual savings would be diminished by 17*s*. 4*d*.; what are the man's weekly wages, a year being taken as 52 weeks exactly?

L.

67. Find the value of.

$$\text{(i)}\ \frac{15\frac{3}{4}-\frac{14}{3}\times 1\frac{5}{6}}{\frac{1}{5}\times 23\frac{1}{3}+\frac{91}{36}}; \qquad \text{(ii)}\ \frac{\frac{1}{7}-\frac{1}{10}+\frac{4}{11}-\frac{1}{3}}{9-\left(\frac{1}{7}-\frac{1}{10}\right)\left(\frac{4}{11}-\frac{1}{3}\right)}.$$

68. A traveller intends to walk to a station, which is 2 miles off, in 40 minutes. What is this speed (i) in miles per hour, (ii) in feet per second?

After going half a mile he remembers that he has left a parcel behind and hurries back to fetch it, keeping up the same new rate of speed all the way back and then to the station. What must this speed be (in miles an hour) if he is to reach the station at the time originally planned? (C.S.)

69. A watch is offered for sale at £5. 15*s*.; and if that price is reduced by 5 per cent., the dealer who is selling it will still make $9\frac{1}{4}$ per cent. profit; how much did the watch cost him?

70. If the population of a certain county is 10 % more than it was 10 years ago, and 5 % more than it was 4 years ago, what is the increase in the last 10 years, given that the population 4 years ago was twenty-two millions?

71. If a kilogram $= 2{\cdot}205$ lbs., and a gallon of water weighs 10 lbs., compare the volumes of a quart and a litre. Hence draw a graph from which the equivalent of any number of quarts may be expressed approximately in litres.

Express $2{\cdot}5$ litres in quarts, and $4{\cdot}7$ quarts in litres.

72. A new railway, 110 kilometres long, in Holland will cost 6,441,000 florins. Find the whole cost, and the cost per mile, in English money to the nearest thousand pounds. Assume that 1 Km. $= \frac{5}{8}$ mile; £1 $=$ 12 florins 11 cents; 1 florin $=$ 100 cents. (C.S.)

M.

73. Find, correct to a penny, the interest on £874 for 220 days at $4\frac{1}{2}$ per cent. per annum.

74. A man's income from $2\frac{1}{2}$ per cent. Consols (after income tax at 1*s*. in the pound has been deducted) is £539. 12*s*. How much stock does he hold?

75. Of two square fields, one contains 10 acres, while the other is broader by 1 per cent. Find the difference in area, expressed in yards, feet, and decimals of a foot. (C.S.)

76. Find, within the hundredth part of an inch, the length which represents on a map, whose scale is 25 inches to the mile, the side of a square field which contains 30 acres.

77. If the manufacturer makes a profit of 20 %, the wholesale dealer a profit of 25 %, and the shopkeeper a profit of 40 %, what was the cost of the manufacture of an article bought at a shop for 17*s*. 6*d*.?

78. The net profits of a business, of which the capital is £500,000, amount in a certain year to £30,562. 9*s*. 6*d*. To this is added a balance of £1050. 19*s*. 0*d*. brought forward from the previous year. The sum of £5000 is placed on the reserve fund. A dividend is paid to the shareholders which is as great a whole number of pounds per £100 of capital as possible, and the remainder is carried forward.

What is the rate per cent. of the dividend, and what is the amount carried forward? (C.S.)

N.

79. It was calculated that, in boring the Simplon tunnel, the rock yielded a centimetre of its substance to the drill per minute. The whole length of the tunnel is $19{\cdot}8$ kilometres. How long would it have taken to bore the tunnel if the work had been entirely uninterrupted?

Answer to be given to the nearest tenth of a year. (C.S.)

80. If 80 Kg. of tea at $2{\cdot}75$ fr. the kilogram are mixed with 20 Kg. at $9{\cdot}5$ fr., at what price per kilogram should the mixture be sold to gain 10 per cent.? (C.S.)

81. A cubical box of external dimensions 17 inches each way would contain crushed ore of the value of £421. 17*s.* 6*d.* if it were made of material 1 inch thick; but by mistake it has been made of thicker material, and the difference in the value of the ore which it will hold is consequently £78. 17*s.* 6*d.*; what is the real thickness of the material?
(C.S.)

82. If £12,100 is lent on condition that £1 per day is paid as interest, find (correct to two decimal places) what rate per cent. per annum the lender receives for his money?

83. What maximum start in a mile race ought a cyclist whose record is 3 minutes 15 seconds to give to another whose record is 3 minutes 20 seconds, assuming that the pace of each is uniform over the course?
(C.S.)

84. A sum is invested at compound interest payable annually. The interest in two successive years was £112. 10*s.* and £115. 6*s.* 3*d.* Find the rate of interest and the value of the principal at the beginning of the first of these years.
(C.S.)

O.

85. Shew that, in finding by Practice the price of a number of articles at £11. 18*s.* 4*d.* each, or the price of a number of articles at £2. 12*s.* 6*d.* each, one line of multiplication and one line of division is sufficient.
(C.S.)

86. What sum, correct to the nearest pound, will amount to £4548. 12*s.* in 63 days at $2\frac{1}{4}$ per cent.?

87. A cube of metal, each edge of which measures five-eighths of an inch, weighs 0·625 lb. What is the length of each edge of a cube of the same metal which weighs 40 lbs.?
(C.S.)

88. Two sets of telegraph wires are carried on opposite sides of a railway on posts whose distances apart are 275 feet in one case, and 135 feet in the other. An engine starts from a point where two posts are exactly opposite one another, runs an exact number of quarter-miles, and stops at a point where two posts are again exactly opposite one another. Find the least distance which the engine can have travelled.

89. A dealer has two sorts of tea, one of which he could sell at 1*s.* 8*d.* per lb. and make 25 per cent. on his outlay, the other at 2*s.* 3*d.* and make $12\frac{1}{2}$ per cent. What profit per cent. will he make if he mixes them in equal quantities and sells the mixture at 1*s.* 11*d.* per lb.?
(C.S.)

✓ **90.** A man's income is derived from £19,250 held in 3 per cent. stock. How much must he sell out at 99 so that by reinvesting in 4 per cent. stock at 104 he may increase his income by 10 per cent.?

91. If 24 men can reap a field of 29 acres in a given time, find roughly by means of a graph the number of acres which could be reaped in the same time by 15, 33, and 42 men respectively.

92. *A* sells an article to *B* at a profit of 20 per cent. *B* sells it to *C* at a profit of 5 per cent. If *C* pays 70 shillings, what did it cost *A* (to the nearest shilling)? (C.S.)

P.

93. Find, correct to the nearest penny, the interest on £2378 for 68 days at $2\frac{3}{4}$ per cent.

94. Assuming that 1 metre = 39·37 inches, and that £1 is worth 25 fr. 20 c., express a price of 4*s.* 11*d.* per yard in francs per metre.

95. How many days will a stream discharging 1000 gallons a minute take to fill a reservoir 100 yards square and 20 feet deep?

[A gallon is 0·16 of a cubic foot.]

96. A man buys 120 eggs for 5*s.* and pays 1*s.* 8*d.* for carriage. If 20 eggs are broken in transit, and the remainder are to be sold so as to make a profit of half a crown, draw a graph to shew the price of any number of eggs. Find to the nearest penny the price of 3 dozen eggs: find also the number of eggs which would fetch 5*s.* 6*d.*

97. A square room, whose floor measures 32 sq. yds. 1 sq. ft. has a height of 11 ft. 6 in.; find the expense of painting its walls and ceiling at $8\frac{1}{2}d.$ per square yard. Allowance is to be made for a door 7 ft. 6 in. by 3 ft. 4 in., and a window 5 ft. 4 in. by 3 ft. 9 in.

98. A man pays for insurance on his life 15 per cent. of his gross income; and after paying income tax on the remainder at 6*d.* in the pound, he has £513. 16*s.* 6*d.* left. What is his gross income?

99. A cylindrical glass has an internal diameter of 2 cm., and is filled with water to about 1 cm. from the top. It is found that 58 steel pellets can be dropped in before the glass is quite full. Find in c. mm. the volume of a pellet; that of a cylinder of diameter *d* cm., and height *h* cm., being $d^2 \times h \times 0{\cdot}7854$ cubic centimetres. (C.S.)

100. If a bar of uniform section be supported horizontally at its extremities A, B, it is known that the load which, placed at any point P, will just break the bar is inversely proportional to the product AP × BP. If the bar be 10 feet long and the breaking load at its middle point be 12 cwt., find the loads which will just break the bar when placed 1, 2, 3, and 4 feet from either end. (C.S.)

CHAPTER XX.

CONTINUED FRACTIONS.

FURTHER NOTES ON APPROXIMATE METHODS.

334. FRACTIONS of the form

$$\text{(i)}\quad 3+\cfrac{1}{2+\cfrac{1}{4+\frac{1}{3}}},\qquad\text{or}\qquad\text{(ii)}\quad \cfrac{1}{2-\cfrac{4}{7-\frac{2}{3}}}$$

are known as **Continued Fractions.** The simplification of such fractions might have been included in the section on Complex Fractions in Chap. v., Art. 131, but (for reasons which will appear later) continued fractions have been reserved for separate treatment here.

EXAMPLE. *Simplify*

$$\text{(i)}\quad \cfrac{5}{8+\cfrac{6}{8-\frac{10}{11}}};\qquad \text{(ii)}\quad 3-\cfrac{3\frac{1}{8}}{1+\cfrac{1}{1+\frac{1}{7}}}.$$

We begin with the lowest part and work upwards, step by step: thus

(i)

$$\frac{6}{8-\frac{10}{11}}=\frac{6\times 11}{88-10}=\frac{6\times 11}{78}=\frac{11}{13};$$

$$\frac{5}{8+\frac{11}{13}}=\frac{5\times 13}{104+11}=\frac{5\times 13}{115}=\frac{13}{23}.$$

(ii) Here the simplification at each step may be done mentally.

$$3-\cfrac{3\frac{1}{8}}{1+\cfrac{1}{1+\frac{1}{7}}}=3-\frac{3\frac{1}{8}}{1+\frac{7}{8}}=3-\frac{25}{15}$$

$$=3-\frac{5}{3}=1\tfrac{1}{3}.$$

EXAMPLES XX. a.

Simplify the following continued fractions :

1. $\cfrac{1}{2+\cfrac{3}{4+\frac{1}{2}}}$.

2. $\cfrac{3}{1+\cfrac{2}{6+\frac{1}{5}}}$.

3. $2+\cfrac{4}{1+\cfrac{1}{1+\frac{2}{7}}}$.

4. $3-\cfrac{1}{2+\cfrac{2}{3-\frac{1}{3}}}$.

5. $\cfrac{6}{2+\cfrac{5}{6+\frac{1}{2}}}$.

6. $\cfrac{10}{1+\cfrac{8}{3+\frac{7}{3}}}$.

7. $7-\cfrac{2}{8-\cfrac{1}{1-\frac{1}{8}}}$.

8. $\cfrac{11}{3+\cfrac{1}{3-\frac{2}{5}}}$.

9. $5-\cfrac{2}{4-\cfrac{3\frac{1}{2}+\frac{2}{3}}{4-\frac{2}{3}}}$.

10. $\cfrac{2+\cfrac{1}{3-1\frac{3}{5}}}{6+\cfrac{7}{8+\frac{10}{11}}}$.

11. $\cfrac{6+\cfrac{1}{6-\frac{1}{6}}}{4-\cfrac{1}{4-\frac{1}{4}}}\times 10\frac{8}{9}$.

12. $\cfrac{2+\cfrac{1}{3\frac{4}{5}}}{2+\cfrac{1}{3+\cfrac{1}{1+\frac{1}{4}}}}$.

13. $2-\cfrac{3}{9-\cfrac{7}{1+\cfrac{1}{2-\frac{3}{8}}}}$.

14. $2-\cfrac{2}{8-\cfrac{7}{1+\cfrac{1}{2+\frac{1}{9}}}}$.

335. The above Exercise will furnish sufficient practice in simplifying continued fractions such as the pupil is likely to meet with in elementary Arithmetic. The rest of the present section will be devoted to the consideration of continued fractions of the form

$$a+\cfrac{1}{b+\cfrac{1}{c+\dots}}, \quad \text{or} \quad \cfrac{1}{a+\cfrac{1}{b+\cfrac{1}{c+\dots}}}$$

For the sake of saving space it is usual to write such fractions in the more compact forms :

$$a+\frac{1}{b+}\,\frac{1}{c+}\dots, \qquad \frac{1}{a+}\,\frac{1}{b+}\,\frac{1}{c+}\dots.$$

It is to be noticed that *the letters a, b, c, ... all denote integral numbers, the signs are all positive, and each of the numerators is unity.*

336. The method of the following example, which is quite general, will shew that any simple fraction or decimal can be expressed in the form of a continued fraction *with unit numerators* and *with all the signs positive.*

EXAMPLE. *Express* $\frac{215}{93}$ *as a continued fraction.*

(i) $\frac{215}{93}=2+\frac{29}{93}=2+\cfrac{1}{\frac{93}{29}}$

(ii) $=2+\cfrac{1}{3+\frac{6}{29}}=2+\cfrac{1}{3+\cfrac{1}{\frac{29}{6}}}$

(iii) $=2+\cfrac{1}{3+\cfrac{1}{4+\frac{5}{6}}}=2+\cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{\frac{6}{5}}}}$

$$=2+\cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{1+\frac{1}{5}}}}.$$

It will readily be seen that the process corresponds to the method of finding the H.C.F. of 215 and 93. Thus

$$\begin{array}{r|r|r|l} \mathbf{3} & 93 & 215 & \mathbf{2} \\ \mathbf{1} & 6 & 29 & \mathbf{4} \\ & 1 & 5 & \mathbf{5} \end{array}$$

The successive quotients are 2, 3, 4, 1, 5.

337. From the successive lines of work in the above example, it appears that

(i) 2 is a first approximation to the value of $\frac{215}{93}$, and is *too small* because the part $\frac{29}{93}$ has been omitted.

(ii) $2+\frac{1}{3}$ is a second approximation, and is *too great* because the *denominator* 3 is *less* than the full denominator $3+\frac{6}{29}$.

(iii) $2+\cfrac{1}{3+\frac{1}{4}}$ is a third approximation, and is *too small* because the part of the *denominator* $\frac{1}{4}$ is *greater* than $\cfrac{1}{4+\frac{5}{6}}$; and so on.

338. The approximate fractions obtained by stopping at the first, second, third, ... quotients of a continued fraction are called the first, second, third, ... **convergents.**

Thus in the example of Art. 336 the successive convergents are

$$2,\quad 2+\frac{1}{3},\quad 2+\frac{1}{3+}\,\frac{1}{4},\quad 2+\frac{1}{3+}\,\frac{1}{4+}\,\frac{1}{1},\quad \text{and}\quad 2+\frac{1}{3+}\,\frac{1}{4+}\,\frac{1}{1+}\,\frac{1}{5},$$

the last of which is of course equal to the value of the original fraction.

Their values when simplified are

$$2,\quad \frac{7}{3},\quad \frac{30}{13},\quad \frac{37}{16},\quad \frac{215}{93}.$$

339. We shall now state some important properties of the convergents of a continued fraction. These may be tested in particular cases, but for formal proof the pupil must consult works on Algebra. [See Hall and Knight's *Higher Algebra*, Chap. XXV.]

(i) After the first two convergents have been found, each of the remaining convergents can be obtained from the two which immediately precede it.

Thus in the example of Art. 338, the successive quotients are

$$2, \quad 3, \quad \mathbf{4}, \quad \mathbf{1}, \quad 5.$$

The first two convergents are $\frac{2}{1}$, $\frac{7}{3}$.

$$\text{The 3rd convergent} = \frac{(\text{num}^{\text{r}} \text{ of } 2^{\text{nd}}) \times 3^{\text{rd}} \text{ quot.} + (\text{num}^{\text{r}} \text{ of } 1^{\text{st}})}{(\text{den}^{\text{r}} \text{ of } 2^{\text{nd}}) \times 3^{\text{rd}} \text{ quot.} + (\text{den}^{\text{r}} \text{ of } 1^{\text{st}})}$$

$$= \frac{7 \times \mathbf{4} + 2}{3 \times \mathbf{4} + 1} = \frac{30}{13}.$$

$$\text{The 4th convergent} = \frac{(\text{num}^{\text{r}} \text{ of } 3^{\text{rd}}) \times 4^{\text{th}} \text{ quot.} + (\text{num}^{\text{r}} \text{ of } 2^{\text{nd}})}{(\text{den}^{\text{r}} \text{ of } 3^{\text{rd}}) \times 4^{\text{th}} \text{ quot.} + (\text{den}^{\text{r}} \text{ of } 2^{\text{nd}})}$$

$$= \frac{30 \times \mathbf{1} + 7}{13 \times \mathbf{1} + 3} = \frac{37}{16};$$

and so on, using each new quotient with the numerators and denominators of the two preceding convergents.

(ii) The value of the continued fraction lies between the values of any two consecutive convergents.

This has been illustrated in Art. 337.

(iii) The difference between any two consecutive convergents is the reciprocal of the product of their denominators.

Thus the difference between $\frac{37}{16}$ and $\frac{30}{13}$ is $\frac{1}{16 \times 13}$,

and the difference between $\frac{215}{93}$ and $\frac{37}{16}$ is $\frac{1}{93 \times 16}$.

(iv) Each convergent is a nearer approximation to the value of the fraction than any of the preceding convergents.

Example 1. *Calculate the successive convergents of the continued fraction* $\frac{1}{3+}\,\frac{1}{2+}\,\frac{1}{4+}\,\frac{1}{1+}\,\frac{1}{3}$.

The first two convergents are $\frac{1}{3}$ and $\frac{2}{7}$.

By the rule in (i) above, the others are

$$\frac{2 \times 4 + 1}{7 \times 4 + 3}, \text{ or } \mathbf{\frac{9}{31}}; \qquad \frac{9 \times 1 + 2}{31 \times 1 + 7}, \text{ or } \mathbf{\frac{11}{38}}; \qquad \frac{11 \times 3 + 9}{38 \times 3 + 31}, \text{ or } \mathbf{\frac{42}{145}}.$$

Thus the five convergents are

$$\frac{1}{3}, \quad \frac{2}{7}, \quad \frac{9}{31}, \quad \frac{11}{38}, \quad \frac{42}{145}.$$

$\frac{42}{145}$ is the value of the continued fraction in its simplest form, and of the preceding convergents each will be found to be a closer approximation than any which precedes it.

EXAMPLE 2. *The value of π, that is the ratio of the circumference of a circle to its diameter, is 3·14159 to six significant figures. Find a series of fractions approximating to this ratio, and shew that the error in taking $\frac{355}{113}$ instead of the true value is less than $\frac{1}{212 \times 113}$.*

Write 3·14159 in the form $3\frac{14159}{100000}$; then dealing only with the fractional part, the process of finding the H.C.F. of 14159 and 100000 gives the successive quotients

$$7, \; 15, \; 1, \; 25, \; 1, \; 7, \; 4.$$

Thus $$3{\cdot}14159 = 3 + \frac{1}{7+}\,\frac{1}{15+}\,\frac{1}{1+}\,\frac{1}{25+}\,\frac{1}{1+}\,\frac{1}{7+}\,\frac{1}{4}.$$

The successive convergents are

$$\frac{3}{1}, \quad \frac{22}{7}, \quad \frac{333}{106}, \quad \frac{355}{113}, \; \ldots\,.$$

Now 3·14159 lies between $\frac{333}{106}$ and $\frac{355}{113}$, and is nearer to $\frac{355}{113}$ than to $\frac{333}{106}$. Again since the difference between these convergents is equal to $\frac{1}{106 \times 113}$, it is evident that 3·14159 differs from the closer approximation by *less than half this fraction.*

That is $\frac{355}{113}$ differs from 3·14159 by less than $\frac{1}{2 \times 106 \times 113}$.

340. It is easy to infer the general statement that a continued fraction differs from *the second of two consecutive convergents* by a fraction less than

$$\frac{1}{2(\text{prod}^{\text{t}} \text{ of their denominators})}.$$

341. The last example furnishes a good illustration of the utility of continued fractions in approximate work. In cases where a high degree of accuracy is of less importance than ready calculation it is convenient to replace 3·14159 by some simpler number. By converting 3·14159 into a continued fraction, the convergents give a series of approximate values from which we may select one suited to our purpose.

EXAMPLES XX. b.

Calculate the successive convergents to

1. $2+\frac{1}{3+}\,\frac{1}{1+}\,\frac{1}{2+}\,\frac{1}{5}$. **2.** $3+\frac{1}{4+}\,\frac{1}{2+}\,\frac{1}{1+}\,\frac{1}{3+}\,\frac{1}{5}$.

3. $\frac{1}{2+}\,\frac{1}{2+}\,\frac{1}{3+}\,\frac{1}{1+}\,\frac{1}{4+}\,\frac{1}{2+}\,\frac{1}{6}$.

Express the following quantities as continued fractions, and find the fourth convergent to each.

4. $\frac{79}{291}$. **5.** $\frac{97}{625}$. **6.** $\frac{313}{233}$. **7.** $\frac{1128}{349}$.

8. 1·139. **9.** 1·414. **10.** 0·393. **11.** 0·3029.

12. In Example 3 shew that the error in taking $\frac{43}{105}$ as the equivalent of the continued fraction is less than $\frac{1}{4620}$.

13. Assuming a metre to be approximately equal to 39·37 inches, shew by the theory of continued fractions that 32 metres is very nearly equal to 35 yards.

14. Two scales of equal length are divided into 162 and 209 equal parts respectively; if they are placed so that their zero points coincide, shew that the 31st division of one nearly coincides with the 40th division of the other.

15. Shew that $\frac{99}{70}$ differs from 1·41421 by a quantity less than $\frac{1}{70\times 169}$.

16. Among the convergents to

$$1+\frac{1}{3+}\,\frac{1}{5+}\,\frac{1}{7+}\,\frac{1}{9+}\,\frac{1}{11},$$

find the first approximation which differs from the true value of the continued fraction by less than $\frac{1}{3600}$.

17. Find an improper fraction lying between 3 and 4 which when converted into a continued fraction has 3, 1, 2, 9, 4 as successive quotients. Shew that by taking either of the fractions $\frac{36}{11}$, $\frac{337}{103}$ instead of the true value the error is less than 0·001.

18. Find a series of fractions approximating to the square root of 10.

[$\sqrt{10}$ lies between 3 and 4. Put $\sqrt{10}=3+x$, so that $(3+x)^2=10$.

Hence $6x+x^2=1$, or $x(6+x)=1$; whence $x=\frac{1}{6+x}=\frac{1}{6+}\,\frac{1}{6+}\cdots$.]

19. A man has to exchange 145 francs into English money when the exchange is at the rate of 25·20 fr. for £1. Shew by continued fractions that by changing 5 francs for 4 shillings, or 29 francs for 23 shillings the loss on either side will not amount to 1*s*.

20. Find by convergents an approximation to the square root of 26 which shall differ from the exact value by less than $\frac{1}{2040 \times 10301}$. (C.S.)

21. The true tropical year contains 365·242218 mean solar days, and the civil year contains 365 days. Shew by means of a continued fraction that to add to the civil year 8 days in 33 years would be a better correction than that furnished by the introduction of leap years. If the correction were adopted, find in how many years approximately the accumulated error would amount to one day. (C.S.)

342. Approximation by Aliquot Parts. When an operating factor is of frequent occurrence it is very convenient to be able to replace it by a formula consisting of simple fractions which are easy to apply. For instance, in Art. 300 we have shewn how the multiplier $\frac{1}{73}$, which occurs so often in certain questions in Interest, may be superseded by the use of the "third, tenth, and tenth rule." We shall now give some further instances of the use of aliquot parts as an aid to rapid calculation.

343. The numerical value of π, which expresses the ratio of the circumference of a circle to its diameter, is 3·14159265.... In the preceding pages the following approximations have been quoted and made use of

$$\tfrac{22}{7},\quad 3{\cdot}1416,\quad \tfrac{355}{113},$$

which are true to 3, 5, and 6 significant figures respectively.

Of these $\frac{22}{7}$ is very easy to apply in the form $3 + \frac{1}{7}$, but it is not a sufficiently close approximation to admit of its frequent use. It can, however, be 'corrected' in the following way so as to give results as accurate as those obtained by using 3·1416. It is not often that a closer approximation than this is required.

Since $3 + \frac{1}{7} = 3{\cdot}142857\ldots$, it exceeds 3·1416 by ·00125 nearly.

Hence $3 + \frac{1}{7} - \frac{1}{800} = 3{\cdot}1416$ to 5 significant figures.

Again $3\frac{1}{7}$, or 3·142857..., exceeds 3·1416 by ·001257... ; and it will be found that $3{\cdot}142857 \times {\cdot}0004 = {\cdot}001257$ to 6 significant figures.

Thus, approximately, $3\frac{1}{7}$ exceeds 3·1416 by ·0004 of itself,

that is, $$3\tfrac{1}{7}(1 - {\cdot}0004) = 3{\cdot}1416.$$

It thus appears that in any calculation involving π as a multiplier, true to 5 significant figures, we may first use $\pi = 3\frac{1}{7}$, and then make a correction by deducting 4 ten-thousandths of the result.

344. Since 1 metre $= 39{\cdot}370113$ inches

$= 1{\cdot}093614$ yards, approximately,

in order to convert a length given in metres to an equivalent in yards, we have to use the multiplier 1·093614.

Now if we write down 1, $\frac{1}{12}$, and $\frac{1}{96}$ in decimal form, we find that their sum to 7 significant figures is 1·093750, which is only ·000136 in excess of the true value.

$$\begin{array}{rr} & 1{\cdot}000000 \\ \frac{1}{12} & {\cdot}083333 \\ \frac{1}{96} & \underline{{\cdot}010417} \\ & 1{\cdot}093750 \end{array}$$

To 5 significant figures $1+\frac{1}{12}+\frac{1}{96}$ differs only to the extent of $\frac{1}{10000}$ from the true value of the multiplier, and is a sufficiently close approximation for all practical purposes.

EXAMPLE. *Find, to 4 significant figures, the number of yards in the circumference of a circle whose diameter is* 740·24 *metres.*

We have *circumference = diameter* × π.

$$\begin{array}{lr} & 740{\cdot}24 \text{ metres} \\ \frac{1}{12} & 61{\cdot}69 \\ \frac{1}{96} & \underline{7{\cdot}71} \\ & 809{\cdot}64 \text{ yards} \\ & \underline{3\frac{1}{7}} \\ & 2428{\cdot}92 \\ \frac{1}{7} & \underline{115{\cdot}66} \\ & 2544{\cdot}58 \\ \text{deduct} & \underline{1{\cdot}02} = 4 \text{ ten-thousandths} \\ & 2543{\cdot}56 \end{array}$$

Here we first use the multiplier $1+\frac{1}{12}+\frac{1}{96}$ to convert the diameter into yards. We then multiply the result by $3\frac{1}{7}$, and correct the product by deducting 4 ten-thousandths of itself [Art. 345].

Thus required circumference = 2544 yards.

[These reduction formulæ and those in the following examples were suggested by Professor A. Lodge in the *Mathematical Gazette* of April 1894.]

345. It is sometimes required to find in decimal form, to a given degree of accuracy, the value of a series consisting of fractions each of which is formed from the preceding according to some fixed law. The following example will illustrate the method of dealing with such cases.

EXAMPLE. *Find the value of the series*

$$1+\frac{1}{2}+\frac{1}{2.3}+\frac{1}{2.3.4}+\frac{1}{2.3.4.5}+\dots$$

correct to five significant figures.

It is easily seen that each of the successive denominators is formed from the preceding by introducing a new factor.

$$\begin{array}{r|l}
1 = 1{\cdot}0000 & 00 \\
\frac{1}{2} = {\cdot}5000 & 00 \\
\frac{1}{2\,.\,3} = {\cdot}1666 & 66 \\
\frac{1}{2\,.\,3\,.\,4} = {\cdot}0416 & 66 \\
\frac{1}{2\,.\,3\,.\,4\,.\,5} = {\cdot}0083 & 33 \\
\frac{1}{2\,.\,3\,.\,4\,.\,5\,.\,6} = {\cdot}0013 & 88 \\
\frac{1}{2\,.\,3\,.\,4\,.\,5\,.\,6\,.\,7} = {\cdot}0001 & 98 \\
\frac{1}{2\,.\,3\,.\,4\,.\,5\,.\,6\,.\,7\,.\,8} = {\cdot}0000 & 25 \\
\frac{1}{2\,.\,3\,.\,4\,.\,5\,.\,6\,.\,7\,.\,8\,.\,9} = {\cdot}0000 & 03 \\
\frac{1}{2\,.\,3\,.\,4\,.\,5\,.\,6\,.\,7\,.\,8\,.\,9\,.\,10} = {\cdot}0000 & 00 \\
\hline
1{\cdot}7182 & 79
\end{array}$$

Each term is expressed in *decimal form* before attempting to find their sum. For the 2nd term we divide the 1st by 2; for the 3rd term we divide the 2nd by 3; for the 4th we divide the 3rd by 4; and so on. A line is drawn after the *fourth* decimal figure, and we retain two extra columns for carrying purposes. The work is continued until we arrive at a term beginning with six ciphers. This and all subsequent terms will not affect the first six decimal places.

Thus to five significant figures the result is 1·7183.

NOTE. It will be seen that if we had kept only one extra column to the right of the line, the 5th decimal figure would have been 4 instead of 7, and the result would not have been correct to 5 significant figures.

EXAMPLES XX. c.

1. Use the formulæ of Art. 343 to find the values of

(i) 2753π; (ii) $32{\cdot}7 \times 16{\cdot}8 \times \pi$

correct to the nearest integer.

2. Find in square metres, correct to two decimal places, the area of a circle the radius of which is 62·3 metres.

3. Shew that $\frac{1}{\pi} = 0{\cdot}31830989\ldots$, and that the expression

$$\frac{1}{3} - \left(\frac{1}{100} + \frac{1}{200} + \frac{1}{50,000}\right)$$

is a true equivalent of this decimal to 5 significant figures.

Find to the nearest metre the diameter of a circular field whose circumference is 291·85 metres.

4. By the formula of Art. 344 express in yards, to the nearest unit,

(i) 3724 metres; (ii) 561·41 metres.

5. Assuming 1 yard $=\frac{32}{35}$ metre, approximately, shew that yards may be converted into metres by means of the multiplier $1-\frac{1}{10}+\frac{1}{70}$.

Express in metres, to the nearest unit,

(i) 3650 yards ; (ii) 5 miles.

6. A surveyor roughly uses his chain as the equivalent of 20 metres. Assuming 1 yard $=0\cdot91439$ metre, shew that in measuring a mile he is liable to an error of about 9·4 metres in defect.

Shew also that, by using the formula,

$$\frac{1\text{ chain}}{1\text{ metre}}=20+\frac{1}{10}+\frac{1}{70},$$

his equivalent of a mile in metres would be correct to the nearest unit.

7. Approximately, $\frac{1\text{ inch}}{1\text{ cm.}}=2\cdot54=2\tfrac{1}{2}+\frac{4}{100}=\frac{10}{4}+\frac{4}{100}$.

Use this formula to find the metric equivalents of the following lengths, to the nearest centimetre,

(i) 5 ft. 3 in. ; (ii) 39·375 in.

8. Shew that to six significant figures

$$\sqrt{2}=1\cdot4\left(1+\frac{1}{100}+\frac{1}{10{,}000}+\frac{1}{20{,}000}\right).$$

Use this formula to find the diagonal of a square enclosure each side of which is 353·55 metres.

9. Find the value of the following series, each correct to five significant figures :

$$\text{(i)}\quad \frac{1}{5}+\frac{1}{5^2}+\frac{1}{5^3}+\frac{1}{5^4}+\frac{1}{5^5}+\dots\,;$$

$$\text{(ii)}\quad \frac{1}{7}+\frac{1}{7^2}+\frac{1}{7^3}+\frac{1}{7^4}+\frac{1}{7^5}+\dots\,.$$

10. Express

$$\frac{1}{2}+\frac{1}{2\,.\,4}+\frac{1}{2\,.\,4\,.\,6}+\frac{1}{2\,.\,4\,.\,6\,.\,8}+\frac{1}{2\,.\,4\,.\,6\,.\,8\,.\,10}+\frac{1}{2\,.\,4\,.\,6\,.\,8\,.\,10\,.\,12}$$

as a decimal to six places.

11. Find the value of

$$\frac{1}{1\,.\,3}+\frac{1}{3\,.\,3^3}+\frac{1}{5\,.\,3^5}+\frac{1}{7\,.\,3^7}+\frac{1}{9\,.\,3^9}+\dots$$

in the form of a decimal to six places.

12. Find the value of

$$16\left\{\frac{1}{5}-\frac{1}{3\times5^3}+\frac{1}{5\times5^5}-\frac{1}{7\times5^7}\right\}-\frac{4}{239}$$

correct to five significant figures.

Further Notes on Decimal Approximation.

346. In Chap. x. we discussed examples involving approximate data, and it was there shewn how in results derived from such data the limits of error can be approximately estimated. But there are some cases in which the work is facilitated by the use of algebraical formulæ.

Suppose M and N are two numbers whose product is required. Also suppose M and N are liable to small errors, in excess or defect, not exceeding x and y respectively; then

$$\begin{aligned}\text{the maximum value of product} &= (M+x)(N+y)\\ &= MN+(My+Nx)+xy,\\ \text{,, minimum ,, ,,} &= (M-x)(N-y)\\ &= MN-(My+Nx)+xy.\end{aligned}$$

If x and y are both small, *xy will generally be very small in comparison with the other quantities*, and may be neglected; thus the real value of the product of M and N lies approximately between

$$MN+(My+Nx) \text{ and } MN-(My+Nx).$$

That is, the limits of error are $\pm(My+Nx)$.

NOTE. This result may be illustrated graphically as in Art. 57. A rectangle, whose adjacent sides are $M+x$, $N+y$ units respectively, is made up of four smaller rectangles containing MN, My, Nx, and xy units of area. If x and y are small compared with M and N, it will easily be seen that the rectangle xy is very small in comparison with the other three.

Let us apply the formula to Example 1, Art. 204. Then $M=60{\cdot}31$, $N=3{\cdot}14$, each being true only to the 2nd decimal figure, so that x and y are the same, each being equal to 0·005. Also $x^2=0{\cdot}000025$, and may be neglected.

Thus the product of 60·31 and 3·14 is liable to an errcr of

$$\pm(60{\cdot}31+3{\cdot}14)\times0{\cdot}005,$$

that is, of $\pm(63{\cdot}45\times0{\cdot}005)$, or $\pm0{\cdot}32$ nearly,

which agrees with the result in Art. 204.

347. In division of one approximate number by another the upper limit of the quotient is obtained by giving to the *dividend* its *greatest* possible value, and to the *divisor* its *least* possible value. Thus with the same notation as before the greatest possible value of the quotient $\frac{M}{N}$ is $\frac{M+x}{N-y}$.

Similarly the least possible value of the quotient is $\frac{M-x}{N+y}$.

Thus the limits of error are

$$\frac{M+x}{N-y}-\frac{M}{N} \text{ and } \frac{M}{N}-\frac{M-x}{N+y}.$$

The first of these gives $\frac{(M+x)N-M(N-y)}{N(N-y)}$,

that is, $$\frac{MN+Nx-MN+My}{N(N-y)}, \text{ or } \frac{Nx+My}{N^2-Ny}.$$

As y is small, the product Ny *will be small compared with* N^2; if we neglect this small quantity, the limit becomes

$$\frac{Nx+My}{N^2}.$$

It will be found that the other limit reduces to the same value; thus, $\frac{M}{N}$ is liable to an error of $\frac{Nx+My}{N^2}$ in excess or defect.

EXAMPLE 1. *Determine the error per cent. in the area of a field if it is calculated from measurements which give the length and breadth respectively as* 45 *chains and* 15·8 *chains, each correct to the last significant figure.*

The length and breadth are liable to errors of 0·5 chain and 0·05 chain respectively. Hence the maximum error in the area is approximately

$$(45\times 0\cdot 05+15\cdot 8\times 0\cdot 5) \text{ square chains,}$$

that is, $$(4\cdot 5+15\cdot 8)\times 0\cdot 5, \text{ or } 20\cdot 3\times 0\cdot 5 \text{ square chains.}$$

$$\therefore \text{ the percentage error}=\frac{10\cdot 15\times \overset{5}{\cancel{100}}}{\underset{9}{\cancel{45}}\times \underset{4}{\cancel{16}}}, \text{ nearly,}$$

$$=\frac{50\cdot 75}{9\times 4}=\frac{5\cdot 66}{4}$$

$$=1\cdot 4, \text{ to two significant figures.}$$

EXAMPLE 2. *From the following data,*

1 cubic foot of water weighs 62·3 lbs.,
1 ,, copper ,, 547 lbs.,

each being true only to the last significant figure, shew that the specific gravity of copper may be found with an error less than 0·016.

$$\text{Required specific gravity} = \frac{547}{62{\cdot}3},$$

the numerator being liable to an error of 0·5, and the denominator to an error of 0·05.

∴ by the formula above, the approximate error in the quotient

$$= \frac{547 \times 0{\cdot}05 + 62{\cdot}3 \times 0{\cdot}5}{(62{\cdot}3)^2}$$

$$= \frac{(54{\cdot}7 + 62{\cdot}3) \times 0{\cdot}5}{(62{\cdot}3)^2}, \text{ or } \frac{117{\cdot}0 \times 0{\cdot}5}{(62{\cdot}3)^2}.$$

Now $\dfrac{117{\cdot}0 \times 0{\cdot}5}{(62{\cdot}3)^2}$ is less than $\dfrac{58{\cdot}5}{60^2}$, or $\dfrac{0{\cdot}585}{6 \times 6}$;

that is, the error is less than $\dfrac{0{\cdot}097}{6}$, or 0·016.

348. The formulæ of Arts. 346, 347 do not in themselves furnish a method of procedure more generally serviceable than the methods explained in Arts. 200–205, but they admit of modifications which lead up to other formulæ of great use in special cases.

The errors spoken of in Arts. 346, 347 are *absolute* errors. We shall now discuss the corresponding cases of *relative* errors [Art. 190].

In Art. 346 we have established the following principle:

If M *and* N *are two approximate numbers known to be liable to small absolute errors of the maximum values* x *and* y *respectively, then in the product of* M *and* N, *the limit of absolute error, in excess or defect, is approximately* $My + Nx$.

Now suppose that r and s are the *relative* errors corresponding to *absolute* errors x and y respectively; then $x = Mr$, $y = Ns$ [Art. 190].

Thus by substituting for x and y in the above formula, the limit of relative error, in excess or defect, is

$$\frac{M \,.\, Ns + N \,.\, Mr}{MN}, \text{ or } r + s, \text{ approximately.}$$

Thus it appears that *when two approximate numbers are multiplied together, the maximum relative error is the sum of the maximum relative errors to which these numbers are liable, provided the errors are small.*

Since *percentage* error $=$ *relative* error $\times 100$, it follows that the same principle is true of percentage errors.

As an illustration we may give the following solution of Ex. 1, Art. 347.

In the product of 45 and 15·8

$$\text{the relative error} = \frac{0\cdot5}{45} + \frac{0\cdot05}{15\cdot8} \qquad \text{[Art. 190]}$$

$$= 0\cdot011 + 0\cdot003, \text{ approximately,}$$

$$= 0\cdot014.$$

Thus $\qquad$ the percentage error $= 0\cdot014 \times 100 = 1\cdot4$,

which agrees with the result on page 420.

349. In Art. 347 we have shewn that $\frac{\mathsf{M}}{\mathsf{N}}$ is liable to a maximum *absolute* error of $\frac{\mathsf{N}x + \mathsf{M}y}{\mathsf{N}^2}$.

If r and s have the same meanings as before, this expression

$$= \frac{\mathsf{N} \times \mathsf{M}r + \mathsf{M} \times \mathsf{N}s}{\mathsf{N}^2} = \frac{\mathsf{M}r + \mathsf{M}s}{\mathsf{N}} = \frac{\mathsf{M}}{\mathsf{N}}(r+s).$$

Hence the corresponding *relative* error is

$$\frac{\mathsf{M}}{\mathsf{N}}(r+s) \div \frac{\mathsf{M}}{\mathsf{N}}, \text{ or } r+s.$$

Thus *in the quotient of one approximate number divided by another the maximum relative error is the sum of the maximum relative errors to which the dividend and divisor are liable, provided the errors are small.*

350. The following approximate equivalents should here be noticed.

When r, s, t are small quantities such that their squares, and the products of any two of them may be neglected,

(i) $(1+r)(1+s) = 1 + (r+s)$, $\quad (1+r)(1-s) = 1 + (r-s)$.

Hence $\quad (1+r)^2 = 1 + 2r$. $\quad$ Also $(1-r)^2 = 1 - 2r$.

(ii) $(1+r)(1+s)(1+t) = 1 + (r+s+t)$.

Hence $\quad (1+r)^3 = 1 + 3r$. $\quad$ Also $(1-r)^3 = 1 - 3r$.

(iii) $\sqrt{1+r} = 1 + \frac{1}{2}r$, $\quad \sqrt{1-r} = 1 - \frac{1}{2}r$.

(iv) $\frac{1}{1+r} = 1 - r$, $\quad \frac{1}{1-r} = 1 + r$.

(i) and (ii) depend only on the multiplication formula explained in Art. 57, and already used several times in this chapter. (iii) and (iv) are easy to remember and apply, but cannot be proved here.

EXAMPLE 1. *Find approximately the values of*

$$\text{(i)}\ 1{\cdot}0078 \times {\cdot}9954\,; \qquad \text{(ii)}\ \frac{(1{\cdot}0032)^3}{({\cdot}9983)^2} \times 1{\cdot}0005.$$

(i) We have
$$\begin{aligned} 1{\cdot}0078 \times {\cdot}9954 &= (1 + {\cdot}0078)(1 - {\cdot}0046) \\ &= 1 + {\cdot}0078 - {\cdot}0046 \\ &= 1 + {\cdot}0032. \end{aligned}$$

The error here is the neglected product ${\cdot}0078 \times {\cdot}0046$, or ${\cdot}00003588$; thus the result true to 5 significant figures is $1{\cdot}0032$.

(ii) We have
$$\begin{aligned} (1{\cdot}0032)^3 &= (1 + {\cdot}0032)^3 = 1 + {\cdot}0032 \times 3 = 1 + {\cdot}0096\,; \\ ({\cdot}9983)^2 &= (1 - {\cdot}0017)^2 = 1 - {\cdot}0017 \times 2 = 1 - {\cdot}0034. \end{aligned}$$

$$\begin{aligned} \therefore\ \text{req}^{\text{d}}\ \text{product} &= \frac{1 + {\cdot}0096}{1 - {\cdot}0034} \times 1{\cdot}0005 \\ &= (1 + {\cdot}0096)(1 + {\cdot}0034)(1 + {\cdot}0005) \\ &= 1 + ({\cdot}0096 + {\cdot}0034 + {\cdot}0005) \\ &= 1 + {\cdot}0135 = 1{\cdot}0135. \end{aligned}$$

EXAMPLE 2. *Obtain a rule for finding approximately the percentage error in area from a given small percentage error in linear measurements. Justify it in a case where the given error in linear measurements is 1·8 per cent.* (C.S.)

Suppose that linear measurements are liable to a percentage error represented by p. Then the true length of a measured distance of l units is $l \pm \frac{p}{100} \times l$, or $l\left(1 \pm \frac{p}{100}\right)$.

Let A be the area obtained from dimensions inaccurately measured; then, since areas vary directly as the squares of their linear dimensions,

$$\frac{\text{true area}}{\mathsf{A}} = \frac{l^2\left(1 \pm \frac{p}{100}\right)^2}{l^2} = \left(1 \pm \frac{p}{100}\right)^2.$$

That is,
$$\begin{aligned} \text{true area} &= \mathsf{A}\left(1 \pm \frac{p}{100}\right)^2 \\ &= \mathsf{A}\left(1 \pm \frac{2p}{100}\right), \text{ when } p \text{ is small,} \\ &= \mathsf{A} \pm \mathsf{A} \times \frac{2p}{100}. \end{aligned}$$

That is, *the percentage error in area, in excess or defect, is twice the percentage error in linear measurement.*

If the percentage error in linear measurements is 1·8 *in excess*, every linear unit becomes $1+\frac{1{\cdot}8}{100}$, or 1·018.

Hence every unit of area becomes

1·018 × 1·018, or 1·036 approximately.

Thus the percentage error in area is

$$0{\cdot}036 \times 100 = 3{\cdot}6,$$

which is double of the given percentage error.

$$\begin{array}{r|l} 1{\cdot}018 & \\ 1{\cdot}018 & \\ \hline 1{\cdot}018 & \\ 10 & 2 \\ 8 & 1 \\ \hline 1{\cdot}036 & \end{array}$$

A percentage error of 1·8 *in defect* may be dealt with similarly.

EXAMPLES XX. d.

1. Find approximately the limits of error in the following products, each number being correct to the last significant figure.

(i) 16·15 × 8·023; (ii) 25·012 × 3·96.

2. If the numbers are only correct to the last figure given in each case, find roughly what the error may amount to in evaluating the following fractions:

(i) $\frac{8{\cdot}556}{1{\cdot}021}$; (ii) $\frac{11{\cdot}247}{14{\cdot}95}$; (iii) $\frac{14{\cdot}2317}{7{\cdot}008}$.

3. If 15·231 and 4·1241 are each correct to 5 significant figures, shew that the error in the quotient of the first divided by the second is less than 0·0001.

4. Shew that in dividing 247·4 by 8·0023, each number being true to the last figure given, the quotient cannot be trusted beyond the first decimal figure.

5. A side of a square is represented by 27·02 m. correct to the nearest centimetre. Find the area, and state whether it can be trusted to the nearest square metre.

6. Two adjacent sides of a rectangle are measured and found to contain 1·762 m. and 0·538 m. respectively. Find its area. If in each measurement there may be an error of 2 mm. find to the nearest tenth of a square centimetre the possible error in the area as calculated from the above measurements.

7. The area of a rectangular field is known to be exactly half an acre, and one side is found by measurement to be 38·5 yards to one decimal figure. Find from this the length of the other side, and say how far it may be in error.

Find by the formulæ of Art. 350 the approximate values of

8. $1{\cdot}000189 \times 1{\cdot}00024$.

9. $(1{\cdot}005) \times (1{\cdot}0002)^2$.

10. $1{\cdot}00064 \div 1{\cdot}0078$.

11. $1{\cdot}0058 \div 0{\cdot}9982$.

12. $(1{\cdot}00018)^2 \times (1{\cdot}00004)^3$.

13. $1{\cdot}007 \div (0{\cdot}995)^2$.

14. $\dfrac{1}{(0{\cdot}98)^3}$.

15. $\dfrac{1}{\sqrt{0{\cdot}997}}$.

16. $\dfrac{\sqrt{0{\cdot}9987}}{(1{\cdot}0003)^2}$.

17. Determine the percentage error in the area of a rectangular field when the measurements of its sides give 25·0 and 17·5 chains, to the nearest tenth of a chain.

18. In consequence of the inaccurate length of his chain a surveyor records the dimensions of a rectangular field as 275 yards and 188 yards. If the true area is 10·4 acres, give the percentage error in the area, and also in the length of his chain. (C.S.)

19. The relative errors in the edges of a cuboid inaccurately measured are r, s, and t. Shew that the relative error in the volume is $r+s+t$ approximately. Estimate roughly the relative error in the volume of a cuboid as calculated from the lengths of its edges which are found by measurement to be 1·234 m., 0·562 m., and 0·237 m., to the nearest millimetre.

20. What is the percentage error in the volume of a cuboid when in measuring each of its edges there is a small percentage error represented by p?

21. In each of three consecutive years there is a percentage increase of 0·8 in the population of a certain village. Find approximately the total percentage increase in the three years.

22. The measured edges of a rectangular block are 27·5 cm., 16·3 cm., and 25·0 cm.; find its volume from these measurements. If the *absolute* error in the last is 2 mm., and the *percentage* error in each is the same, by how much per cent. may the calculated volume be wrong?

23. A bar of copper, 1 unit in length, increases by 0·0000168 of its length for a rise of temperature of 1 degree Centigrade. Find the expansion in area of a rectangular copper plate 55 cm. long and 27 cm. wide when the temperature is raised 10 degrees. The metal is assumed to expand equally in all directions.

[The decimal 0·0000168 is called the **coefficient of linear expansion** for copper.]

24. The area of a zinc roof is 10,000 square feet at 0° C.; find its area, to the nearest square foot, at 30° C.

[The coefficient of linear expansion for zinc $= 0{\cdot}0000298$.]

25. A rectangular block of copper occupies a volume of 8 cubic feet at 0° C. Find approximately the volume at 100° C.

CHAPTER XXI.

LOGARITHMS.

351. DEFINITION. If a number N can be expressed in the form a^x, the index x is called **the logarithm of the number N to the base a.**

EXAMPLES.

(i) Since $81=3^4$, the logarithm of 81 to base 3 is **4**.

(ii) Since $2^8=256$, the logarithm of 256 to base 2 is **8**.

(iii) Since $10^1=10$, $10^2=100$, $10^3=1000$, ...
the natural numbers **1**, **2**, **3**, ... are respectively the logarithms of 10, 100, 1000, ... to base 10.

352. The logarithm of N to base a is usually written $\log_a \mathrm{N}$, so that the same relation between the number and its logarithm is expressed by the two equations:

$$a^x=\mathrm{N}, \quad x=\log_a \mathrm{N}.$$

353. Any number might be taken as the base of a system of logarithms, but in arithmetical calculations the system in use is that which has 10 for its base. The advantages of this system will appear later.

When a particular system is in use, the suffix denoting the base may be omitted. Thus we usually write log 2, log 3, ... instead of $\log_{10} 2$, $\log_{10} 3$, ...

Logarithms to the base 10 are known as **Common Logarithms;** this system was first introduced in 1615 by Briggs, a contemporary of Napier the inventor of Logarithms.

[*Articles* 354, 355, 356 *may be omitted by pupils who have read the Theory of Indices in Algebra.*]

354. Since every logarithm is an *index*, it follows that the rules which govern the use of logarithms are deducible from the laws of indices. The use of indices in previous chapters has been of a simple kind, and the examples have been such as could be easily dealt with from the definition of a positive integral index. It is now necessary to examine the fundamental laws of indices more closely.

(i) Since, by definition of an index,

$$2^3 = 2 \times 2 \times 2, \text{ and } 2^5 = 2 \times 2 \times 2 \times 2 \times 2;$$
$$\therefore\ 2^3 \times 2^5 = (2 . 2 . 2) \times (2 . 2 . 2 . 2 . 2)$$
$$= 2^8, \text{ since the factor 2 occurs 8 times,}$$
$$= 2^{3+5};$$

that is, the index of 2 in the product is the **sum** of the indices of 2 in the factors of the product.

Similarly
$$2^3 \times 2^5 \times 2^4 = 2^8 \times 2^4$$
$$= 2^{12}.$$

In general, using letters instead of numbers,

$$a^m \times a^n = a^{m+n},$$
$$a^m \times a^n \times a^p = a^{m+n+p},$$

and so on, for any number of factors, so long as m, n, and p are positive whole numbers.

(ii) By definition,
$$2^5 \div 2^3 = \frac{2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2}$$
$$= 2 \times 2 = 2^2$$
$$= 2^{5-3};$$

that is, the index of 2 in the quotient is the **difference** of the indices of 2 in the dividend and divisor.

And generally, $a^m \div a^n = a^{m-n}$, where m and n stand for any positive whole numbers, and m is greater than n.

(iii) By definition,
$$(2^3)^4 = 2^3 \times 2^3 \times 2^3 \times 2^3$$
$$= 2^{3+3+3+3} \text{ by (i),}$$
$$= 2^{12}$$
$$= 2^{3 \times 4};$$

that is, the index of 2 in the result is the **product** of the indices of the powers in the expression $(2^3)^4$.

And generally, $(a^m)^n = a^{mn}$, where m and n stand for any positive whole numbers.

355. Collecting the above results we have

$$\text{(i)}\quad a^m \times a^n = a^{m+n};$$
$$\text{(ii)}\quad a^m \div a^n = a^{m-n};$$
$$\text{(iii)}\quad (a^m)^n = a^{mn}.$$

These are the fundamental laws of indices and they have been proved on the supposition that m and n are positive whole numbers, and in the case of (ii) m is supposed to be greater than n. But it is found convenient to use expressions involving fractional and negative indices such as $10^{\frac{2}{3}}$, 3^{-7}, or, more generally, $a^{\frac{p}{q}}$, a^{-n}. Further it is important that the fundamental laws shall be applicable to indices of all kinds without distinction. It can be proved algebraically (see Hall and Knight's *Elementary Algebra*, Chap. xxx.) that fractional and negative indices will conform to the general laws provided that we interpret them as follows:

$$a^{\frac{p}{q}} \text{ is equivalent to } \sqrt[q]{a^p}, \text{ or the } q^{\text{th}} \text{ root of } a^p.$$

Thus $$81^{\frac{1}{4}} = \text{the fourth root of } 81 = 3.$$
$$8^{\frac{2}{3}} = \sqrt[3]{8^2} = \sqrt[3]{64} = 4.$$

Again, $$a^{-n} \text{ is equivalent to } \frac{1}{a^n}.$$

Thus $$2^{-5} = \frac{1}{2^5} = \frac{1}{32}; \qquad 10^{-3} = \frac{1}{10^3} = 0{\cdot}001;$$
$$8^{-\frac{1}{3}} = \frac{1}{8^{\frac{1}{3}}} = \frac{1}{\sqrt[3]{8}} = \frac{1}{2}.$$

356. One case deserves special attention.

If $a^m \times a^n = a^{m+n}$ is to be true for *all* values of the indices, we have by putting 0 in the place of m,

$$a^0 \times a^n = a^{0+n} = a^n:$$
$$\therefore \ a^0 = \frac{a^n}{a^n}$$
$$= 1.$$

Hence *any number* with zero index is equivalent to 1.

357. If we give these interpretations to negative and fractional indices, the fundamental laws may be applied to all indices without restriction.

EXAMPLES.

(i) $10^3 \times 10^{-5} = 10^{3-5} = 10^{-2} = \frac{1}{10^2} = 0{\cdot}01.$

(ii) $27^2 = (3^3)^2 = 3^6.$

(iii) $27^{\frac{2}{3}} = \sqrt[3]{27^2} = \sqrt[3]{3^6} = 3^2 = 9,$

or $27^{\frac{2}{3}} = (27^{\frac{1}{3}})^2 = 3^2 = 9.$

(iv) $4^{-\frac{3}{2}} = \frac{1}{4^{\frac{3}{2}}} = \frac{1}{\sqrt{4^3}} = \frac{1}{\sqrt{64}} = \frac{1}{8}.$

EXAMPLES XXI. a.

Read off the values of

1. $2^4\times2^{-3}$.
2. $2^3\times2^{-4}$.
3. $2^5\times2^{-5}$.
4. $3^4\times3^{-5}$.
5. $3^4\times3^{-4}$.
6. $5^3\times5^{-1}$.
7. $6^{-4}\times6^2$.
8. $2^{-3}\times2^{-1}$.
9. $4^2\div4^3$.
10. $3\div3^4$.
11. $8\div2^4$.
12. $10^2\div10^4$.
13. $36^{\frac{1}{2}}$.
14. $27^{\frac{1}{3}}$.
15. $16^{\frac{1}{4}}$.
16. $8^{\frac{2}{3}}$.
17. $4^{\frac{3}{2}}$.
18. $64^{\frac{1}{3}}$.
19. $9^{\frac{3}{2}}$.
20. $32^{\frac{2}{5}}$.
21. 2^{-4}.
22. 4^{-2}.
23. 3^{-1}.
24. a^{-1}.
25. 4^{-3}.
26. a^{-5}.
27. 3^0.
28. x^0.

29. From the statement $10^{m-n}=10^m\times10^{-n}$, what do you infer by putting m equal to n?

30. State in words the meaning of $x^{\frac{m}{n}}$. Find its numerical value when $x=27$, $m=2$, $n=3$.

Read off as decimals or whole numbers

31. 10^{-3}.
32. $10^{-5}\times10^2$.
33. $10^3\div10^6$.
34. 1000×10^{-6}.
35. $2{\cdot}143\times10^3$.
36. $2{\cdot}143\times10^{-2}$.
37. $2{\cdot}143\times10^5$.
38. $2{\cdot}143\times10^{-4}$.
39. $2{\cdot}143\div10^3$.

358. The following general propositions are applicable to all logarithms independently of any particular base.

359. *The logarithm of* 1 *is* 0.

For $a^0=1$ for all values of a; therefore $\log_a 1=0$, whatever the base may be.

360. *The logarithm of the base itself is* 1.

For $a^1=a$; therefore $\log_a a=1$.

361. *To find the logarithm of a product.*

Let M and N be two numbers such that $M=a^x$, $N=a^y$.

Then $x=\log_a M,\qquad y=\log_a N.$

The product $MN=a^x\times a^y=a^{x+y}$;

whence, by definition, $\log_a MN=x+y$

$$=\log_a M+\log_a N.$$

Similarly $\log MNP=\log M+\log N+\log P$;

and so on for any number of factors.

EXAMPLE. $\log 42=\log(2\times3\times7)=\log 2+\log 3+\log 7$.

362. *To find the logarithm of a quotient, or a fraction.*

As before suppose $\mathsf{M}=a^x$, $\mathsf{N}=a^y$;

so that $x=\log_a \mathsf{M}$, $y=\log_a \mathsf{N}$.

The fraction $$\frac{\mathsf{M}}{\mathsf{N}}=\frac{a^x}{a^y}=a^{x-y};$$

whence, by definition, $$\log_a\frac{\mathsf{M}}{\mathsf{N}}=x-y$$
$$=\log_a \mathsf{M}-\log_a \mathsf{N}.$$

EXAMPLE. $$\log\left(2\tfrac{1}{7}\right)=\log\tfrac{15}{7}=\log 15-\log 7.$$
$$=\log(3\times 5)-\log 7$$
$$=\log 3+\log 5-\log 7.$$

363. *To find the logarithm of a number raised to any power, integral or fractional.*

Suppose $\mathsf{M}=a^x$, so that $x=\log_a \mathsf{M}$, and suppose it is required to find the value of $\log_a(\mathsf{M}^p)$.

We have $$\mathsf{M}^p=(a^x)^p=a^{px};$$

whence, by definition, $$\log_a(\mathsf{M}^p)=px$$
$$=p\log_a \mathsf{M}.$$

Similarly $$\log_a\left(\mathsf{M}^{\frac{1}{r}}\right)=\frac{1}{r}\log_a \mathsf{M}.$$

EXAMPLE. $$\log\frac{3^5\cdot\sqrt[3]{2}}{\sqrt[4]{5}}=\log\left(3^5\cdot 2^{\frac{1}{3}}\right)-\log 5^{\frac{1}{4}}$$
$$=\log 3^5+\log 2^{\frac{1}{3}}-\log 5^{\frac{1}{4}}$$
$$=5\log 3+\tfrac{1}{3}\log 2-\tfrac{1}{4}\log 5.$$

364. The results we have proved may be summarised as follows:

(i) the logarithm of a **product** is equal to the **sum** of the logarithms of its factors;

(ii) the logarithm of a fraction is equal to the logarithm of the numerator **minus** the logarithm of the denominator;

(iii) the logarithm of the p^{th} **power** of a number is equal to the logarithm of the number **multiplied by** p;

(iv) the logarithm of the r^{th} **root** of a number is equal to the logarithm of the number **divided by** r.

Thus by the use of logarithms the operations of **multiplication** and **division** may be replaced by those of **addition** and **subtraction**; the operations of **involution** and **evolution** by those of **multiplication** and **division**.

365. The following examples will serve to familiarise the pupil with the laws of operation above established. The rest of the chapter will be devoted to the practical applications of Common Logarithms.

EXAMPLE 1. *Express*

(i) $2^8=256$ *in the logarithmic form* (*e.g.* $x=\log_a \mathrm{N}$).

(ii) $5=\log_3 243$ *in the index form* (*e.g.* $a^x=\mathrm{N}$).

(i) By definition of Art. 351, 8 is the logarithm of 256 to base 2; that is, $8=\log_2 256$.

(ii) Here 5 is the index to which the base 3 must be raised in order to equal 243. Hence $3^5=243$.

EXAMPLE 2. *Express* $\log \frac{144}{\sqrt[5]{128}}$ *in terms of* $\log 2$ *and* $\log 3$.

$$\begin{aligned}\log \frac{144}{\sqrt[5]{128}} &= \log 144 - \log 128^{\frac{1}{5}}\\ &= \log(16\times 9) - \tfrac{1}{5}\log 128\\ &= \log(2^4\times 3^2) - \tfrac{1}{5}\log 2^7\\ &= \log 2^4 + \log 3^2 - \tfrac{1}{5}(7\log 2)\\ &= 4\log 2 + 2\log 3 - \tfrac{7}{5}\log 2\\ &= \tfrac{13}{5}\log 2 + 2\log 3.\end{aligned}$$

EXAMPLES XXI. b.

Read off each of the following equations in the logarithmic form.

1. $2^5=32$. **2.** $3^7=2187$. **3.** $5^4=625$. **4.** $7^3=343$.

Read off the following logarithmic equations in the index form.

5. $\log_2 128=7$. **6.** $\log_4 1024=5$. **7.** $\log_3 729=6$. **8.** $\log_a b=c$.

Express in terms of $\log a$, $\log b$, and $\log c$

9. $\log \frac{a}{bc}$. **10.** $\log \frac{a^2}{bc^3}$. **11.** $\log \frac{a^{\frac{1}{2}}b^{\frac{2}{3}}}{c^5}$. **12.** $\log \frac{\sqrt[4]{b}\cdot\sqrt[3]{c}}{\sqrt{a^3}}$.

Express in terms of $\log 2$ and $\log 3$

13. $\log 36$. **14.** $\log \frac{1}{108}$. **15.** $\log \sqrt{648}$. **16.** $\log \sqrt{54}\times\sqrt[3]{243}$.

Shew that

17. $\log\left(\frac{217}{38}\div\frac{31}{266}\right)=2\log 7.$ **18.** $\log\frac{26}{51}+\log\frac{119}{91}=\log 2-\log 3.$

19. If $\mathsf{A}=\mathsf{PR}^n$, shew that $n=\frac{\log \mathsf{A}-\log \mathsf{P}}{\log \mathsf{R}}.$

20. From the formula $\mathsf{V}=\frac{\pi d^3}{6}$, shew that

$$\log d=\tfrac{1}{3}(\log \mathsf{V}+\log 6-\log\pi).$$

Common Logarithms.

366. Since $10^1=10,\ 10^2=100,\ 10^3=1000,\ 10^4=10000,\ \dots$
we see that the numbers 1, 2, 3, 4, ...
are the logarithms of 10, 100, 1000, 10000, ..., respectively.

Also since $10^{-1}=\frac{1}{10}=\cdot 1,\ 10^{-2}=\frac{1}{10^2}=\cdot 01,\ 10^{-3}=\frac{1}{10^3}=\cdot 001,\ \dots$
we see that the logarithms of ·1, ·01, ·001, ...
are respectively −1, −2, −3, ...

NOTE. Since $10^0=1$, the logarithm of 1 is 0.

Thus it appears that the logarithms of all numbers which are exact powers of 10 are integers either positive or negative. In the case of numbers which are not exact powers of 10 the logarithms will always lie between two consecutive integers and will therefore be partly integral and partly fractional.

The integral part of a logarithm is called the **characteristic**, and the fractional part when expressed as a decimal is called the **mantissa.**

367. We shall now shew that the characteristics may always be determined by inspection.

A number which has one digit in its integral part, such as 8·27, is greater than 10^0 and less than 10^1;

∴ its logarithm lies between 0 and 1; that is, *its logarithm is a fraction.*

A number which has two digits in its integral part, such as 34·062, is greater than 10^1 and less than 10^2;

∴ its logarithm lies between 1 and 2, and may be expressed as

1 + *a fraction.*

A number with three digits in its integral part, such as 137·5, is greater than 10^2 and less than 10^3;

∴ its logarithm lies between 2 and 3, and may be expressed as

$$\mathbf{2}+a \text{ fraction}.$$

Similarly log 6154·73 = **3** + *a fraction*; and so on. From these examples we deduce the following rule:

The characteristic of the logarithm of a number greater than unity is less by one than the number of digits in its integral part, and is positive.

EXAMPLE. The characteristics of

	log 314,	log 87·263,	log 2·78,	log 3500	
are	**2**,	**1**,	**0**,	**3**,	respectively.

368. Again, a decimal with no ciphers immediately after the decimal point, such as ·327, is greater than ·1 and less than 1, that is, greater than 10^{-1} and less than 10^0.

∴ its logarithm lies between -1 and 0, and may be written

$$-\mathbf{1}+a \text{ fraction}.$$

A decimal with 1 cipher after the decimal point, such as ·0654 is greater than 10^{-2} and less than 10^{-1};

∴ its logarithm lies between -2 and -1, and may be written

$$-\mathbf{2}+a \text{ fraction}.$$

A decimal with 2 ciphers after the decimal point, such as ·003878, is greater than 10^{-3} and less than 10^{-2};

∴ its logarithm lies between -3 and -2, and may be written

$$-\mathbf{3}+a \text{ fraction}.$$

Hence the following rule:

The characteristic of the logarithm of a number less than one is negative, and is greater by one than the number of ciphers immediately after the decimal point.

EXAMPLE. The characteristics of

	log ·4,	log ·3748,	log ·000135,	log ·08	
are	$-$**1**,	$-$**1**,	$-$**4**,	$-$**2**,	respectively.

369. *The mantissæ are the same for the logarithms of all numbers which have the same significant digits.*

For if any two numbers have the same sequence of digits, differing only in the position of the decimal point, one must be equal to the other multiplied or divided by some integral power of 10. Hence their logarithms must *differ by an integer*. In other words, their decimal parts or mantissæ are the same.

EXAMPLES. (i) $\log 32700 = \log(3{\cdot}27 \times 10^4) = \log 3{\cdot}27 + \log 10^4$
$= \log 3{\cdot}27 + \mathbf{4}.$

(ii) $\log {\cdot}0327 = \log(3{\cdot}27 \times 10^{-2}) = \log 3{\cdot}27 + \log 10^{-2}$
$= \log 3{\cdot}27 - \mathbf{2}.$

(iii) $\log {\cdot}000327 = \log(3{\cdot}27 \times 10^{-4}) = \log 3{\cdot}27 + \log 10^{-4}$
$= \log 3{\cdot}27 - \mathbf{4}.$

Thus log 32700, log ·0327, log ·000327 differ from log 3·27 only in the *integral* part; that is the mantissa is the same in each case.

NOTE. Here, by introducing a power of 10, the numbers have been expressed in *standard form* [Art. 153] with the decimal point after the first significant figure. When this method is adopted *the index of the power of* 10 *is the characteristic of the logarithm.*

370. The logarithms of all integers from 1 to 20000 have been found and tabulated. In Chambers' Mathematical Tables they are given to seven places of decimals, but for many practical purposes sufficient accuracy is secured by using four figure logarithms (available for all numbers from 1 to 9999), such as are contained in the Tables given on pages 444–447.

371. Advantages of Common Logarithms. It will now be seen that it is unnecessary to tabulate the characteristics, since they can always be written down by inspection [Arts. 367, 368]. Also the Tables need only contain the mantissæ of the logarithms of integers [Art. 369].

In order to secure these advantages it is convenient *always to keep the mantissa positive*, and it is usual to write the minus sign over a negative characteristic and not before it, so as to indicate that the characteristic alone is negative.

Thus $\bar{5}{\cdot}4771$, which is the logarithm of ·00003, is equivalent to $-5 + {\cdot}4771$, and must be distinguished from $-5{\cdot}4771$, in which both the integer and the decimal are negative.

372. In the course of work we sometimes meet with a logarithm wholly negative. In such a case a rearrangement is necessary in order to write the logarithm with a positive mantissa. A result such as $-3{\cdot}5229$ may be transformed by subtracting 1 from the integral part and adding 1 to the decimal part.

Thus $-3{\cdot}5229 = -3 - 1 + (1 - {\cdot}5229)$
$= -4 + {\cdot}4771$, or $\bar{4}{\cdot}4771$.

373. The following examples dealing with negative characteristics should be carefully studied.

EXAMPLE 1. *From the sum of* $\bar{3}\cdot9605$ *and* $1\cdot2135$ *subtract*

(i) $3\cdot7234$, (ii) $\bar{4}\cdot7234$.

$$\begin{array}{llll} & \bar{3}\cdot9605 & & \\ & 1\cdot2135 & & \\ \cline{2-2} \text{(i)} & \bar{1}\cdot1740 & \text{(ii)} & \bar{1}\cdot1740 \\ & 3\cdot7234 & & \bar{4}\cdot7234 \\ \cline{2-2} \cline{4-4} & \bar{5}\cdot4506 & & 2\cdot4506 \end{array}$$

Here after adding the decimal figures we have 1 to carry. Thus at the first stage the characteristic is the algebraic sum of 2 and -3, which is written $\bar{1}$.

In (i) when we get to the integral part we have to subtract $3+1$ from -1; the result is -5, and is written $\bar{5}$.

In (ii) we have to subtract $-4+1$ (or -3) from -1. The result is 2.

EXAMPLE 2. (i) *Multiply* $\bar{1}\cdot8173$ *by* 3. (ii) *Divide* $\bar{4}\cdot8134$ *by* 3.

$$\begin{array}{lr} \text{(i)} & \bar{1}\cdot8173 \\ & 3 \\ \cline{2-2} & \bar{1}\cdot4519 \end{array}$$

On multiplying 8 by 3 we have to carry 2 to the product of $-\bar{1}$ and 3.

Hence the characteristic is $-3+2$, or -1.

(ii)
$$\begin{aligned} \bar{4}\cdot8134 &= \bar{6}+2\cdot8134; \\ \therefore\ \tfrac{1}{3}(\bar{4}\cdot8134) &= \tfrac{1}{3}(\bar{6}+2\cdot8134) \\ &= \bar{2}\cdot9378 \end{aligned}$$

Here we cannot divide $\bar{4}\cdot8134$ as it stands by 3. By a suitable addition and subtraction the characteristic is adjusted so that its negative part is a multiple of the given divisor. A similar artifice is always employed in dividing a logarithm with a negative characteristic.

EXAMPLES XXI. c.

1. Find by inspection the characteristics of the logarithms of

3174, $625\cdot7$, $3\cdot502$, $\cdot4$, $\cdot374$, $\cdot000135$, $23\cdot29$.

2. The mantissa of log 3754 is $\cdot5745$: write down the logarithms of $37\cdot54$, $\cdot003754$, 3754000, $\cdot3754$.

3. The logarithm of $8\cdot061$ is $0\cdot9064$: write down the logarithms of $806\cdot1$, $8\cdot061\times10^4$, $8\cdot061\times10^{-5}$.

Also write down the numbers whose logarithms are

$5\cdot9064$, $1\cdot9064$, $\bar{3}\cdot9064$, $\bar{1}\cdot9064$.

Find (to four decimal figures) the values of

4. $\bar{1}\cdot3681\times3$.
5. $\bar{2}\cdot0068\times7$.
6. $\bar{4}\cdot9832\times12$.
7. $\bar{2}\cdot4320+\bar{1}\cdot3971$.
8. $\bar{3}\cdot6583-\bar{4}\cdot7241$.
9. $\bar{2}\cdot4871+4\cdot3970$.
10. $4\cdot5885-2\cdot9347$.
11. $\bar{1}\cdot6989-\bar{3}\cdot3010$.
12. $\bar{4}\cdot5703\div5$.
13. $\frac{1}{6}(\bar{3}\cdot8123)$.
14. $\frac{2}{5}(\bar{1}\cdot5632)$.
15. $\frac{3}{4}(\bar{2}\cdot1305)$.

Use of Four-Figure Tables.

374. *To find the logarithm of a given number from the Tables.*

Example 1. *Find* log 38, log 380, log ·0038.

We first find the number 38 in the left hand column on p. 444. Opposite to this we find the digits 5798. This, with the decimal point prefixed, is the mantissa for the logarithms of all numbers whose significant digits are 38. Hence, prefixing the characteristics, we have

$$\log 38 = 1{\cdot}5798, \quad \log 380 = 2{\cdot}5798, \quad \log {\cdot}0028 = \bar{3}{\cdot}5798.$$

Example 2. *Find* log 3·86, log ·0386, log 386000.

The same line as before will give the mantissa of the logarithms of all numbers which begin with **38**. From this line we choose the mantissa which stands in the column headed 6. This gives ·5866 as the mantissa for all numbers whose significant digits are 386. Hence, prefixing the characteristics, we have

$$\log 3{\cdot}86 = {\cdot}5866, \quad \log {\cdot}0386 = \bar{2}{\cdot}5866, \quad \log 386000 = 5{\cdot}5866.$$

375. Similarly the logarithm of any number consisting of not more than 3 significant digits can be obtained directly from the Tables. When the number has 4 significant digits, use is made of the principle that *when the difference between two numbers is small compared with either of them, the difference between their logarithms is very nearly proportional to the difference between the numbers.* It would be out of place to attempt any demonstration of the principle here. It will be sufficient to point out that differences in the logarithms corresponding to small differences in the numbers have been calculated, and are printed ready for use in the *difference columns* at the right hand of the Tables. The way in which these differences are used is shewn in the following example.

Example. *Find* (i) log 3·864; (ii) log ·003868.

Here, as before, we can find the mantissa for the sequence of digits 386. This has to be *corrected* by the addition of the figures which stand underneath 4 and 8 respectively in the difference columns.

$$\begin{array}{llr} \text{(i)} & \log 3{\cdot}86 & = 0{\cdot}5866 \\ \text{diff. for} & \quad 4 & 5 \\ \hline \therefore & \log 3{\cdot}864 & = 0{\cdot}5871 \end{array} \qquad \begin{array}{llr} \text{(ii)} & \log {\cdot}00386 & = \bar{3}{\cdot}5866 \\ \text{diff. for} & \quad 8 & 9 \\ \hline \therefore & \log {\cdot}003868 & = \bar{3}{\cdot}5875 \end{array}$$

Note. In printing the differences non-significant ciphers are omitted. Thus the differences used above are really ·0005 and ·0009. This accounts for the position of the digits 5 and 9 in making the necessary 'correction.' With a little practice the correction from the difference columns can be performed mentally.

376. The number corresponding to a given logarithm is called its **antilogarithm.** Thus in the last example 3·864 and ·003868 are respectively the numbers whose logarithms are 0·5871 and $\bar{3}$·5875.

Hence antilog 0·5871 = 3·864 ; antilog $\bar{3}$·5875 = ·003868.

377. *To find the antilogarithm of a given logarithm.*

In using the Tables of antilogarithms on pages 446, 447, it is important to remember that we are seeking *numbers* corresponding to *given logarithms.* Thus in the left hand column we have the first two digits of the given *mantissæ*, with the decimal point prefixed. The characteristics of the given logarithms will fix the position of the decimal point in the numbers taken from the Tables.

EXAMPLE. *Find the antilogarithm of* (i) 1·583 ; (ii) $\bar{2}$·8249.

(i) We first find ·58 in the left hand column on page 447, and pass along the horizontal line and take the number in the vertical column headed by 3. Thus ·583 is the mantissa of the logarithm of a number whose significant digits are 3828. Hence antilog 1·583 = 38·28.

(ii) antilog $\bar{2}$·824 = ·06668
diff. for 9   14
∴ antilog $\bar{2}$·8249 = ·06682

Here corresponding to the first 3 digits of the mantissa we find the sequence of digits 6668, and the decimal point is inserted in the position corresponding to the characteristic $\bar{2}$. To the number so found we add 14 from the difference column headed 9.

378. The following examples illustrate the use of logarithms in abbreviating arithmetical calculations.

EXAMPLE 1. *Find the product of* 72·38 *and* ·5689 *to four significant figures, by the use of the Tables.*

log 72·3 = 1·8591
diff. for 8   5
log ·568 = $\bar{1}$·7543
diff. for 9   7
log *product* = 1·6146

antilog 1·614 = 41·11
diff. for 6   6
antilog 1·6146 = 41·17

Thus the required product is 41·17.

NOTE. It is important to observe that accuracy beyond four significant figures can never be secured with four-figure logarithms. Moreover we cannot always rely on the accuracy of the last figure. In the present case, if the product of 72·38 and ·5689 is obtained by contracted multiplication it will be found that the result to four significant figures is 41·18.

EXAMPLE 2. *Find the value of* $\dfrac{3{\cdot}274 \times {\cdot}0059}{14{\cdot}83 \times {\cdot}077}$ *to four significant digits.*

By Art. 362, log *fraction* = log *numerator* − log *denominator*.

Numerator.	**Denominator.**
$\log 3{\cdot}27 = 0{\cdot}5145$	$\log 14{\cdot}8 = 1{\cdot}1703$
diff. for 4 = 5	diff. for 3 9
$\log {\cdot}0059 = \bar{3}{\cdot}7709$	$\log {\cdot}077 = \bar{2}{\cdot}8865$
log *numerator* = $\bar{2}{\cdot}2859$	log *denominator* = $0{\cdot}0577$
$\bar{2}{\cdot}2859$	antilog $\bar{2}{\cdot}228 = {\cdot}01690$
subtract $0{\cdot}0577$	diff. for 2 1
log *fraction* = $\bar{2}{\cdot}2282$	antilog $\bar{2}{\cdot}2282 = {\cdot}01691$

Thus $$\frac{3{\cdot}274 \times {\cdot}0059}{14{\cdot}83 \times {\cdot}077} = {\cdot}01691.$$

EXAMPLE 3. *Find the cube root of* ·02748.

Let $x = \sqrt[3]{{\cdot}02748}$, or $({\cdot}02748)^{\frac{1}{3}}$;

then $$\log x = \tfrac{1}{3}\log({\cdot}02748)$$
$$= \tfrac{1}{3}(\bar{2}{\cdot}4391), \text{ from Table of Logs.,}$$
$$= \bar{1}{\cdot}4797;$$
$$\therefore\ x = {\cdot}3018, \text{ from Table of Antilogs.}$$

EXAMPLES XXI. d.

Find by means of the Tables the values (to four significant digits) of the following products:

1. $\cdot 2834 \times 17{\cdot}62$. **2.** $8{\cdot}034 \times {\cdot}1893$. **3.** $470{\cdot}8 \times 6{\cdot}39$.

4. $3{\cdot}7 \times 8{\cdot}9 \times {\cdot}023$. **5.** $31{\cdot}9 \times 1{\cdot}51 \times 9{\cdot}7$. **6.** $43 \times 8{\cdot}07 \times {\cdot}0392$.

Find the value of

7. $\dfrac{17{\cdot}3}{294{\cdot}8}$. **8.** $\dfrac{487}{6398}$. **9.** $\dfrac{2{\cdot}179}{{\cdot}8973}$. **10.** $\dfrac{{\cdot}01254}{{\cdot}4105}$.

11. $\dfrac{2{\cdot}38 \times 3{\cdot}901}{4{\cdot}83}$. **12.** $\dfrac{14{\cdot}72 \times 38{\cdot}05}{387{\cdot}9}$. **13.** $\dfrac{925{\cdot}9 \times 1{\cdot}597}{74{\cdot}03}$.

14. $\sqrt{5{\cdot}1}$. **15.** $\sqrt[3]{11}$. **16.** $({\cdot}097)^4$. **17.** $\sqrt[4]{10{\cdot}15}$.

379. In using four-figure logarithms for purposes of practical calculation care must be taken not to attempt a greater degree of accuracy than is obtainable from the Tables. In some cases the first step of the work will be to adapt the *data* to the Tables.

380. In scientific work which records very large or very small approximate results it is often convenient to express them by writing the significant digits in standard form [Art. 153], and multiplying or dividing by a power of 10.

Thus 4037000, correct to the nearest thousand, may be written $4{\cdot}037 \times 10^6$.

And ·004037, correct to the nearest millionth, may be written $4{\cdot}037 \times 10^{-3}$.

Or again, if it is known that the distance of the Earth from the Sun is 92,000,000 miles, true to the nearest million, this approximate distance may be represented by $9{\cdot}2 \times 10^7$ miles.

EXAMPLE 1. *Find as accurately as possible from four-figure Tables the product of* 37·848 *and* 40869.

Here the data must first be adapted to the Tables.

Now $37{\cdot}848 = 37{\cdot}85$ correct to four significant figures

and $40869 = 40870$ ....

$$\begin{aligned} \log 37{\cdot}85 &= 1{\cdot}5781 \\ \log 40870 &= 4{\cdot}6115 \\ \hline \log \text{product} &= 6{\cdot}1896 \end{aligned}$$

$$\text{Now antilog } {\cdot}1896 = 1{\cdot}547\,;$$

$$\therefore \text{ the required } \textit{product} = 1{\cdot}547 \times 10^6, \text{ or } 1547000,$$

the fourth significant figure being open to doubt ; and this is the closest approximation that can be obtained by the use of four-figure Tables.

EXAMPLE 2. *Find the value of* $\dfrac{\left(330 \times \frac{1}{49}\right)^4}{\sqrt[3]{22 \times 6{\cdot}9}}$ *to the nearest integer.*

Denote the expression by x, then

$$\log x = 4(\log 330 - \log 49) - \tfrac{1}{3}(\log 22 + \log 6{\cdot}9),$$

$$\begin{array}{rr} \log 330 = 2{\cdot}5185 & \log 22 = 1{\cdot}3424 \\ \log 49 = 1{\cdot}6902 & \log 6{\cdot}9 = 0{\cdot}8388 \\ \hline 0{\cdot}8283 & 3\,\underline{\big|\,2{\cdot}1812} \\ 4 & 0{\cdot}7271 \\ \hline 3{\cdot}3132 & \\ \text{subtract } 0{\cdot}7271 & \\ \hline \end{array}$$

$$\log x = 2{\cdot}5861 = \log 385{\cdot}6, \text{ from the Tables.}$$

$$\therefore\ x = 386, \text{ to the nearest integer.}$$

EXAMPLES XXI. d. *(Continued.)*

Find from the Tables, as accurately as possible, the values of the following expressions, giving the results in standard form.

18. $\dfrac{153\cdot76 \times \cdot0137}{276 \times \cdot0038}$.   **19.** $\dfrac{3302\cdot7 \times 14\cdot3}{\cdot0561 \times \cdot387 \times \cdot0091}$.

20. 5^8.   **21.** 11^6.   **22.** 7^7.   **23.** 13^5.

24. $\sqrt[3]{82\cdot558}$.   **25.** $17^3 \times 29^2$.   **26.** $(2\cdot301)^5$.   **27.** $(\cdot089)^{\frac{4}{7}}$.

28. $\dfrac{5^3 \times 19^2}{6^5}$.   **29.** $\dfrac{1}{(3\cdot47)^4}$.   **30.** $\sqrt{\dfrac{\cdot01367 \times \cdot0296}{873\cdot47}}$.

31. Find the value of $\sqrt{\dfrac{\cdot678 \times 9\cdot01}{\cdot0234}}$ to the nearest integer.

[*Some of the Miscellaneous Examples on Contracted Work may here be worked by the use of logarithms. See page* 199.]

Applications of Logarithms.

381. In Chap. XVIII. reference was made to the use of logarithms in connection with examples in Compound Interest. If £P amounts to £A in n years at r per cent., we have

$$\text{A} = \text{PR}^n, \text{ or } \text{P}\left(1 + \frac{r}{100}\right)^n; \qquad \text{[Art. 313.]}$$

whence

$$\log \text{A} = \log \text{P} + n \log \text{R}.$$

Thus any of the four quantities involved in the formula may be found when the other three are known. The Tables should be used in all cases where r or n is required, and their use will also be found convenient in finding A or P whenever the number of years is large.

EXAMPLE 1. *In how many years at compound interest will* £342 *amount to* £1000 *at* 3 p.c. *per annum?*

Let n denote the number of years; then

$$1000 = 342(1\cdot03)^n.$$

Hence

$$\log 1000 = \log 342 + n \log 1\cdot03,$$

or

$$\begin{aligned} n &= \frac{\log 1000 - \log 342}{\log 1\cdot03} \\ &= \frac{\cdot4660}{\cdot0128} \\ &= 36\cdot4. \end{aligned}$$

$$\begin{aligned} \log 1000 &= 3\cdot0000 \\ \log 342 &= \underline{2\cdot5340} \\ & 0\cdot4660 \end{aligned}$$

Thus the required time is about $36\frac{1}{2}$ years.

Example 2. *The gas-service pipe to a house 75 feet from the main is $\frac{7}{8}$ in. in diameter; for how many burners, each taking 5 cubic feet of gas per hour, will this serve? The number of cubic feet per hour delivered by a pipe on that main is* $1000\sqrt{\dfrac{d^5}{0{\cdot}45L}}$, *where* d *is the diameter of the pipe in inches, and* L *is the length of the pipe in yards.*

Each burner consumes 5 cu. ft. of gas per hour, hence, putting $d=0{\cdot}875$, $L=25$, we have to find the value of $200\sqrt{\dfrac{(0{\cdot}875)^5}{0{\cdot}45\times 25}}$.

Let x denote the value of this expression; then

$$\log x = \log 200 + \tfrac{1}{2}[\log(0{\cdot}875)^5 - (\log 0{\cdot}45 + \log 25)]$$

$$= 2{\cdot}3010 + \bar{1}{\cdot}3294$$

$$= 1{\cdot}6304;$$

$\therefore\ x = 42{\cdot}70$, from the Tables.

Thus the pipe supplies a little more than enough for 42 burners.

$$\begin{array}{rl} 5\log 0{\cdot}875 = & \bar{1}{\cdot}9420\times 5 \\ = & \bar{1}{\cdot}7100 \\ \text{subtract} & 1{\cdot}0511 \\ 2 \,\big) & \bar{2}{\cdot}6589 \\ & \bar{1}{\cdot}3294 \end{array} \qquad \begin{array}{rl} \log 0{\cdot}45 = & \bar{1}{\cdot}6532 \\ \log 25 = & 1{\cdot}3979 \\ \hline & 1{\cdot}0511 \end{array}$$

EXAMPLES XXI. e.

Find, to the nearest pound, the amount at Compound Interest of

1. £370 in 25 years at 4 % per annum.

2. £250 in 7 years, at 5 % per annum, the interest being paid annually for the first five years, and half-yearly for the last two.

3. Find, to the nearest pound, what sum will amount to £3000 in 15 years at $3\frac{1}{2}$ % compound interest.

4. Find in how many years £1130 will amount to £3000 at 5 % compound interest.

5. A man leaves to his son aged 8, a sum of £270 to accumulate at compound interest, at 4 % per annum, until it amounts to £450. How old will the son be when he receives the legacy?

6. The population of Ireland, to the nearest thousand, was 4,459,000 at the census of 1901, and was decreasing at a rate such that in x years from the census it would fall to $(0{\cdot}9477)^x$ of the population at the census. Estimate, as nearly as possible with four-figure Tables, the population 3 years after the census.

[*Some of the Examples in* XVIII. g. *may here be worked by the aid of the Tables, bearing in mind the limitations named in Art.* 379.]

7. Find a mean proportional between 2·87 and 30·08; and a third proportional to 0·0238 and 7·805. [See Art. 268.]

8. If $s=\frac{1}{2}ft^2$, find f when $s=289·3$, $t=3\frac{7}{8}$.

9. Find the weight, to the nearest kilogram, of an iron girder which is 5·4 m. long, 0·36 m. wide and 0·22 m. thick, having given that a cubic centimetre of iron weighs 7·76 grams.

10. The time of oscillation, measured in seconds, of a pendulum, l cm. long, is $\pi\sqrt{\frac{l}{g}}$, where $\pi=3·1416$ and $g=981$. Find in seconds the time of oscillation of a pendulum 150 cm. long. (C.S.)

11. The volume of a sphere of radius r is given by the formula $V=\frac{4}{3}\pi r^3$; find the radius of a sphere the volume of which is 248·6 cubic centimetres.

12. A cubical block of metal, each edge of which is 18·2 cm., is melted down into a sphere. Find the diameter of the sphere as correctly as possible.

13. When $m=9·17$, $v=17·64$, find the value of $\frac{1}{2}mv^2$.

14. If $F=\frac{mv^2}{gr}$, find F when

$$m=24·7, \quad r=8·4, \quad v=60, \quad g=32·19.$$

15. If $\frac{v^2}{r}=\frac{g}{289}$, calculate v, having given that

$$r=4000, \quad g=\frac{32·2}{5280}.$$

Also shew that the value of $\frac{2\pi r}{v\times60\times60}$, where

$$\pi=3·1416, \text{ is approximately } 24.$$

16. A garden roller of 13 inches internal radius and 3 feet long, is four-fifths filled with water to make it heavier. Find, to the nearest pound, the weight of water, given that the internal volume of a roller of r inches radius, and l inches long, is $\pi r^2 l$ cubic inches. [1 cubic foot of water weighs 62·3 lbs.] (C.S.)

17. A litre measure is 14·2 cm. high. What must be the height (to the nearest millimetre) of a half-litre measure of the same shape? Among measures of the same shape the volume varies as the cube of the height. (C.S.)

18. The height of a pint pot of a certain shape is 12·6 cm., to the nearest tenth. A litre being 1·76 pints to the nearest hundredth, find the height of pots of the same shape that will hold 3 decilitres, 5 decilitres, and 1 litre. (C.S.)

19. If a person 35 years old pays a single premium of £50. 7*s.* 6*d.* into the Post Office Savings Bank, he assures that the sum of £100 will be paid on his death.

Assuming that interest is reckoned at $2\frac{1}{2}$ per cent., and is added annually to the capital, find to the nearest year what is the expectation of life at that age, that is, find the number of years in which the sum paid will amount to £100. (C.S.)

20. In order to find the diameter of a tube of uniform bore some mercury was poured into it and the height of the column measured. The weight of the mercury was 25·6 grams, the height of the column 15·3 cm. What was the diameter of the tube? A cubic cm. of mercury weighs 13·6 grams.

[The volume of a cylinder of height h, on a circular base of diameter d, $= 0{\cdot}7854 h \times d^2$.] (C.S.)

21. Find, as nearly as possible, how many metres of wire, 0·3 mm. in diameter, can be drawn from 450 Kg. of copper, of which the specific gravity is 8·88.

22. If a water-pipe is L yards long, d inches in diameter, and one end is H feet higher than the other, then $\sqrt{(3d)^5 \times \mathsf{H} \div \mathsf{L}}$ gallons of water will flow through the pipe in a minute. Use this formula to find how many gallons per minute will flow through a pipe a mile long, $4\frac{1}{4}$ inches in diameter, one end being 38 feet higher than the other. (C.S.)

23. An air-pump with a cylinder of volume A is used to exhaust a vessel of volume B. After n strokes of the pump the pressure of the air in the vessel is $\left(\frac{\mathsf{B}}{\mathsf{A}+\mathsf{B}}\right)^n \times p$, where p is the atmospheric pressure. Taking p as 30 inches of mercury, A as 17 cubic inches, and B 130 cubic inches, find the pressure in inches of mercury after 15 strokes and after 50 strokes. (C.S.)

24. The area of the opening under the arch of a bridge is sometimes calculated from the formula $\frac{4\mathsf{V}}{3}\sqrt{(0{\cdot}626\mathsf{V})^2 + \mathsf{C}^2}$, where V is the height of the arch and C half the distance between the ends of the arch. Calculate this area to two significant figures when $\mathsf{V} = 12$ feet and $\mathsf{C} = 16$ feet. (C.S.)

Logarithms.

No.	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	715	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7

Logarithms.

No.	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

Antilogarithms.

Log.	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
·09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

Antilogarithms.

Log.	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
·51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
·52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
·53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
·54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
·55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
·58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
·69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

MISCELLANEOUS EXAMPLES.

382. Some Applications of Graphs. When two quantities x and y are so related that a change in one produces a proportional change in the other, their variations can always be expressed by an equation of the form $y = ax$, where a is some constant quantity. Hence in all such cases the graph which exhibits their variations is *a straight line through the origin*, so that in order to draw the graph it is only necessary to know the position of one other point on it. Such examples as deal with work and time, distance and time (when the speed is uniform), quantity and cost of material, principal and simple interest at a given rate per cent., may all be illustrated by linear graphs through the origin.

Example 1. *At* 8 a.m. *A starts from* P *to ride to* Q *which is* 48 *miles distant. At the same time B sets out from* Q *to meet A. If A rides at* 8 *miles an hour, and rests half an hour at the end of every hour, while B walks uniformly at* 4 *miles an hour, find graphically*

(i) *the time and place of meeting;*
(ii) *the distance between A and B at* 11 *a.m.;*
(iii) *at what time they are* 14 *miles apart.*

In Fig. 14 on the opposite page, let the position of P be chosen as origin; let time be measured horizontally from 8 a.m. (1 inch to 1 hour), and let distance be measured vertically (1 inch to 20 miles). Thus each division on the horizontal axis represents 6 minutes and each division on the vertical axis stands for 2 miles.

In 1 hour A rides 8 mi.; therefore if D is taken 0·4 inch above the point which marks 9 p.m., PD is the graph of A's motion for the first hour. In the next half hour he makes *no advance towards* Q; therefore the corresponding portion of the graph is DE. Proceeding in the same way, we may complete the broken line PDEFGHKX giving the details of A's motion between 8 a.m. and 1 p.m.

On the vertical axis mark PQ to represent 48 mi. and mark the hours on the horizontal line through Q. At 9 a.m. B has walked 4 mi. towards P. Measuring a distance to represent 4 mi. *downwards* we get the point R, and QR produced is the graph of B's motion. It cuts A's graph at X. Hence the point of meeting is X, which is 28 mi. from P, and the time is 1 p.m.

The distance between A and B at any time is shewn by the difference of the ordinates. Thus at 11 a.m. their distance apart is MG, which represents 20 mi.

Lastly, to find when A and B are 14 mi. apart, slide a graduated ruler parallel to the vertical axis till the difference of the ordinates of the two graphs is found to be 14. This is shewn by NT; thus the time is 11.30 a.m.

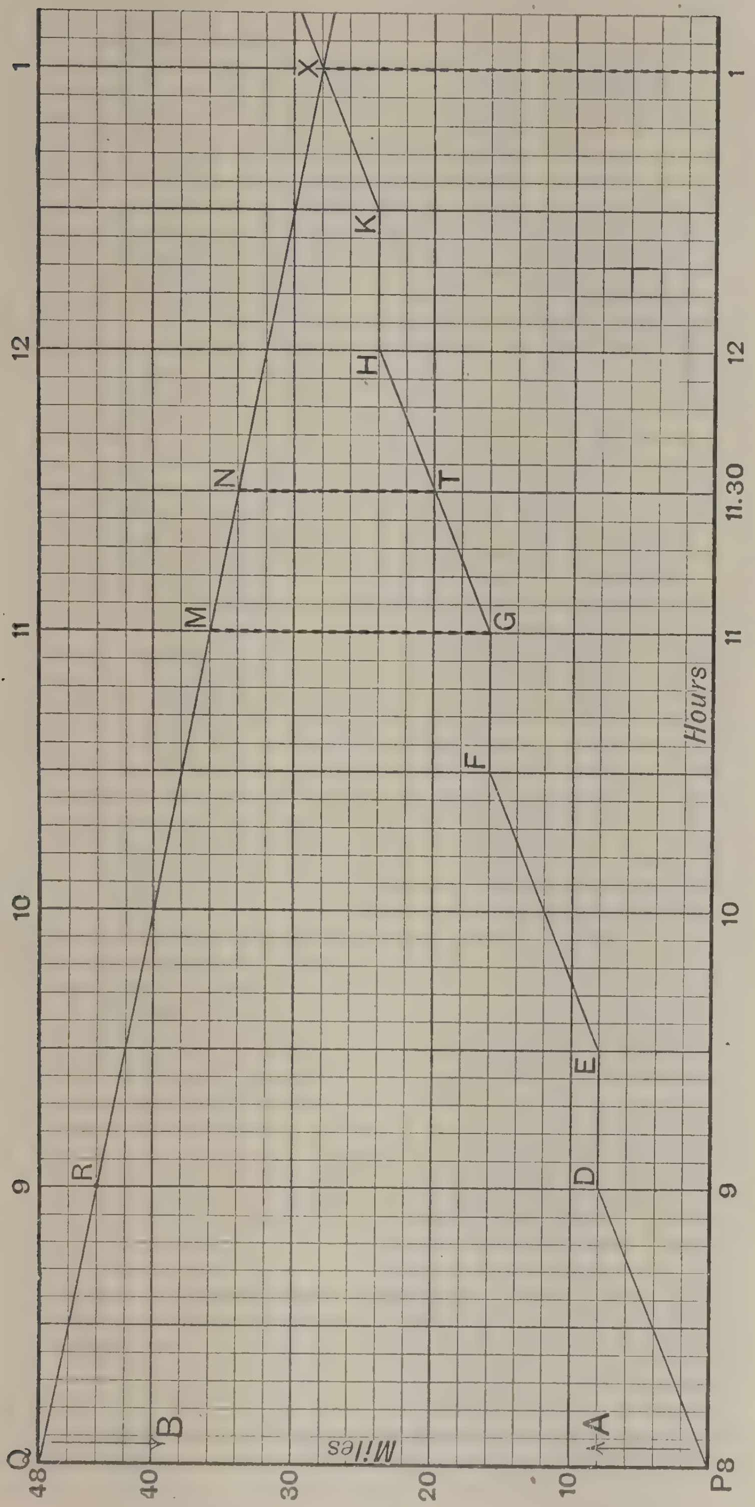


FIG. 14

NOTE. In examples of this type the advantage of the graphical method lies in the fact that a graph records a number of allied results which can be interpreted at a glance instead of making each a subject of separate calculation. For instance, the diagram on the preceding page gives the relative positions of A and B at any time between 8 a.m. and 1 p.m.

EXAMPLE 2. *A, B, and C run a race of* 300 *yards. A and C start from scratch, and A covers the distance in* 40 *seconds, beating C by* 60 *yards. B, with* 12 *yards' start, beats A by* 4 *seconds. Supposing the rates of running in each case to be uniform, find graphically the relative positions of the runners when B passes the winning post.*

Find also by how many yards B is ahead of A when the latter has run three-fourths of the course.

In Fig. 15 on the opposite page, let time be measured horizontally (1 inch to 20 seconds), and distance vertically (1 inch to 60 yards). Thus each division on the horizontal axis represents 2 seconds, and each division on the vertical axis stands for 6 yards.

O is the starting point for *A* and *C*; and if we take OP equal to 0·2 inch, representing 12 yards, on the vertical axis, P will represent *B*'s starting point.

Since *A* runs 300 yards in 40 seconds, the graph of his course is found by joining O to the point which marks 40 seconds on the time-axis.

From this point measure a vertical distance of 1 inch downwards to Q. Then since 1 inch represents 60 yards, Q is *C*'s position when *A* is at the winning post, and OQ is the graph of *C*'s course.

Along the time-axis take 1·8 inch to R, representing 36 seconds; then since *B* completes his course in 36 seconds, PR represents the graph of *B*'s running.

Through R draw a vertical line to meet the graphs of *A* and *C* in S and T respectively. Then S and T mark the positions of *A* and *C* when *B* passes the winning post.

By inspection RS and ST represent 30 and 54 yards respectively.

Thus *B* is 30 yards ahead of *A*, and *A* is 54 yards ahead of *C*.

Again, since *A* runs three-fourths of the course in 30 seconds, the difference of the corresponding ordinates of *A*'s and *B*'s graphs after 30 seconds will give the distance between *A* and *B*. By measurement we find VW = 0·45 inch, which represents 27 yards.

Thus *B* is 27 yards ahead of *A* when the latter has run three-fourths of the course.

NOTE. The solutions of the two preceding examples have been given very fully to illustrate and enforce the general principles on which the linear graphs depend. Solutions may usually be presented with less detail, and the results quickly obtained from a well-drawn diagram on a suitable scale.

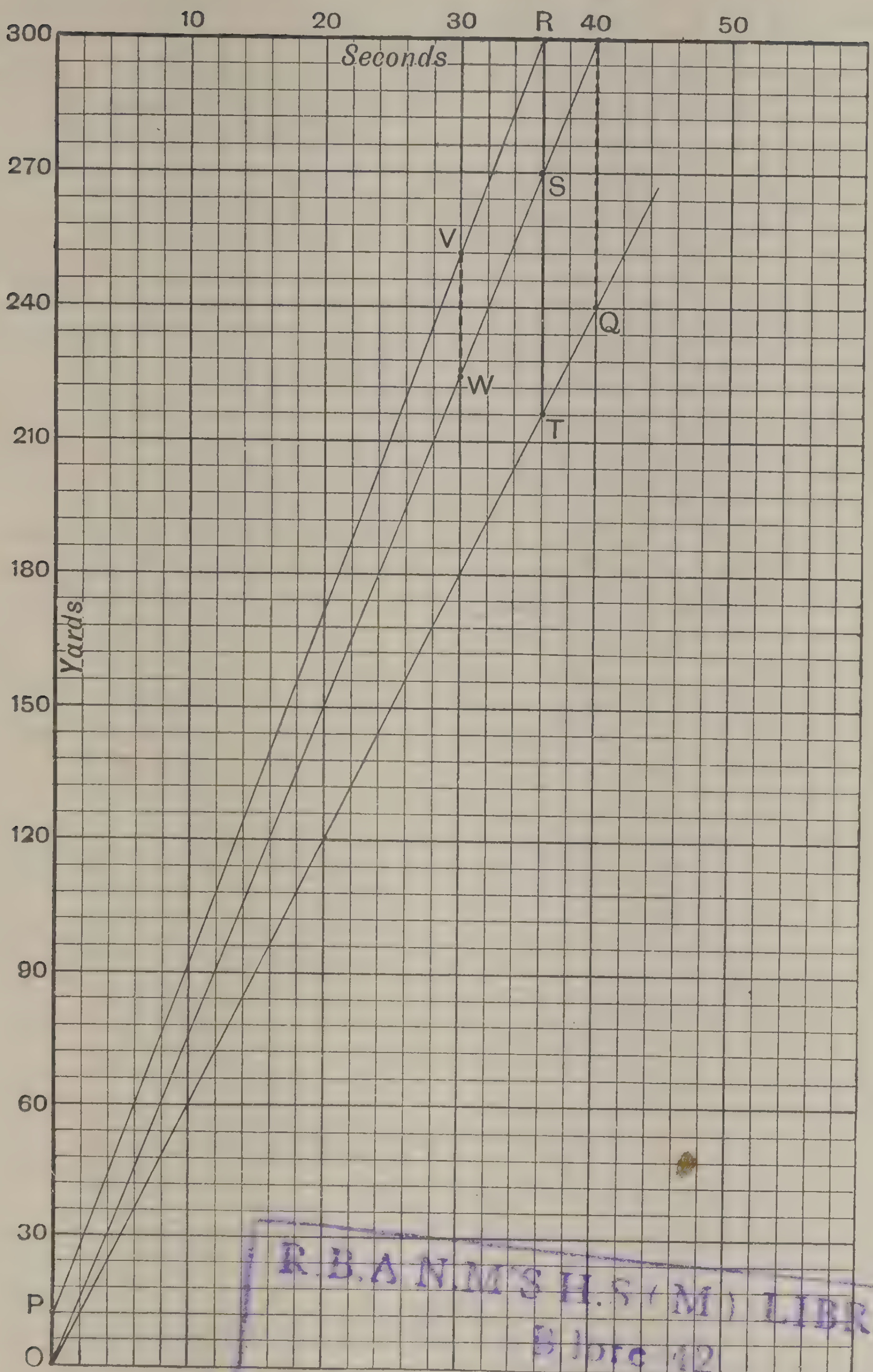


FIG. 15.

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383. When a variable quantity y is partly constant and partly proportional to a variable quantity x, the algebraical relation between x and y is of the form $y=ax+b$, where a and b are constant. The corresponding graph will therefore be a straight line; and since a straight line is completely determined when the positions of two points are known, it follows that, in all problems which can be illustrated by linear graphs, it is sufficient if the data furnish for each graph two independent pairs of simultaneous values of the variable quantities.

Example 1. *In a certain establishment the clerks are paid an initial salary for the first year, and this is annually increased by a fixed bonus. the initial salary and the bonus being different in different departments. A receives £130 in his 10th year, and £220 in his 19th. B, in another department, receives £140 in his 5th year and £180 in his 13th.*

Draw graphs to shew their salaries in different years.

In what year do they receive equal salaries?

Also find in what year A earns the same salary as that received by B for his 21st year.

In Fig. 16 let each horizontal division represent 1 year; and let the salaries be measured vertically, beginning at 130, with 1 division to represent £2.

Suppose the initial salary is denoted by £a, and the yearly bonus by £b; then the salary for the first year is £a; for the second it is £$(a+b)$; for the third it is £$(a+2b)$; for the fourth it is £$(a+3b)$: and so on.

Hence if the salary *after* x years is denoted by £y. it is evident that in each case we have a relation of the form $y=a+bx$, where a and b are constant. Thus the variations of time and salary may be represented by linear graphs.

After 9 years *A* receives £130, and after 18 years he receives £220. That is, when $x=9$, $y=130$, and when $x=18$, $y=220$. Thus the points P and Q are determined, and by joining them we have the graph for *A*'s salary.

Similarly the graph for *B*'s salary is found by joining P′ (4, 140) and Q′ (12, 180).

To find in what year *A* and *B* have equal salaries we must note the point L where the two graphs have the same ordinate and abscissa. Here $x=16$, $y=200$.

Thus *A* and *B* have the same salary when each have served 16 years, that is in their 17th year.

Again *B*'s salary at the end of 20 years is given by the ordinate of M, which is the same as that of Q which represents *A*'s salary after 18 years.

Thus *A*'s salary for his 19th year is equal to *B*'s salary for his 21st year.

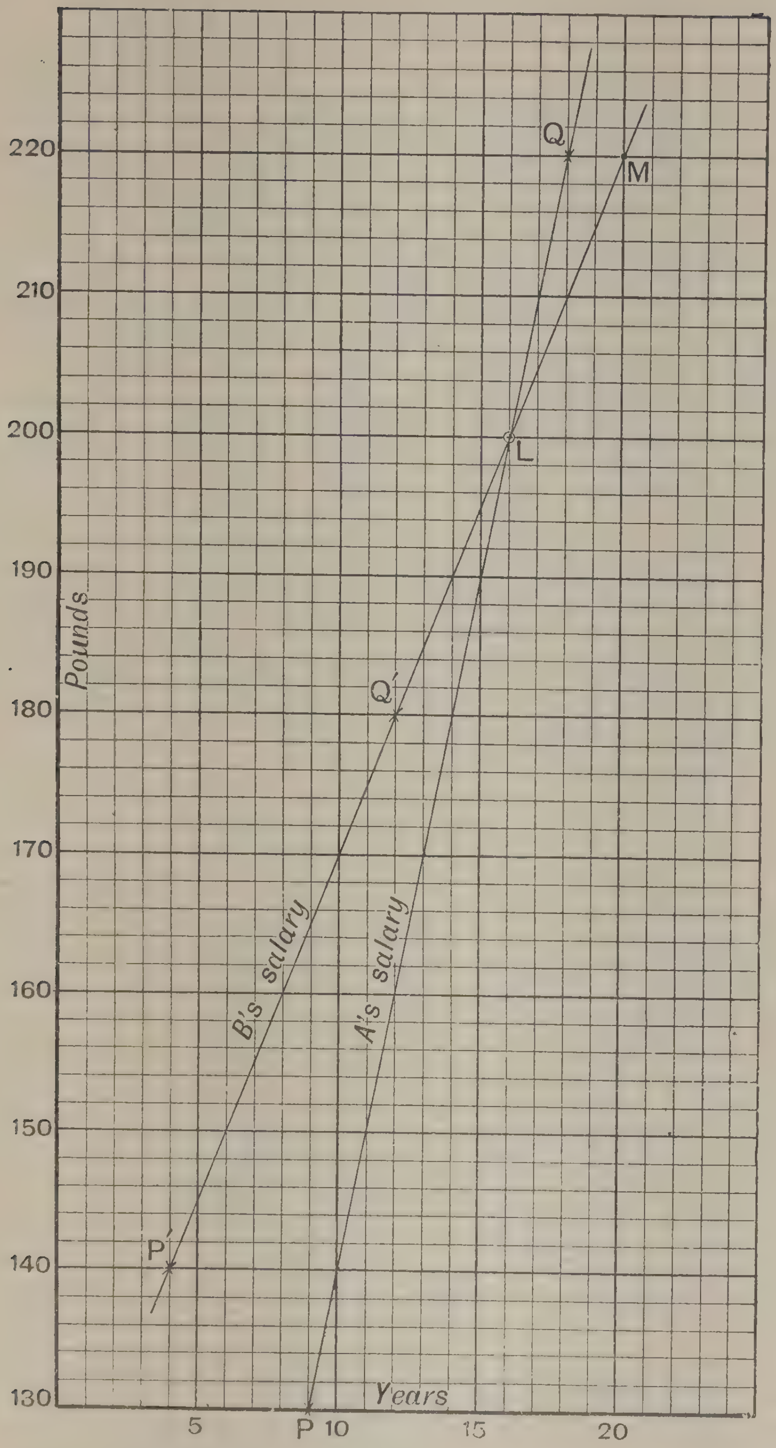

220
210
200
190
180
170
160
150
140
130
Pounds
Q
M
L
Q′
B's salary
A's salary
P′
Years
5
P 10
15
20

FIG. 16.

MISCELLANEOUS EXAMPLES VI.

1. By measuring time along OX (1 inch for 1 hour) and distance along OY (1 inch for 10 miles) shew how to draw lines

(i) from O to indicate distance travelled towards Y at 12 miles an hour;

(ii) from Y to indicate distance travelled towards O at 9 miles an hour.

If these are the rates of two men starting at noon to ride towards each other from places 60 miles apart, find from the graphs when they are first 18 miles from each other. Also find (to the nearest minute) their time of meeting.

2. At noon *A* starts to walk at 6 miles an hour, and at 1.30 p.m. *B* follows on horseback at 8 miles an hour. When will *B* overtake *A*? Also find

(i) when *A* is 5 miles ahead of *B*;

(ii) when *A* is 3 miles behind *B*.

3.✓ Two bicyclists ride to meet each other from two places 95 miles apart. *A* starts at 8 a.m. at 10 miles an hour, and *B* starts at 9.30 a.m. at 15 miles an hour. Find graphically when and where they meet, and at what times they are $37\frac{1}{2}$ miles apart.

[Take 25 mi. to the inch vertically, and 1 hr. to the inch horizontally.]

4. The salary of a clerk is increased each year by a fixed sum. After 6 years' service his salary is raised to £128, and after 15 years to £200. Draw a graph from which his salary may be read off for any year, and determine from it (i) his initial salary, (ii) the salary he should receive for his 21st year.

5.✓ A man gains 12 per cent. by selling articles at 16*s*. 4*d*. each: what price must he charge for each article in order to make a profit of £10. 18*s*. 9*d*. on the sale of 100 articles?

6. The table given below shews the approximate population and the average daily supply of water per head in the places named. Estimate the average supply per head for the three cities taken together:

	Approx. population.	Gallons per head.
Manchester, - -	610,000	16·5
Liverpool, - -	730,000	17·9
Birmingham, - -	520,000	15·3

7. A man's income from $2\frac{1}{2}$ per cent. Consols (after income tax at 1*s.* in the £ has been deducted) is £448. 8*s.* How much stock does he hold?

8. *A* takes 9 strides to *B*'s 7; *A*'s strides are $3\frac{1}{2}$ ft., and *B*'s $4\frac{1}{4}$ ft. Find how far *A* must run to catch *B* who has a start of 25 yards.

9. X and Y are two towns 35 miles apart. At 8.30 p.m. *A* starts to walk from X to Y at 4 miles an hour; after walking 8 miles he rests for half an hour, and then completes his journey on horseback at 10 miles an hour. At 9.48 a.m. *B* starts to walk from Y to X at 3 miles an hour; find when and where *A* and *B* meet. Also find at what times they are $6\frac{1}{2}$ miles apart.

10. *A* can beat *B* by 20 yards in 120, and *B* can beat *C* by 10 yards in 50. Supposing their rates of running to be uniform, find graphically how much start *A* can give *C* in 120 yards so as to run a dead heat with him. If *A*, *B*, and *C* start together, where are *A* and *C* when *B* has run 80 yards?

11. *A*, *B*, and *C* run a race of 200 yards. *A* gives *B* a start of 8 yards, and *C* starts some seconds after *A*. *A* runs the distance in 25 seconds and beats *C* by 40 yards. *B* beats *A* by 1 second, and when he has been running 15 seconds, he is 48 yards ahead of *C*. Find graphically how many seconds *C* starts after *A*. Shew also from the graphs that if the three runners started level they would run a dead heat.

[Take 1 inch to 40 yards, and 1 inch to 10 seconds.]

12. The rainfall on a roof 20 ft. long and 12 ft. wide is collected in a covered rectangular cistern 5 ft. long, 4 ft. wide, and 8 ft. deep; what depth of water will be found in the cistern after a fall of 2 inches of rain?

13. When the exchange between Paris and London is $25{\cdot}10\frac{1}{2}$ francs for £1, express 430 fr. 50 c. in £. *s.* *d.* to the nearest penny.

14. Find, to the nearest penny, the difference between the simple and compound interest on £401. 13*s.* 4*d.* for 3 years at 5 per cent.

15. It cost £42 to carpet the whole floor of a room 30 ft. long and 28 ft. wide; if a space 1 ft. 3 in. wide is left uncovered all round, what is the saving in cost?

16. Standard gold consists of 11 parts by weight of pure gold and 1 part of copper. If a sovereign weighs 123 grains, find how many can be coined from a bar of standard gold containing 1 lb. avoirdupois of pure gold.

17. *A* and *B* ride to meet each other from two towns X and Y which are 60 miles apart. *A* starts at 1 p.m., and *B* starts 36 minutes later. If they meet at 4 p.m., and *A* gets to Y at 6 p.m., find, by means of a graph, the time when *B* gets to X. Also find the times when they are 22 miles apart. When *A* is half-way between X and Y, where is *B*?

18. Two men, setting out at the same time from places A and B, twelve miles apart, meet at a distance of four miles from B. If they had both started together from A, at the same rates as before, and the faster one had gone to B and immediately turned back, how far from B would he have met the other? Verify your solution graphically.

19. A cyclist has to ride 75 miles. He rides for a time at 9 mi. an hour and then alters his speed to 15 mi. an hour, covering the distance in 7 hours. Find graphically at what time he changed his speed.

20. A county contains three districts, A, B, C. The population of A is 185000 and is increasing at the rate of 0·482 per cent. per annum; that of B is 235000 and is increasing at the rate of 0·324 per cent. per annum; that of C is 325000 and is increasing at the rate of 0·516 per cent. per annum. What is the rate of increase per cent. per annum for the whole county?

21. In the first four months the takings of a business are respectively £335. 2*s*. $1\frac{1}{2}$*d*., £371. 15*s*. 11*d*., £401. 11*s*. $5\frac{1}{2}$*d*., and £446. 11*s*. 6*d*. What must be the average takings for the remaining months in order that the receipts for the year may amount to £5000?

22. Express 765 and 7931520 in prime factors. Find all the pairs of numbers which have 765 for their H.C.F. and 7931520 as their L.C.M.

23. The cost of turfing a rectangular grass plot of which the sides are in the ratio of 2 : 3 is £14. 8*s*., at the rate of 4*d*. per square foot. Find the lengths of the sides.

24. The capital of a railway is six millions sterling; one-third is borrowed on mortgage at $4\frac{1}{2}$ per cent., the remainder is held in shares. The working expenses are 40 per cent. of the gross receipts: find, to the nearest penny, what the average weekly receipts must be to pay the shareholders 5 per cent.

25. A man buys eggs at 18 for a shilling, and sells them at $10\frac{1}{2}$*d*. per dozen; find his gain per cent.

26. By using decimal contracted methods find, to the nearest penny, the value of 40 tons 2 cwt. 12 lbs. at £17. 5*s*. 7*d*. per ton.

27. Three substances are mixed together: their volumes are proportional to 3, 4, 5 respectively, and the weights of equal volumes are proportional to 4, 5, 6 respectively. What is the weight of each substance if the weight of the mixture is 5 lbs. 13 oz.?

28. The distance from London to Bristol is 119 miles; if I were to set out at noon to cycle from London, riding 26 miles the first hour and decreasing my pace by 3 miles each successive hour, find graphically how long it would take me to reach Bristol. Also find approximately the time at which I should reach Faringdon, which is 48 miles from Bristol.

29. The following table shews the number of acres planted with tea in India and Ceylon in three different years:

	1895.	1900.	1905.
Number of acres planted with tea, -	720,717	914,487	913,290

It has been estimated that the increase of acreage between 1895 and 1900 caused an increase of crop of 48 million lbs. Find how much tea (to the nearest lb.) this gives to the acre.

Find also, at the same rate, the decrease of crop owing to the decrease of acreage between 1900 and 1905. (C.S.)

30. Find graphically at what distance from London, and at what time, a train which leaves London for Rugby at 2.33 p.m., and goes at the rate of 35 miles an hour, will meet a train which leaves Rugby at 1.45 p.m. and goes at the rate of 25 miles an hour, the distance between London and Rugby being 80 miles.

Also find at what times the trains are 24 miles apart, and how far apart they are at 4.9 p.m.

[Take 1 inch to 20 mi., and 1 inch to 2 hrs.]

31. A rectangular plot of building land with a frontage of 34 feet and a depth of 125 feet was let on a building lease at the rate of £60 per acre. Find the rent of the plot to the nearest penny.

32. A tradesman marks his goods at such a price that he can deduct 10 per cent. for cash and yet make 15 per cent. profit. What is the marked price of an article which cost him £3. 12*s.*?

33. *A* can do a piece of work in $2\frac{1}{2}$ days which *B* can do in $3\frac{1}{3}$ days. If *A*'s wages are £1. 16*s.* 8*d.* a week and *B*'s wages are £1. 8*s.* 9*d.*, what would *A* have charged for doing a piece of work for which *B* received £11. 10*s.*?

34. In France, when the population was 33·4 millions, the number of acres under wheat was 13,224,000, and the average yield per acre 15·2 bushels; when the population had grown to 38·2 millions, the acres under wheat were 17,198,000 and the average yield in bushels per acre 18·0.

Find at each period the total wheat crop in bushels (to the nearest million bushels) and the amount of wheat raised per head of population (to the nearest bushel). (C.S.)

35. The expenses of a school are partly constant and partly proportional to the number of boys. The expenses were £650 for 105 boys, and £742 for 128. Draw a graph to represent the expenses for any number of boys; find the expenses for 115 boys, and the number of boys that can be maintained at a cost of £710.

[See Art. 383. Take 1 inch to £100 on the axis of y, 1 inch to 10 boys on the axis of x; and begin measuring ordinates at 600, abscissæ at 100.]

36. The annual expenses of a Convalescent Home are partly constant and partly proportional to the number of inmates. The expenses were £384 for 12 patients and £432 for 16. Draw a graph to shew the expenses for any number of patients, and find from it the cost of maintaining 15.

In a rival establishment the expenses were £375 for 5, and £445 for 15 patients. Find graphically for what number of patients the cost would be the same in the two cases.

37. At the last census in a certain town it was found that the males were 42 per cent. of the population, and in another town 46 per cent. of the population was male. The population of the first town is to that of the second as 3 : 8. What percentage (to the nearest integer) of the combined populations of the two towns is male? (C.S.)

38. A closed cubical cistern, made of metal 1 centimetre thick, has an internal capacity of 1 cubic metre. Assuming the metal to be 7 times as heavy as water, find the weight of the cistern in kilograms.

39. If the decimal fraction 0·567 of a pound were taken to be equal to 567 farthings, express the error that would be made as a percentage of the true value.

Find, to the nearest whole number, what the error would be in farthings. (C.S.)

40. Two casks originally contain 60 gallons of wine and 30 gallons of water respectively. On three successive occasions 12 gallons of liquid are drawn from each cask and placed in the other. Express in gallons and decimals of a gallon the quantity of wine now in each cask.

41. A piece of work can be done by 3 men and 4 boys in 6 days, by 3 men and 1 boy in 8 days, and by 4 women and 8 boys in 5 days. How long would a woman take to complete the work single-handed?

42. Calculate to the nearest centime, and using as few figures as possible, the value of £32. 17*s.* 11$\frac{1}{4}$*d.* in French money, at the rate of 25 francs 11 centimes for £1.

43. What is the ground-rent per acre, when an estate of 9 acres 3 roods 25 poles yields £166. 8*s.* 6*d.* in ground-rents?

44. A ventilating tube whose sectional area is 44 square inches is delivering a current with the velocity of 1·24 ft. per sec. into a hall whose dimensions are 52 feet by 35 feet by 21 feet. Find, to the nearest second, how long it will take for the air of the hall to be renewed in this way.

45. A man travels 60 miles in 3 hours, partly by rail and partly by coach. If he had gone all the way by rail he would have arrived at his destination an hour earlier, and would have saved two-fifths of the time he was on the coach. How far did he travel by coach?

46. A merchant borrows £34,481 on the condition that at the end of each year he is to pay back part of the principal, with interest at 5% on the amount standing unpaid during the year. Prove that by an annual payment of £9724. 1*s.* he can clear off the debt in four years.

47. If 31 Napoleons contain 180 grams, and 6231 sovereigns 704103 grains, of gold, find the smallest whole number of Napoleons that are equal to a whole number of sovereigns, 5 lbs. being equivalent to 2·268 kilograms.

48. A grocer mixes two qualities of sugar at $3\frac{1}{2}d.$ and $5d.$ per lb. respectively, and gains 2 per cent. on his outlay by selling the mixture at $4\frac{1}{2}d.$ In what ratio does he mix the two qualities? (C.S.)

49. Prove, with as little calculation as possible, that

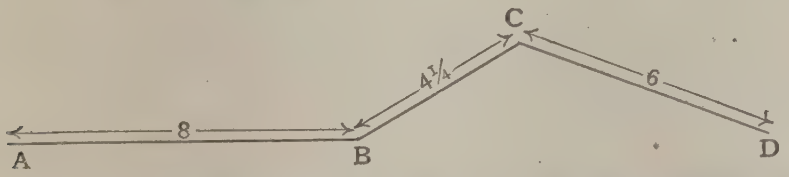
$$39{\cdot}94771 \times 2{\cdot}499531 \div 1249835$$

is greater than 0·00007 and less than 0·00009. (C.S.)

50. If a fall of £2·3375 in the selling price of certain goods converts the seller's profits of $5\frac{1}{2}$ per cent. on his outlay to a loss of $5\frac{1}{2}d.$ in the £, what did he pay for the goods? (C.S.)

51. French and English wall papers are cut into pieces, the former 18 inches wide and 9 yards in length, the latter $22\frac{1}{2}$ inches wide and 12 yards in length. A paperhanger's book contains patterns of both which are priced by the piece. How would you reduce the prices of the French pieces (given in English money) so as to compare them with the prices of the English pieces? (C.S.)

52. Two pedestrians, *X* and *Y*, start from A and D respectively at the same time, their rates of walking being as 4 : 3. *X*'s rate on the level is 4 miles an hour, uphill 3 miles an hour, and down hill 5 miles an hour. Where will they meet? (C.S.)



[The numbers indicate miles; A to B is 8 miles, B to C $4\frac{1}{4}$ miles, and C to D 6 miles.]

53. *A* walks $7\frac{1}{2}$ yds. while *B* walks 6. If they start from the same point walking in the same direction, and *B* has a start of 20 yds., find graphically *A*'s position relatively to *B*

(i) when *B* has walked 50 yards;

(ii) when *A* has walked 130 yards.

How far will *B* have walked when he is 4 yards behind *A*?

54. I row against a stream flowing $1\frac{1}{2}$ miles an hour to a certain point, and then turn back, stopping two miles short of the place whence I originally started. If the whole time occupied in rowing is 2 hrs. 10 mins. and my uniform speed in still water is $4\frac{1}{2}$ miles an hour, find graphically how far upstream I went.

[Take 1·2 of an inch horizontally to represent 1 hour, and 1 inch to 2 miles vertically.]

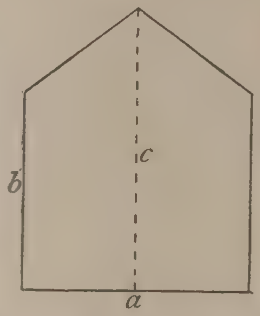
55. A pint measure is 12·4 cm. high to the nearest mm. Find what must be the height of a litre measure of the same shape. Among measures of the same shape the volume varies as the cube of the height. A litre is 1·76 pints to the nearest hundredth. (C.S.)

56. In making out his balance-sheet a manufacturer reckons that his machinery (if there have been no renewals) deteriorates in a year by 5 per cent. of its value at the beginning of the year. If he buys machinery that costs £2740, at what value will it stand in his balance-sheet 6 years afterwards? (C.S.)

57. If 10 men, 8 women, and 6 boys can complete a piece of work in 25 days, for how many days will 19 men, 20 women, and 30 boys be occupied upon three times the work, the work of a man, woman, and boy being to one another as 5 to 3 to 2?

58. A recipe gives directions to mix 4 parts of substance A with 7 parts of substance B. These parts ought to be taken by weight but, by mistake, they are taken by volume. Find the error in the percentage of weight of A in the mixture if 117 cubic inches of A weigh as much as 151 cubic inches of B. (C.S.)

59. A haystack 30 ft. long has throughout its length the section shewn in the margin to a scale of 1 inch to 6 yds. The area of a section which is a ft. wide, b ft. high at the sides, and c ft. high at the ridge is $a \times \frac{b+c}{2}$ sq. ft. Find (to the nearest ton) the weight of hay in the stack, taking a cubic foot of hay as weighing 10 lbs.



60. A rectangular stone trough is 3 ft. 9 ins. long, 2 ft. 7 ins. broad and 1 ft. 2 ins. deep externally, and weighs 874 lbs. when empty. The interior surface is irregular, but it is found by trial that the trough holds 38 gallons. Find, to one place of decimals, how many times the stone is as heavy as water, bulk for bulk. [1 cubic foot of water weighs 62·3 lbs., 1 gallon of water weighs 10 lbs.] (C.S.)

61. A man has a lease of a house, which has 3 years to run, and the rent is £150 a year, to be paid at the end of each year. In consideration, however, of a sum of money paid down he gets the rent reduced to £100 for the rest of his term. Find, to the nearest shilling, what sum he ought to pay, the current rate of investments being 3 per cent.

62. At the age of 20 a clerk commenced at £100 a year for the first three years. His salary was then raised £20 each year for 10 consecutive years, and afterwards remained stationary. He has just retired, and reckons that, on the average, his salary has been £250 a year. What is his present age? (C.S.)

63. There was a question in an arithmetic paper:—To find the L.C.M. of three given numbers. One of them was 3425, another 1829, and the third was an even number of 5 figures. One candidate, by mistake, copied down 3245 instead of 3425 but obtained the correct answer. What was the third given number? (C.S.)

64. I have two kinds of foolscap paper ruled, so that on the one I find that 10 intervals measure 3·3 inches, and on the other 8 intervals measure 2·7 inches. Express, as the vulgar fraction of an inch, how much an interval on the one paper differs in size from an interval on the other paper.

Also find the shortest distance which is equal to an exact number of intervals on both kinds of paper. (C.S.)

65. Assuming that 32 metres = 105 feet, and that 4 litres = 7 pints, find how many litres of water would be contained in a trench 48 metres long, 2 metres wide, when a trench of the same depth, 35 ft. long and 4 ft. wide, contains 2400 gallons.

66. *A* starts 3 min. after *B* for a place $4\frac{1}{2}$ miles distant. *B* on reaching his destination, immediately returns, and after walking a mile meets *A*. If *A*'s speed is a mile in 18 minutes, what is *B*'s speed?

67. *A*, *B*, and *C* entered into partnership, and provided capitals of £1100, £1300, and £1700 respectively. Some months later, £500 extra capital being needed, it was supplied by *B*. At the end of 12 months the total profit was £2513. 14*s*., and *A*'s share thereof £623. 14*s*. When did *B* supply the extra capital?

68. If 1 metre = 39·37 inches; 1 gram = 15·43 grains, find by how much per cent. a pressure of 15 lbs. per square inch exceeds a pressure of 1 kilogram per square centimetre.

69. A man's expenditure each year is to his income for that year as 5 is to 4; and his income for any year is to his income for the previous year as 9 is to 10. His expenditure for the year 1902 exceeded by 50 his income for the year 1900. What was his income in 1900?

70. A wine merchant buys 200 gallons of wine at 10*s*. per gallon; at the end of 4 years he finds that 25 gallons of it have been lost by leakage; at what price per gallon must he sell the remainder so that he may have 5 per cent. compound interest on his capital?

71. One cubic centimetre of concentrated sulphuric acid weighs 1·842 grams. If 70 parts by volume of this acid are mixed with 30 parts of water, it is found that one cubic centimetre of the mixture weighs 1·615 grams. Find how much per cent. the mixture has shrunk. (C.S.)

72. Three athletes run a three-mile race round a course of 528 yards. Their rates are as 32 : 30 : 22. Find the positions of the other two when the winner comes in.

73. A man buys some property for £2800, and lets it on lease at a rent of £155 per annum; if the property costs £14 a year for repairs, what interest does he get on his investment?

If the rent be collected by an agent for a commission of $2\frac{1}{2}$ per cent., what is the net interest obtained by the landlord?

74. The third-class return fare from London to Plymouth, a distance of 234 miles, is 32*s.* 10*d.* Express this in terms of centimes per kilometre, having given that 1 yard = 0·9144 metre, and £1 = 25·17 francs.

75. A motor-car, travelling at the rate of 40 miles an hour, starts from a place P to travel, on a road running beside a railway, to a place Q, a distance of 150 miles. Fifteen minutes after the motor-car has started, a train, travelling at the rate of 60 miles an hour, leaves P to go to Q. If the train stops for five minutes at a station 35 miles from P, find graphically:

(i) At what distances from P the train and motor-car are together.

(ii) The length of time between the arrivals at Q of the train and motor-car. (C.S.)

76. From a vessel containing a gallons of spirits, b gallons are taken and replaced by water. From the mixture b gallons are taken and replaced by water; shew that the vessel now contains $a\left(\frac{a-b}{a}\right)^2$ gallons of spirits.

77. A, B, and C together can do a piece of work in 60 days. After they had worked together for 10 days, A withdraws and B and C work together for 20 days. B then withdraws and C completes the work in 96 days more, working one-third longer each day. Working at his former rate C could alone do the whole work in 222 days. Find how long B would take to do the work alone.

78. A sum of money is accumulating at compound interest at a certain rate per cent. If simple interest instead of compound were reckoned, the interest for the first two years would be diminished by 13*s.* 4*d.*, and that for the first three years by £2. 0*s.* 4*d.*; what is the sum?

79. Express $\sqrt{25+\sqrt{125}}$ as a decimal correct to 4 significant figures.

80. The elder of two brothers inherited from his father two-fifths more money than the other. Since their father's death, the elder brother has increased his capital to the extent of three-sevenths of its original value, and the younger has lost £707. He now finds that he has only £43 for every £100 that his elder brother has. How much had each at first?

81. A rectangular roof 10·54 metres long by 8·2 metres broad is covered with sheet lead whose thickness is 8 millimetres. Find the weight of the covering in kilograms. [Specific gravity of lead = 11·35.]

82. Using contracted decimal work find the value of 5 tons 3 cwt. $26\frac{1}{2}$ lbs. at £1. 17*s*. $5\frac{1}{2}d$. per cwt.

83. A train starting at 12 o'clock on a 25 mile journey passes the points given in the annexed table at the stated times:

Time,	12	h. m. 12.4	h. m. 12.8	h. m. 12.13	Arrives h. m. 12.20, stops 3 mins.	h. m. 12.27	h. m. 12.31	h. m. 12.38 Stop.
Distance in miles,	0	2	5	10	15	17	20	25

Plotting times horizontally and distance vertically, draw the curve shewing the speed of the train throughout, and find from it at about what distance from the start the speed is greatest, and that speed in miles per hour. (C.S.)

84. The weight of a foot-length of cast-iron pipe when the metal is $\frac{1}{2}$ inch thick is given by the following table:

D	1	1·5	2	2·5	3	3·5	4	4·5	5
W	7·4	9·8	12·3	14·7	17·2	19·6	22·1	24·5	27·0

D being the diameter of the bore in inches and W the weight of the pipe in lbs. Draw a graph to represent this relation, and from it give the weight of a foot-length when the diameter of the bore is $3\frac{1}{4}$ inches. Represent 1 inch by 4 cm., and 1 lb. by 1 cm., and shew the scales clearly on the figure.

To check your result calculate the weight of the pipe for a bore of $3\frac{1}{4}$ inches by means of the formula

$$W = 1{\cdot}56\pi(D + 0{\cdot}5).$$

(C.S.)

85. A gramophone with 5 rolls of music costs £7; the same gramophone with 20 rolls of music costs £9. How much should be paid for the gramophone with 50 rolls of music? (C.S.)

86. The following rules for conversion of units are sometimes employed:

(i) to convert ounces to grams multiply by $\frac{85}{3}$;

(ii) to convert pounds to kilograms multiply by $\frac{5}{11}$.

Having given 1 kilogram = 2·204 pounds, find the percentage error in each case. (C.S.)

87. If a sheet of lead 10·9 metres long and 19 decimetres wide weighs 1747 kilograms, and 1 cubic centimetre of lead weighs 11·4 grams, what is the thickness of the sheet to the nearest tenth of a millimetre?

88. Three persons contribute sums of £250, £500, and £750 respectively towards a venture, on the understanding that the profits shall be divided in such a way that the *rate* of interest which each receives shall be in proportion to the amount of his contribution. If the profits for a year amount to £245, how much will each of them receive? (C.S.)

89. An astronomical clock has its dial divided in 24 divisions instead of 12, and the small hand goes round in 24 hours, the large hand going round once every hour. The 24 hour is noon. Find when the hands are at right angles between 24 and 1, and also the interval between two successive meetings of the hands.

90. In a hundred yards race A can beat B by 4 yards; in a quarter of a mile race C can beat A by 11 yards; by how much can C beat B in a mile race, supposing that the average speeds of each man when running a hundred yards, a quarter of a mile, and a mile are proportional to 9 : 8 : 7?

91. Find the cost of making a road half-a-mile long, 36 ft. wide; the soil being first excavated to a depth of one foot at a cost of 1*s*. per cubic yard, rubble being then laid in, 9 inches deep, at a cost of 1*s*. 6*d*. per cubic yard, and 3 inches of gravel, at 3*s*. 3*d*. per cubic yard, being laid on the top, and the whole consolidated by a steam roller at a cost of 2*d*. per square yard.

92. In the year 1905 the total British and Australian coinage was as follows:

Gold.	Silver.	Bronze.
£18,588,000	£510,490	£100,325

Express each of these totals as a percentage of the total coinage correct to three significant figures.

93. A fundholder directs his broker to purchase £100 shares in a certain mine, quoted at $272\frac{3}{8}$ per share. To accomplish this he authorises him to sell out £850 in 3 p.c. stock at $95\frac{5}{8}$, and £1300 in $4\frac{1}{2}$ p.c. stock at $105\frac{3}{8}$. The broker's charge on each of the three transactions is $\frac{1}{8}$ per cent. on the nominal value of the stock; how much does he receive for the whole transaction?

94. An armourer undertakes to supply 2000 swords at 17*s*. 3*d*. each: and he estimates that if 5 per cent. fail to stand the required tests and are worthless, the profit will be 15 per cent. on his whole outlay. At the trial 35 per cent. of the swords prove worthless: how much does the armourer lose by the contract?

95. The population of a certain town was 5000 in 1850 and its rates of increase for successive periods of 10 years from 1850 to 1900 were 8, 10, 13, 14, and 15 per cent. Find the population at the end of each period, and plot a curve shewing the growth of population during the 50 years. From the curve estimate the population in the years 1875 and 1895. (C.S.)

96. The following is an extract from Bradshaw :

Distance from London in miles.	
Willesden, - - - -	$5\frac{1}{2}$
Northampton, - - -	$65\frac{3}{4}$
Rugby, - - - -	$82\frac{1}{2}$

An evening train	p.m.
Leaves Willesden, - -	7.12
Arrives at Northampton,	8.22
Leaves Northampton, -	8.25
Arrives at Rugby, - -	8.52

Find, to the nearest 10th of a mile, the speed in miles per hour between Willesden and Northampton, and between Northampton and Rugby ; supposing the speed uniform on each run.

Give a graph shewing the distance of the train from London at any time between 7.12 and 8.52. Use 1 cm. to represent 10 minutes and 2 cm. to represent 10 miles. From the graph read off the distance from London at 8.15 p.m., and the time at which the distance is 70 miles. (C.S.)

97. The rent of a house is £200 per annum, due by half-yearly payments on Jan. 1st and July 1st, and no payment has been made since the beginning of last year. What sum to the nearest penny will be due at the beginning of next year, interest being added to arrears at the beginning of each half year, at the rate of 5 per cent. per annum?

98. Cycling from A to B I ride at 15·75 kilometres an hour except up one hill 0·75 kilometre long, where I dismount and push the machine at 4·5 kilometres an hour. On returning I ride all the way at 15 kilometres an hour and take the same time over the journey. What is the distance from A to B? (C.S.)

99. *A* and *B* start together from the same point on a circular track, and walk in the same direction till they both again arrive at the starting point. If *A* performs a circuit in 224 seconds and *B* in 364 seconds, how many times will *A* have passed *B*?

100. Find correct to 4 significant figures the value of $(1{\cdot}031485)^5$, stating with reasons what figures may be disregarded at any stage of the work.

101. A rectangular field is 127·35 metres long and 98·27 metres wide. Express the area as the decimal of an acre, having given that 1 yd. = 0·9144 metre.

102. A man invests £1000 in 3 p.c. stock at 105. At the end of the first year he invests the year's dividend in the same stock at 107 ; at the end of the second year he invests the whole dividend for that year in the same stock at 108. What is his dividend for the third year, correct to the nearest penny?

103. From a quarter-cask (27 gallons) of sherry a dishonest servant abstracts half-a-pint daily, replacing it by half-a-pint of water. Find an expression to shew the number of pints of sherry left after 20 days.

104. There are two cogged wheels, with 5 and 24 teeth respectively, working into each other. If the first makes 16 complete revolutions in 10 seconds, how many revolutions will the second make in 1 minute? (C.S.)

105. The average annual depth of rainfall at a certain place was 24·98 inches for the years 1899, 1900, 1901; the succeeding three years it was 29·62. The year 1903 was the wettest, when there fell 4·8 inches more than in 1902, 6·36 inches more than in 1904, and 7·47 more than in 1900. The fall in the year 1901 was short of the preceding year by only 0·17 inch. Find the depth of rainfall that fell in each of the six years. (C.S.)

106. In a flat race *A* beats *B* by 15 yards, and *C* by 29 yards. When *B* and *C* run over the course together *B* wins by 15 yards; find the length of the course.

107. A scholarship is founded from the investment of £2000 in $4\frac{1}{2}$ p.c. stock at $114\frac{1}{2}$: find the annual value of the scholarship after income tax at 11*d.* in the £ has been deducted.

108. Steel wire is made to bear a strain of 215,000 lbs. per square inch of section. Find, to the nearest cwt., the weight that can be suspended from such a steel wire of diameter 0·104 of an inch.

The area of the section is $0·785 \times d^2$, where d is the diameter. (C.S.)

109. A man bought certain goods of which he sold $\frac{1}{3}$ at a profit of 14 per cent., $\frac{3}{5}$ at a profit of $17\frac{1}{2}$ per cent., and the remainder at a profit of 20 per cent. What was his profit per cent. on the whole?

110. A man lends his capital on mortgage at $3\frac{3}{4}$ per cent. per annum. After 6 months the money is repaid, with an additional month's interest in lieu of notice. The man then uses the capital and interest to purchase 3 per cent. stock at $108\frac{3}{4}$. He sells this stock at $106\frac{3}{4}$ at the end of the year, having received one half-yearly dividend. If he has to pay a brokerage of $\frac{1}{4}$ per cent. on the nominal value of the stock both when buying and when selling it, what is his net gain per cent. on his capital for the year?

111. Three persons, *A*, *B*, and *C*, hold a grazing farm in common at a yearly rent of £570. *A* puts in 126 oxen for three months; *B* puts in 162 for 5 months; and *C* puts in 72 for 12 months. How much of the rent should each person pay?

112. A Parliamentary grant is made at the rate of 5*s.* per head for all the children at elementary schools. If this grant is distributed at the rate of 5*s.* 9*d.* per child in town and 3*s.* 3*d.* per child in country schools, what percentage of the total number of children are in each class of school? (C.S.)

113. A bicycle was sold at a loss of 40 per cent. on the cost price, and a second bicycle was bought with the proceeds when 5 guineas had been added. This second bicycle was sold at a loss of 35 per cent., and a third bicycle was bought with the proceeds after the addition of £5. 15*s.* 3*d.* This third bicycle cost £16; what was the price of the first?

114. Shew that a hectare is approximately 2·471 acres, having given that a metre is 3·28 feet.

115. Knowing the number of pounds in a cubic inch of a substance, you can find the number of kilograms in a cubic centimetre by multiplying by $\frac{0{\cdot}4536}{(2{\cdot}54)^3}$. Express this multiplier as a decimal to 3 places. If steel weighs 488 lbs. per cubic foot, how many kilograms per cubic centimetre does it weigh?

116. The surface of a sphere of radius r inches is $4\pi r^2$ square inches, where $\pi = 3{\cdot}14159$. If π is taken, instead, to be equal to $\frac{22}{7}$, find, roughly in square miles, the difference that this will make in the calculated area in the earth's surface, the earth being a sphere of radius 4000 miles. (C.S.)

117. A company borrows the sum of £1,658,775 on the understanding that at the end of each year a portion of the principal is to be paid off, with interest at 4 per cent. per annum on the amount standing unpaid during that year. Prove that the debt can be cleared off in four years by an annual payment of £456,976. (C.S.)

118. A ship strikes on a rock and fires guns at intervals of one minute. A lifeboat's crew start from the shore, rowing $3\frac{1}{3}$ miles an hour, at the instant when they see the flash of one of those guns, and they have rowed 308 ft. when they hear the report of the next gun. If light travels instantaneously, and sound at the rate of 1100 ft. per sec., what is the distance of the ship from the shore?

119. Two trains start at the same time, one from Liverpool to Manchester, and the other in the opposite direction, and running steadily complete the journey in 42 minutes and 56 minutes respectively. How long is it from the moment of starting before they meet? (C.S.)

Verify your result by a graphical solution.

120. If 9 men and 6 boys can do in 2 days what 5 men and 7 boys could do in 3 days, in what time could 2 men and 5 boys do the same?

121. There are two jars, one containing a gallon of brandy and the other a gallon of water. One pint of brandy is taken from the first and placed in the second; then one pint of the mixture is taken from the second and placed in the first. Shew that the amount of water now in the first is equal to the amount of brandy in the second.

122. Assuming that the volume of a sphere is 0·5236 that of a cube whose edge is its diameter, and that a cubic inch of iron weighs 4·48 oz., find to the nearest ounce the weight of a hollow sphere formed of iron $\frac{1}{4}$ in. thick, whose internal diameter is 7 in.

123. Three railway tickets, a 1st, a 2nd, and half a third class were purchased for 16*s*. $10\frac{1}{2}$*d*. The 1st class ticket cost $1\frac{2}{3}$ times as much as the 2nd, and the 2nd class $1\frac{1}{2}$ times as much as a whole 3rd class ticket. The distance travelled was 45 miles. Find the cost of each ticket and the rate per mile for each class. (C.S.)

124. Find a prime number of 3 digits, such that the digit in the units place is equal to the sum of the other two, and if the other two digits be interchanged we still have a prime number of 3 digits. (C.S.)

125. A man starts to row on the still water of a canal, and completes a mile in quarter of an hour. He then reaches a river and rows with the stream $1\frac{1}{3}$ miles in the next quarter of an hour. How long will it take him to return to the starting point if he rows back with the same strength as on his outward course?

126. A man borrows £25,220 which he undertakes to pay back with compound interest at the rate of 5 per cent. per annum in three equal yearly instalments at the end of one, two, and three years. Shew that each instalment is £9261.

127. Two passengers have between them 345 lbs. of luggage and pay on their excess luggage 4*s*. 2*d*. and 6*s*. 8*d*. respectively. If the luggage had belonged to one of them the excess charge would have been 15*s*. How much free luggage is allowed to each passenger?

128. The cost of producing a photogravure is partly fixed, viz. £30 for the plate, and partly variable, viz. 3*s*. 6*d*. per copy. What is the least number of copies that must be sold at 15*s*. each to ensure a profit? Verify the solution graphically.

129. If silk costs 5·60 francs per square metre, find to the nearest penny the value of a piece of silk 14 yards long, 26 inches wide, having given £1 = 25·20 fr., 1 metre = 3·28 ft.

130. The basin of the Severn is 4350 square miles in area, and the yearly rainfall 29 inches. Half of the rain that falls is again evaporated and the rest flows to the sea. Find, to the nearest million gallons, how much water flows to the sea per hour. (A cubic foot of water is 6·23 gallons.) (C.S.)

✓**131.** A party of tourists set out for a station 3 miles distant and go at the rate of 3 miles an hour. After going half a mile one of them has to return to the starting point; at what rate must he now walk in order to reach the station at the same time as the others? (C.S.)

Verify your solution graphically.

132. Tithe Rent Charge in 1901 was of such value that every nominal £100 was really worth only £66. 10*s*. $9\frac{1}{4}$*d*. Find the amount of Tithe Rent Charge which is worth £1. 14*s*. 4*d*. less than its nominal value. (C.S.)

133. The first of a series of cogged wheels, working into each other in a straight line, has a certain number of teeth; the number of teeth in the second is to that of the first as 6 : 7; of the third to the second as 5 : 6; and of the fourth to the second as 2 : 3. If the wheels are set in motion, how many revolutions must each wheel make before they are simultaneously in their original positions? (C.S.)

134. What sum at compound interest will amount to £650 at the end of the first year and £676 at the end of the second year?

135. If a rectangular pathway which measures 787·4 metres in length and 1·53 metres in width is made at a cost of $1\frac{1}{2}$ francs per square metre, find the length to the nearest foot of a similar pathway 6 feet wide which costs 1*s*. $1\frac{1}{2}$*d*. per square yard, the total cost being the same. [Assume £1 = 25·20 fr.]

136. The receipts of a company amounted in one year to £354,000 and the expenses to £251,000. In the subsequent year receipts fell off by $3\frac{1}{2}$ per cent. and expenses increased $\frac{1}{2}$ per cent. Find by what percentage of itself the profit in the first year exceeded the profit in the succeeding year.

137. At the beginning of a year a man invests £9000 in 3 per cent. stock at 117, and £7200 in $2\frac{3}{4}$ per cent. stock at 108. At the beginning of the next year when both stocks are at 111 he sells them. Assuming him to have received one year's interest on each stock, and to have paid a brokerage of 5*s*. per £100 of stock on the latter transaction but not on the former, find how much he has gained on the whole.

138. Find the value of $\sqrt{1 \cdot 009} - \sqrt{0 \cdot 091}$ correct to 4 places of decimals.

139. Two houses exactly alike are built one in 4 months and the second in 7 months. The number of workmen engaged on the first is double that employed on the second, and they work two hours a day overtime, for which they are paid half as much again as for work done in the ordinary working day of 10 hours. If the sum paid for labour is £1740, find the price of each house.

140. A litre of water weighs a kilogram, a litre of another liquid weighs 1·340 kilograms. A mixture of the two weighs 1·270 kilograms per litre. Determine the volume of each in a litre of the mixture. (C.S.)

141. Three trains start from a town A at 12.0, 12.5, and 12.10, and travelling each at a uniform rate by 3 different routes of the same length to a town B, are observed to pass a signal box at B exactly abreast of each other at 12.50. If at 12.20 the sum of the distances traversed by the three trains is 36 miles 7 furlongs, how far from A is the signal box at B? (C.S.)

142. The volume of a segment h cm. high of a sphere of r cm. radius is $\frac{\pi h}{3}\{3rh - h^2\}$ c.cm. How many litres of milk (to the nearest hundredth) will fill to a depth of 12 cm. a hemispherical bowl of 30 cm. diameter? (C.S.)

143. A milkman adulterates his milk as follows:—He takes a pint out of each gallon and replaces it with water. He then takes another pint out of the mixture and replaces that with water; and he repeats the operation a third time. If he then sells his adulterated milk at the price per gallon that the pure milk originally cost him, find his gain per cent. correct to two decimal places. (C.S.)

144. By the use of logarithms find the value of

(i) $\frac{52{\cdot}45 \times 378{\cdot}372 \times 0{\cdot}020863}{87{\cdot}318 \times 0{\cdot}584}$, (ii) $(1{\cdot}03)^2 \times (1{\cdot}025)^4 \times (1{\cdot}05)$,

as accurately as the Tables permit.

145. Find the amount at compound interest of £500 for 4 years, the interest being at the rate of 3 per cent. per annum for the first two years, 10 per cent. paid quarterly for the third year, and 5 per cent. per annum for the fourth year.

146. One gallon of spirit which contains 11 per cent. of water is added to three gallons containing 7 per cent. of water, and to this mixture half a gallon of water is added. Find the percentage of the water in the mixture.

147. The lifting capacity of a balloon is equal to the difference between the weight of the gas it contains and the weight of the same volume of air, less the weight of the balloon. Find the lifting capacity of a spherical balloon 26 ft. in diameter, filled with a gas which is 0·4 times as heavy as air, the weight of the balloon being 1 cwt.

[The volume of a sphere is $\frac{\pi d^3}{6}$, where d is the diameter; a pound of air has a volume of 13·14 cubic ft.] (C.S.)

148. How many shot, $\frac{1}{8}$ in. diameter, will go to a pound if a cubic foot of lead weighs between 708 and 712 lbs.? (C.S.)

149. A certain mixture should be made by dissolving 12 grams of bluestone and 8 grams of lime in 1 litre of water. The mixture is actually made by dissolving 2 oz. of bluestone and $1\frac{1}{4}$ oz. of lime in 1 gallon of water. On this plan how many grams of bluestone and of lime are there to the litre of water? (C.S.)

150. Buttons are made with lead discs 0·8 of an inch across, and 0·09 of an inch thick. The volume of a disc, a inches across and b inches thick, may be calculated from the formula:

$$\text{Volume} = 0{\cdot}785 \times a \times a \times b,$$

and lead weighs 711 lbs. per cubic foot. How much lead (to the nearest pound) is wanted for a thousand buttons? (C.S.)

151. A man takes a boat and rows up stream. When he has gone $1\frac{1}{2}$ miles up stream he passes a bottle floating down with the stream. He goes on up the river for another 18 minutes and then turns. When he gets back to the starting-place he finds that the bottle has just arrived there. Assuming that his speed through the water and the speed of the stream are both uniform, find the speed of the stream in miles per hour. (C.S.)

152. A wine-merchant buys in Bordeaux a cask of claret, and the freight to London is £1. It contains 208 litres of wine at 75 centimes the litre, and for the cask itself he has to pay 12 francs. He has to pay duty 1*s.* 3*d.* per gallon, and bottles the wine at a cost of 2*s.* a dozen bottles. If he sells it at 18*s.* a dozen, what percentage of this price is profit?

(£1 = 25 francs; 6 bottles = 1 gallon.) (C.S.)

153. If a corporation borrows p pounds at the rate of r per cent., the loan and interest can be paid off by n equal annual instalments of a pounds where

$$a = \frac{p \times R^n \times (R-1)}{R^n - 1}, \quad \text{and} \quad R = 1 + \frac{r}{100}.$$

What must be the annual payment if a loan of £5000 at 3% is repaid by 10 equal payments? (C.S.)

154. Through a wooden pipe, whose cross-section is a square on a side of 6 cm., water flows uniformly at the rate of 50 metres a minute. How long will it take to discharge 30,000 litres?

155. A banker invests half the money deposited by his clients in 4 per cent. preference stock at 150, and lends a quarter of it on mortgages at 5 per cent., keeping the remainder in cash at the bank. After paying 2 per cent. interest on deposits, he has an average annual balance of £1000 in his favour. Find the average amount of the deposits to the nearest pound.

156. One train leaves Bristol at 3 p.m. and reaches London at 6 p.m.; a second train leaves London at 1.30 p.m. and arrives at Bristol at 6 p.m.; if both trains are supposed to travel uniformly, at what time will they meet? Shew from a graph that the time does not depend upon the distance between London and Bristol.

157. A mixture is made of two liquids, one of which weighs 0·68 gram per c.cm. and the other 1·04 gram per c.cm., and the mixture weighs 0·82 gram per c.cm. Find the percentage (by volume) of each liquid in the mixture. (C.S.)

158. A plan is to be enlarged so that a plot of ground which occupied 1·5 sq. in. in the original shall occupy at least 1 sq. foot in the enlarged plan. If the original measured 6 in. by 4 in., what must the least dimensions of the enlarged plan be? (C.S.)

159. A manufacturer estimates that he can make a certain patented article for £17 per gross. Find, to the nearest penny, the retail price for each article which must be fixed so that, after paying the patentee a royalty of 10 per cent. on the retail price, and giving the retail tradesman a discount of 40 per cent. on the retail price, the manufacturer may make a profit of 25 per cent. on the cost of making it.

If the retail price were fixed at 6 shillings, what would be the actual amount of profit made by the manufacturer on each article? (C.S.)

160. Where a railway goes round a curve the outer rail is raised to keep the train on the line. When the curve has a radius of R feet, the distance between the two rails is W feet, and the customary speed of the trains is V miles an hour, the outer rail should be $\frac{W \times V^2}{1 \cdot 25 \times R}$ inches higher than the inner rail. Calculate how much higher the outer rail should be on a curve of radius 580 feet, if the rails are 4 ft. $8\frac{1}{2}$ in. apart, and the customary speed is 40 miles an hour. (C.S.)

161. In a certain examination the highest and lowest marks gained in a Latin paper were 153 and 51. These have to be reduced so that the maximum (120) is given to the first candidate, and the minimum (30) to the lowest. This is done by reducing all the marks in a certain ratio, and then increasing or diminishing them all by the same number. In a Greek paper the highest and lowest marks were 161 and 56; after a similar adjustment these become 100 and 40 respectively. Draw graphs from which all the reduced marks may be read off, and find the marks which should be finally given to a candidate who scored 102 in Latin and 126 in Greek.

Shew also that it is possible in one case for a candidate to receive equal marks in the two subjects both before and after reduction. What are the original and reduced marks in this case?

162. From a cylinder of wood with a diameter of 11 inches a rectangular beam is cut. If the breadth of the beam is 8 inches, find the depth from the formula $d=\sqrt{D^2-b^2}$ where b is the breadth and d the depth of the beam and D the diameter of the cylinder.

If the strength of a beam varies as bd^2, compare the strengths of beams 3, 4, 5, 6, 7, 8 ins. broad cut from a cylinder 11 inches in diameter, and so find, roughly, the breadth of the strongest beam which can be cut from the cylinder. (C.S.)

163. A man riding a bicycle, at the rate of 15 miles per hour, towards a fort where minute guns are being fired, finds that he hears the 52nd report just 50 minutes after the first. Find the velocity of sound.

164. Water has to be diverted from a river through a six-inch diameter pipe running full bore at a velocity of one foot per second to irrigate a field of 20 acres. How long will it take to deliver an inch of water over the whole area? [Area of circle $= \pi$ (radius)^2.] (C.S.)

165. On a single line railway, 55 miles long, there are stations at every 5 miles, where it is possible for two trains to pass one another. A fast train starts at noon from one terminus, running at the rate of 50 miles an hour, and stops 2 minutes at the fifth station out. A slow train starts 5 minutes later from the other terminus, running at the rate of 20 miles an hour, and stops 2 minutes at every station. At what station must the slow train lie by for the fast train to pass it? The slow train must be out of the way at least a minute before the fast train is due. (C.S.)

166. A hollow ball, of external diameter 6 centimetres and of thickness half a centimetre, weighs 162 grams when it is filled with liquid. Find to the nearest hundredth of a gram the weight of a cubic centimetre of the liquid, a cubic centimetre of the substance of the ball weighing 2 grams. [If a sphere is inscribed in a cube, then volume of sphere : volume of cube :: 11 : 21.] (C.S.)

167. For a certain machine and for various loads the table shews what effort (P kilograms) was required to move the load (W kilograms):

W	50	100	200	300	400	500	600
P	106	122	152	184	216	246	278

Plot these values, and assuming the relation between P and W to be of the form $P = aW + b$, find the values of a and b. From the graph and also from the equation find to the nearest integer the value of P when $W = 350$, and the value of W when $P = 158$. (C.S.)

168. The number of cubic centimetres in the volume of a cone on a circular base may be found from the formula $0{\cdot}2618 \times d \times d \times h$, where d and h are the numbers of centimetres in the diameter of the base and in the height respectively. Find, to the nearest kilogram, the weight of a solid circular cone of ice the diameter of whose base is 60 cm. and whose height is 70 cm., being given that the weight of a c.c. of ice is 0·918 of the weight of the same volume of water.

What would be the weight (to the nearest kilogram) of another cone of ice of the same shape but only half these dimensions? (C.S.)

169. The following rules are in common use:

(*a*) To calculate the circumference of a circle, from three and one-seventh of the diameter subtract one eight-hundredth part.

(*b*) To calculate the diameter of a circle, from one-third of the cirference subtract one-hundredth and also one two-hundredth part.

Shew that these rules are approximately correct, and apply rule (*a*) to find the circumference of the circle whose diameter is 0·05847″, and rule (*b*) to find the diameter of the circle whose circumference is 382·7″.

Compare your results with those found by applying the usual formula ($C = 2\pi r$). Give all results to four significant figures. (C.S.)

170. When a body floats in a liquid, the weight of the displaced liquid is equal to the weight of the body.

A man whose weight is 10 stone (or 140 lbs.) finds that he can just float in fresh water when he is practically wholly immersed. Calculate his volume in cubic feet. [1 cu. ft. of water weighs 62·3 lbs.]

At a football match in Wales men were crowded together to the utmost extent upon a stand 360 ft. long and 40 ft. wide. Taking the average weight to be 10 stone, and supposing the space occupied by the men's bodies to be 90 per cent. of the space between the stand and 5 ft. 6 in. above it, find the number of men on the stand. (C.S.)

171. At the point where a stream passes a bridge it is 18 metres wide and has a uniform depth of 3·6 metres. If the stream flows uniformly at the rate of 1·75 kilometres per hour, how many litres of water pass through the bridge in one minute? (C.S.)

172. A man has in his workshops 96 ordinary gas burners, each burning $3\frac{1}{2}$ cubic feet of gas per hour for an average of 13 hours a week from October 1st to March 31st.

He adopts a new patent burner said to save 33 per cent. If the price of gas is 2*s.* 9*d.* per 1000 cubic feet, how much ought he to save in the 6 months by using the patent? (C.S.)

173. A company has 3 kinds of capital, £17,904,062 guaranteed stock on which it pays dividend at the rate of 5 per cent. per year, £11,925,808 preference stock on which it pays 5 per cent. per year, and £35,538,259 ordinary stock, the dividend on which varies with the profits. For a certain half-year £1,492,156. 11*s.* 9*d.* is available for dividend.

What dividend to the nearest quarter per cent. for the half-year can it declare on its ordinary stock, £35,538,259, so as to carry over at least £40,000 after paying the dividends on the three descriptions of stock? What amount will it carry over after paying these dividends to the nearest penny? (C.S.)

174. Six bicycle balls of the same size weigh together 14 grams. The steel of which they are made is 7·76 times as heavy as water. A ball of d cm. diameter contains $0·5236 \times d^3$ c.cm. Find whether the diameter of the balls exceeds 8 millimetres. (C.S.)

175. A statuette 10 in. high is to be made of silver; a plaster model 4 ft. in height has been made, and weighs 150 lbs. What will be the weight of the silver statuette, silver being 12·9 times as heavy as plaster? (C.S.)

176. A yard-measure is too long by 0·18 of an inch. If it is used to measure the sides of a rectangle, find by how much per cent. the true area exceeds the calculated area.

177. The weights of two spheres are in the ratio of 8 to 17, and the weights of a cubic foot of the material in the two spheres are in the ratio of 289 to 64: compare their radii.

178. In 1891 the population of three towns was 45,682, 25,408, and 18,960 respectively. In 1901 the two former had increased by 8 per cent. and $4\frac{1}{2}$ per cent. respectively, and the latter had decreased by 10 per cent.; find the average increase per cent. in the population of the three towns.

179. A man buys at £8. 16*s*. 8*d*. per ton a quantity of ore which contains 23 per cent. of copper; 92 per cent. of the copper is extracted at a cost of 2*s*. 8*d*. per cwt. of ore; find the price per ton at which the copper must be sold to ensure a profit of 15 per cent. on the whole.

180. The weight of the projectile of a gun whose calibre (internal diameter) is d inches is kd^3 lbs., where k is constant.

Calculate the calibre of a gun whose projectile is twice as heavy as that of a gun whose calibre is 10 inches. (C.S.)

181. If a circular disc measures a inches across and b inches in thickness its volume is $0·7854 \times a^2 \times b$ cubic inches.

A penny measures 1·23 inches across and a halfpenny 1 inch; if the thicknesses were in the same ratio, what fraction (as a decimal to the nearest hundredth) would the weight of the halfpenny be of the weight of the penny? (C.S.)

182. The diameter of some galvanised wire is 3 millimetres, and a cubic decimetre of the material of the wire weighs 6·7 kilograms. Find, to the nearest metre, the length of wire in a quantity which weighs 10 kilograms. The volume of a length of L metres, of diameter D metres, is $0·7854 \times \mathsf{D}^2 \times \mathsf{L}$ cubic metres. (C.S.)

183. A dishonest tradesman marks his goods at an advance of 5 per cent. on the cost price, but uses a fraudulent balance, whose beam is horizontal when the weight in one scale is one-fifteenth more than the weight in the other. What is his actual gain per cent.? (C.S.)

184. Three vessels, each capable of holding 32 gallons, contain mixtures of wine and water. In the first, which is half full, the quantities of wine and water are equal; in the second, which is three-quarters full, there are 9 gallons of wine; and in the third, which is full, there are only 4 gallons of wine. If the second is now filled up from the third, and then the first filled from the second, what will the quantities of wine and water become in the first?

185. Find, to the nearest kilogram, the weight of a piece of lead piping which is 7 metres long, and has an inside diameter of 8·6 cm. and a thickness of 1·15 cm. The weight W in kilograms of a piece of this piping, 1 metre in length, inside diameter A cm., thickness B cm., is found from the formula

$$\mathsf{W} = 3·523 \times \mathsf{B} \times (\mathsf{A} + \mathsf{B}).$$

What would this piece of lead piping cost at 4*d*. a kilogram? (C.S.)

ANSWERS.

PART I.

I. a. Page 4. **1.** (i) 4002; (ii) 46298. **2.** 1878. **3.** 416.
4. 66. **5.** 11817. **6.** 9123. **7.** 67573. **8.** 19294.
9. 2,662,750. **10.** 14456. **11.** 14496. **12.** 27898.

I. b. Page 8. **1.** 8645. **2.** 334,368.
3. 1,289,377. **4.** 5,211,624. **5.** 43,618,934.
6. 20,826,052. **7.** 353,798,810. **8.** 353,524,691.
9. 635,161,754. **10.** 444,150. **11.** 2,314,400.
12. 26,310,000. **13.** (i) 21375; (ii) 106,875; (iii) 534,375.
14. (i) 480,420; (ii) 2,402,100; (iii) 12,010,500.
15. (i) 357,896; (ii) 3,648,348.
16. (i) 8,385,597; (ii) 4,997,477; (iii) 67,592,994.
18. 212. **19.** 193. **20.** 3856. **21.** 1901.
22. 27276. **23.** 48741. **24.** 2916. **25.** 723.
26. 266. **27.** 11008. **28.** 39892. **29.** 368,372.
30. (i) 39,367,944; (ii) 7,256,359,800.

I. c. Page 11. **1.** 77; *rem.* 33. **2.** 75; *rem.* 48.
3. 19; *rem.* 5. **4.** 142; *rem.* 4. **5.** 150; *rem.* 6.
6. 1050; *rem.* 3. **7.** 44; *rem.* 33. **8.** 58; *rem.* 60.
9. 25; *rem.* 7. **10.** 150; *rem.* 81. **11.** 10; *rem.* 160.
12. 19; *rem.* 468. **13.** 321. **14.** 108; *rem.* 11.
15. 209; *rem.* 24. **16.** 2345. **17.** 432; *rem.* 23.
18. 1030; *rem.* 72. **19.** 871; *rem.* 23. **20.** 607. **21.** 857.
22. 289; *rem.* 41. **23.** 1198; *rem.* 166. **24.** 7126; *rem.* 45.

Miscellaneous Examples I. Page 13. **1.** Ninety-nine millions, four hundred and forty thousand, eight hundred and eighty.
2. Thirty-two thousand and fourteen. **3.** 948,007. **4.** £5375.
5. 3427. **6.** 360, 216, 20736, 4. **7.** 13. **8.** 42.
9. 351,196. **10.** 237,263. **11.** £812. **12.** 25.

13. 60200. **14.** 189. **15.** 246. **16.** £24.
17. Hat 6*s.*, cap 2*s.* **18.** 25*s.*, 11*s.* **19.** 115,411,149.
20. 3,578,234,210. **21.** 1256 ; *rem.* 209. **22.** 987,818,609.
23. 1,000,116. **24.** 58. **25.** 3,182,079.
26. 501. **27.** 5488. **28.** (i) 1,322,972,384 ; (ii) 1,668,152,175.
29. 1293. **30.** (i) 73700 ; (ii) 250,000. **31.** 3785 ; *rem.* 36.
32. 1243. **33.** 465. **34.** 555,111,111,222. **35.** 966,432.
36. 23. **37.** 36. **38.** 35 yrs., 15 yrs. **39.** 10.
40. 80,000,000, **41.** 20. **42.** He put down 5 for 3.
43. The two dividends are 68734 and 13746.
44. 43364. **45.** £23. **46.** 1*s.*
47. Greatest 23,456,708 ; least 20,345,678 ; diff. 3,111,030.
48. *A*, 73 ; *B*, 46 ; *C*, 30. **49.** *A*, 48 ; *B*, 24 ; *C*, 96. Av., 9.
50. 20. **51.** 7 and 6. **52.** Divisor 561 ; quotient 943.

II. a. Page 18. **13.** 1246. **14.** 2420. **15.** 17555.
16. 360. **17.** 7308. **18.** 744. **19.** 6430. **20.** 3903.
21. 9223. **22.** 6430. **23.** 4383. **24.** 8765.
25. £9. 2*s.* 6*d.* **26.** £13. 13*s.* 10*d.* **27.** £19. 6*s.* 4*d.*
28. £7. 5*s.* **29.** £9. 10*s.* $8\frac{1}{2}$*d.* **30.** £17. 5*s.* $1\frac{1}{2}$*d.*
31. £6. 16*s.* $8\frac{1}{4}$*d.* **32.** £9. 11*s.* $6\frac{1}{4}$*d.* **33.** £22. 16*s.* $9\frac{3}{4}$*d.*
34. 1347. **35.** 3481. **36.** 3664. **37.** 5788. **38.** 4493.
39. 15367. **40.** £7. 8*s.* 6*d.* **41.** £10. 0*s.* 3*d.*
42. £19. 1*s.* 4*d.* **43.** £135. 18*s.* **44.** £431. 2*s.* 6*d.*
45. £2030. 7*s.* 6*d.* **46.** 380. **47.** 777.
48. 2776. **49.** £4000. **50.** 937 fr. 50 c.

II. b. Page 20. **1.** £22. 0*s.* 1*d.* **2.** £39. 7*s.* 7*d.*
3. fr. 105,20. **4.** £42. 1*s.* $2\frac{1}{2}$*d.* **5.** £100.
6. fr. 1029,42. **7.** £143. 11*s.* 6*d.* **8.** £1000. 1*s.*
9. fr. 735,60. **10.** £51. 8*s.* **11.** fr. 310,00. **12.** $262,50.
13. (i) £46101. 13*s.* 7*d.* ; (ii) £36705. 13*s.* 9*d.* ; (iii) £30000.
Total, £112,807. 7*s.* 4*d.*

II. c. Page 22. **1.** £9. 16*s.* 8*d.* **2.** £85. 18*s.* $10\frac{1}{2}$*d.*
3. £5. 1*s.* $0\frac{3}{4}$*d.* **4.** fr. 288,38. **5.** fr. 660,75.
6. fr. 1668,59. **7.** £1. 6*s.* 7*d.* **8.** £6. 1*s.* $7\frac{1}{2}$*d.*
9. fr. 6,78. **10.** $300,01. **11.** £785. 3*s.* 2*d.*
12. £372. 12*s.* 4*d.* **13.** £324. 16*s.* 9*d.* **14.** £28. 15*s.* 5*d.*
15. £10. **16.** £90. 5*s.* 6*d.* **17.** £11. 6*s.* 11*d.*
18. £96. 16*s.* 4*d.* **19.** £6. 10*s.* 8*d.* **20.** £3. 10*s.*

II. d. Page 24.

1. (i) £46. 3*s.* 9*d.*; (ii) £107. 15*s.* 5*d.*; (iii) £169. 7*s.* 1*d.*
2. (i) £105. 17*s.* 6*d.*; (ii) £185. 5*s.* 7$\frac{1}{2}$*d.*; (iii) £238. 4*s.* 4$\frac{1}{2}$*d.*
3. (i) 1040 fr. 75 c.; (ii) 1873 fr. 35 c.; (iii) 2497 fr. 80 c.
4. (i) $2502,48; (ii) $3336,64; (iii) $4170,80.
5. £451. **6.** £520. 7*s.* 6*d.* **7.** £1133. 6*s.* 7*d.*
8. £2970. 15*s.* **9.** fr. 64948,40. **10.** $48680,64.
11. (i) £231. 5*s.* 10*d.*, £2312. 18*s.* 4*d.*, £23129. 3*s.* 4*d.*
(ii) 232 fr. 7 c., 2327 fr., 23270 fr.
12. (i) £330. 1*s.* 8*d.*; (ii) £371. 6*s.* 10$\frac{1}{2}$*d.*; (iii) £701. 8*s.* 6$\frac{1}{2}$*d.*
13. £1095. 11*s.* 11*d.* **14.** £1827. 4*s.* 1*d.* **15.** £3332. 12*s.* 8*d.*
16. £8187. 14*s.* 6*d.* **17.** £11635. 8*s.* 9*d.* **18.** £10584. 19*s.* 7*d.*
19. fr. 7117,17. **20.** fr. 23341,21.

II. e. Page 26.

1. £5. 7*s.* 11*d.* **2.** £5. 15*s.* 2*d.*
3. £4. 11*s.* 3*d.* **4.** £26. 0*s.* 4$\frac{1}{2}$*d.* **5.** 18 fr. 32 c.
6. 46 fr. 28 c. **7.** £1. 13*s.* 3*d.* **8.** £3. 9*s.* 7*d.*
9. £7. 6*s.* 4*d.* **10.** £41. 5*s.* 2$\frac{1}{2}$*d.* **11.** fr. 40,25.
12. $90,04. **13.** (i) fr. 73,08; (ii) fr. 730,80.
14. (i) £23. 2*s.* 7*d.*; (ii) £2. 6*s.* 3*d.*; *rem.* 1*d.*
15. £3. 5*s.* 2*d.*; *rem.* 7*s.* 11*d.* **16.** 6 fr. 8 c.; *rem.* 23 c.
17. £23. 2*s.* 7*d.* **18.** £16. 3*s.* 5*d.* **19.** £41. 5*s.* 3*d.*
20. £6. 5*s.* 9*d.*; *rem.* 17*s.* 3*d.* **21.** fr. 18,48; *rem.* fr. 2,54.
22. $20,32; *rem.* 76 ¢. **23.** £82. 10*s.* 5*d.*
24. 17. **25.** 11. **26.** 35. **27.** 23. **28.** 45.
29. 14. **30.** 72. **31.** 111. **32.** 23; *rem.* 10*d.*
33. 13; *rem.* £1. 14*s.* 10*d.* **34.** 72; *rem.* 16*s.* **35.** 10000.

II. f. Page 27.

16. 15*s.* 11*d.* **17.** £1. 5*s.* 1*d.*
18. 2$\frac{1}{4}$*d.* **19.** £19. 7*s.* 6*d.* **20.** £2. 8*s.*
21. £9. 3*s.* 4*d.* **22.** £215. 10*s.* 10*d.* **23.** £89. 3*s.* 2*d.*
24. 17*s.* 6$\frac{1}{2}$*d.* **25.** 10*s.* 2*d.* **26.** 205.
27. 6*d.* **28.** 4493. **29.** 87; 7*d.* over.
30. 218; 1*s.* over. **31.** 54. **32.** 148; $\frac{1}{2}$*d.*
33. 939. **34.** £23. 5*s.* 8*d.*; £35. 10*s.* 11*d.*; £56. 6*s.* 7*d.*
35. 500; 1*s.* 8*d.* **36.** 41. **37.** £535. 2*s.* 11*d.*
38. £166. 15*s.* **39.** £5. 13*s.* 6*d.* **40.** *A*, £40; *B*, £27.
41. *A*, £120; *B*, £80; *C*, £65.
42. *A*, £10. 10*s.*; *B*, £8. 9*s.*; *C*, £7. 3*s.* 6*d.*
43. £38. 19*s.* 2*d.* **44.** £1000. **45.** 2*d.*; £8. 6*s.* 8*d.*
46. 96. **47.** 18*s.* 6*d.* **48.** 2$\frac{3}{4}$*d.*; 9*s.* 2*d.*
49. 12800. **50.** £430. 12*s.* 4$\frac{1}{2}$*d.*

II. g. Page 33. **1.** 101. **2.** 283. **3.** 719.
4. 700. **5.** 80. **6.** 785. **7.** 693. **8.** 603.
9. 690. **10.** 530. **11.** 503. **12.** 53. **13.** 14960.
14. 19610. **15.** 991. **16.** 7000. **17.** 800. **18.** 30.
19. 7830. **20.** 7803. **21.** 7083. **22.** 7830. **23.** 14014.
24. 141,004. **25.** (i) 4 yds. 2 ft. 3 in.; (ii) 13 yds. 2 ft. 1 in.
26. 7 m. 2 dm. 3 cm. **27.** 8 m. 1 dm. 3 cm. **28.** 9 m. 5 cm.
29. 7 m. 2 cm. **30.** 7 dm. 2 mm.
31. 5 Km. 1 Hm. 7 Dm. 3 m. **32.** 5 Km. 2 m.
33. 5 Km. 2 Hm. **34.** 5 Km. 2 Dm.
35. 15840; 27720. **36.** 80; 792; 8 in.
37. 35 yds. 0 ft. 7 in. **38.** 33 m. 7 dm. 1 cm.
39. 33 dm. 7 cm. 1 mm. **40.** 24 mi. 760 yds.
41 25 Km. 520 m. **42.** 3 mi. 2 fur. 209 yds.
43. (i) 11 yds. 2 ft. 3 in.; (ii) 11 m. 9 dm. 1 cm.
44. 2 mi. 780 yds.; 2 Km. 20 m.
45. (i) 49 yds. 2 ft. 3 in.; (ii) 46 m. 5 dm. 3 cm.
46. (i) 3 yds. 8 in.; *rem.* 3 in. (ii) 3 m. 1 dm. 9 cm.; *rem.* 8 cm.
47. 12 mi. **48.** 6 Km. **49.** 528.
50. 10; 5 ft. over. **51.** 4964. **52.** 20 chains; 440 yds.
53. 497. [True run = 497 mi. 800 yds.]
54. 2 mi. 300 yds.; 13 m. **55.** 445 yds.

II. h. Page 36. **1.** 1025. **2.** 9520. **3.** 4872. **4.** 5000.
5. 900. **6.** 3900. **7.** 641. **8.** 641. **9.** 3250.
10. 25 lbs. **11.** 2 tons 13 cwt. 64 lbs. **12.** 44 gals. 1 qt. 1 pint.
13. 4 Hg. 7 g. **14.** 4 Kg. 7 g. **15.** 4 Kg. 7 Hg.
16. 4 Kg. 7 Dg. **17.** 3 Hl. 5 Dl. 7 l. **18.** 29 Hl. 3 l.
19. (i) 2240; (ii) 2200. **20.** 41 cwt. 3 qr. 23 lb.
21. 24 Kg. 5 Dg. **22.** 24 l. 5 cl.
23. 8 cwt. 65 lbs.; 1 Dg. 3 g. **24.** 39 tons 3 cwt. 2 qr.
25. 10 l. 9 dl. 1 cl. **26.** 9 cg.
27. 10 lbs.; 171 lbs. = 12 st. 3 lbs. **28.** 68 yds.; 12 lbs. 12 oz.
29. 2 tons 16 cwt. 1 qr. **30.** 31; 18 lbs. over.
31. 135. **32.** 3.20 p.m. **33.** 88 tons.

II. k. Page 41. **6.** 2700. **7.** 8100. **8.** 96.
9. 480. **10.** 48. **11.** 400. **12.** 1152. **13.** 1476.
14. 9072. **15.** 2200. **16.** 3600. **17.** 5000. **18.** 1400.
19. 140,000. **20.** 141,400. **21.** 51617. **22.** 50607.
23. 122 sq. yds. 2 sq. ft. **24.** 8 sq. ft. 96 sq. in.

25. 560 sq. yds. 2 sq. ft. 36 sq. in. **26.** 56 sq. dm. 48 sq. cm.
27. 56 sq. m. 48 sq. dm. **28.** 56 sq. m. 4 sq. dm. 8 sq. cm.
29. 84 sq. yds. 5 sq. in. **30.** 27 sq. m.
31. (i) 100; (ii) 10000; (iii) 1,000,000.
32. (i) 969 sq. yds. 3 sq. ft.; (ii) 960 sq. m. 84 sq. dm.
33. (i) 12; (ii) 12. **34.** 250 sq. yds. **35.** 83 sq. yds. 3 sq. ft.
36. 2 sq. ft. 108 sq. in. **37.** 3 sq. m. 96 sq. dm.
38. 140 sq. m. **39.** 3 sq. in. **40.** 20 ft.
44. (i) 1; (ii) 3; (iii) $4\frac{1}{2}$; (iv) 6. **45.** (i) 26620; (ii) 37400.
46. 300 ac. **47.** 1 ac. **48.** 29 Ha.
49. (i) 356 ac. 1 r.; (ii) 314 Ha. **50.** 35.

II. 1. Page 47.

3. (i) 600 cu. in.; (ii) 24 cu. ft.; (iii) 3600 cu. ft.; (iv) 125 cu. ft.
4. (i) 46656; (ii) 17900. **5.** 6 cu. m. **6.** 900 tons.
7. (i) 1000; (ii) 1000 Kg. **8.** 224. **9.** 9.
10. 360. **11.** 21; 12 cwt. **12.** 390.

III. a. Page 50.

4. 120. **5.** 125. **6.** 80.
7. 200. **8.** 40. **9.** 102. **10.** 20. **11.** 9.
12. 10. **13.** 42. **14.** 24. **15.** 1. **16.** 24.
17. 60. **18.** 15. **19.** 300. **20.** 45. **21.** 440.
30. 48. **31.** 1. **32.** 700. **33.** 480. **34.** 4.
35. 2. **36.** 3. **37.** 4. **48.** 0. **49.** 0.
50. 60. **51.** 21. **52.** 23. **53.** 0. **54.** 59.
55. 1. **56.** 179. **57.** 971. **58.** 120. **59.** 5.

III. b. Page 54.

1. $-\pounds 12$; $+\pounds 3$. **2.** $\pounds a-\pounds b-\pounds c+\pounds d$.
3. 3 miles S. **4.** $+4$, -2. **5.** 20.
6. -6°. **7.** $+24$, -4. **8.** -4 miles.
9. A, $\pounds a-\pounds x+\pounds y$; B, $\pounds b-\pounds y+\pounds x$.

III. c. Page 58.

13. 9, 24. **14.** 4, 16. **15.** 33.
16. 171. **17.** 32. **18.** 4. **19.** 56. **20.** 43.
21. 10. **22.** 19. **23.** $6a$. **24.** $15b$.
25. $5x$. **26.** $6m$. **27.** $6y$. **28.** $4x$.

III. e. Page 61.

13. 1. **14.** 3. **15.** 6.
16. 5. **17.** 2. **18.** 3. **19.** 5. **20.** 4.
21. 7. **22.** 4. **23.** 2. **24.** 7. **25.** 4.
26. 6. **27.** 2. **28.** 6. **29.** 3.

III. f. Page 63. **1.** 16, 12. **2.** 27, 18. **3.** 5. **4.** 36, 24.
5. *A*, £35; *B*, £20; *C*, £12. **6.** *A*, £14; *B*, £28; *C*, £24.
7. *A*, £15; *B*, £25; *C*, £45. **8.** *A*, £35; *B*, £72; *C*, £81.
9. 40 shillings; 27 sixpences. **10.** *A*, £9. 5*s*.; *B*, £2. 15*s*.
11. *A*, £4. 10*s*.; *B*, £1. 10*s*. **12.** 17*s*.; 13*s*.

IV. a. Page 70. **1.** 3, 4. **2.** 3, 9. **3.** 3, 9.
4. 3, 5, 9. **5.** 11. **6.** 5. **7.** 3, 11.
8. None. **9.** 3, 4, 8, 9, 11. **10.** 11. **11.** 3, 4, 9.
12. 3, 4, 11. **13.** 2, 5, 0, 0, 0. **14.** 2, 2, 6, 7. **15.** 8, 9, 2.
16. (i) 4; (ii) 7. **17.** $3^2 . 5$. **18.** $2^2 . 3 . 7$. **19.** $2^2 . 3 . 11$.
20. $3 . 5 . 7$. **21.** $3^3 . 5$. **22.** $2^5 . 3^2$. **23.** $3^4 . 5$.
24. $2 . 3 . 7 . 11$. **25.** $2^2 . 5 . 7^2$. **26.** $2 . 3^2 . 7 . 11$.
27. $2 . 3^2 . 5^2 . 7$. **28.** $3^2 . 7 . 11 . 13$. **29.** $2^2 . 7^2 . 17$.
30. $2 . 3^2 . 13 . 23$. **31.** $2^3 . 3^2 . 5 . 17$. **32.** $3^2 . 5 . 7 . 23$.
33. $2 . 3^2 . 11 . 97$. **34.** $2^7 . 3^2 . 7 . 31$. **35.** Prime.
36. $3 . 101$. **37.** Prime. **38.** $7 . 97$.
39. Prime. **40.** $2^3 . 89$. **41.** Prime.
42. Prime. **43.** $23 . 29$. **44.** $11 . 971$.
45. (i) $7 . 10^3 + 5 . 10^2 + 4 . 10 + 3$; (ii) $8 . 10^3 + 2 . 10 + 9$.
46. $100p + 10q + r$.
47. $100a + 10b + c$. (i) c must be even; (ii) c must be 5 or 0.
48. $1000a + 100b + 10c + d$.

IV. b. Page 72. **18.** 18. **19.** 24. **20.** No. **21.** No.
22. 48. **23.** No. **24.** 44. **25.** 56. **26.** No. **27.** 84.
28. 2. **29.** 5. **30.** 11. **31.** 7. **32.** 5. **34.** 22.
35. 28. **36.** 36. **37.** 45. **38.** 7056. **39.** 1521.
40. (i) 18; (ii) 12; (iii) 7; (iv) 25.

IV. c. Page 74. **15.** 36. **16.** 46. **17.** 17.
18. 45. **19.** 22. **20.** 16. **21.** 48. **22.** 33.
23. 15. **24.** 12. **25.** 17. **26.** 36. **27.** 37.
28. 35. **29.** 50. **30.** 18. **31.** 135. **32.** 11.
33. $3xy$. **34.** $4abc$. **35.** $5p^2q$. **36.** $8a$. **37.** xy.

IV. d. Page 76. **1.** 13. **2.** 29. **3.** 43.
4. 28. **5.** 17. **6.** 47. **7.** 31. **8.** 7.
9. 4. **10.** 74. **11.** 493. **12.** 7. **13.** 221.
14. 263. **15.** 589. **16.** 31. **17.** 1507. **18.** 772.
19. 1545. **20.** 6732. **21.** 61. **22.** 94. **23.** 31.
24. 2223. **25.** 27. **26.** 852. **27.** 185. **29.** A crown.
30. 7 in. **31.** 385, 525. **32.** 31. **33.** 479, 83.

IV. e. Page 78. **25.** H.C.F. $=5.3$; L.C.M. $=5^3.3^2$.
26. H.C.F. $=2.7$; L.C.M. $=2^4.7^3$. **27.** H.C.F. $=3$; L.C.M. $=2^4.3^4$.
28. H.C.F. $=ab$; L.C.M. $=a^3b^2$. **29.** H.C.F. $=2b$; L.C.M. $=12abc$.
30. H.C.F. $=yz$; L.C.M. $=9xy^2z$. **31.** 120. **32.** 60.
33. 72. **34.** 240. **35.** 756. **36.** 240. **37.** 1452.
38. 1764. **39.** 7200. **40.** 720. **41.** 1260. **42.** 6720.
43. 32130. **44.** 8640. **45.** 9360.
46. H.C.F. $=15$; L.C.M. $=225$. **47.** H.C.F. $=14$; L.C.M. $=558$.
48. H.C.F. $=11$; L.C.M. $=11550$.

IV. f. Page 80. **1.** 9061. **2.** 12673. **3.** 40513.
4. 38399. **5.** 48345. **6.** 5,189,184. **7.** 472,787.
8. £2. 12*s.* 6*d.* **9.** 32. **10.** 39312. **11.** 729.
12. 46046. **13.** 8 gallons. **14.** £20. 12*s.* 6*d.* **15.** 5040 mi.
16. 4 min. 48 sec. **17.** 46 min. 12 sec. **18.** 1309.

V. d. Page 87. **11.** $\frac{1}{5}$. **12.** $\frac{3}{4}$. **13.** $\frac{4}{7}$.
14. $\frac{7}{11}$. **15.** $\frac{13}{14}$. **16.** $\frac{1}{13}$. **17.** $\frac{17}{19}$. **18.** $\frac{1}{23}$.
19. $\frac{1}{17}$. **20.** $\frac{19}{23}$. **25.** (i) $\frac{a}{b}$; (ii) $\frac{b}{x}$; (iii) $\frac{2a}{3}$; (iv) $\frac{2x}{3a}$.
26. $\frac{17}{19}$. **27.** $\frac{23}{24}$. **28.** $\frac{29}{31}$. **29.** $\frac{23}{29}$ **30.** $\frac{13}{16}$.
31. $\frac{11}{19}$. **32.** $\frac{93}{103}$. **33.** $\frac{5}{8}$. **34.** $\frac{4}{9}$. **35.** $\frac{3}{8}$.

V. e. Page 89. **2.** $3\frac{11}{16}$. **3.** $2\frac{14}{19}$. **4.** $8\frac{8}{15}$.
5. $4\frac{5}{17}$. **6.** $5\frac{1}{19}$. **7.** $7\frac{1}{2}$. **8.** $9\frac{2}{5}$. **9.** $8\frac{7}{10}$.
10. $7\frac{11}{23}$. **11.** $14\frac{1}{8}$. **12.** $6\frac{3}{17}$. **13.** $16\frac{1}{7}$. **16.** $\frac{76}{13}$.
17. $\frac{165}{13}$. **18.** $\frac{221}{14}$. **19.** $\frac{465}{23}$. **20.** $\frac{288}{25}$. **21.** $\frac{775}{19}$.
22. $\frac{888}{49}$. **23.** $\frac{575}{21}$. **24.** $\frac{1229}{23}$. **25.** $\frac{1661}{57}$.

V. f. Page 91. **7.** $\frac{1}{2}, \frac{4}{7}, \frac{2}{3}$. **8.** $\frac{1}{3}, \frac{7}{18}, \frac{5}{12}$.
9. $\frac{1}{10}, \frac{1}{5}, \frac{1}{4}, \frac{1}{2}$. **10.** 2nd, by $\frac{4}{63}$. **11.** 1st, by $\frac{3}{56}$.
12. 2nd, by $\frac{11}{144}$. **13.** 2nd, by $\frac{1}{100}$.

V. g. Page 92. **21.** $\frac{1}{25}$. **22.** $\frac{17}{30}$. **23.** $\frac{29}{36}$.
24. $\frac{8}{15}$. **25.** $\frac{1}{8}$. **26.** $\frac{7}{20}$. **27.** $\frac{13}{24}$. **28.** $\frac{5}{36}$.
29. 1. **30.** $\frac{4}{5}$. **31.** $\frac{2}{3}$. **32.** 0. **35.** $\frac{9a}{20}$.
36. $\frac{11m}{12}$. **37.** $\frac{p}{18}$. **38.** $\frac{a+b}{ab}$. **39.** $\frac{1}{6a}$. **40.** $\frac{3m+2n}{mn}$.

41. 1. **42.** $\frac{31}{35}$. **43.** $1\frac{5}{21}$. **44.** 2. **45.** $\frac{2}{3}$.
46. $1\frac{2}{3}$. **47.** $\frac{1}{15}$. **48.** $\frac{13}{21}$. **49.** $\frac{19}{27}$. **50.** $\frac{1}{12}$.

V. h. Page 94. **1.** $4\frac{5}{6}$. **2.** $6\frac{39}{80}$. **3.** $12\frac{13}{70}$. **4.** $11\frac{11}{60}$.
5. 10. **6.** $3\frac{4}{15}$. **7.** $1\frac{2}{5}$. **8.** $1\frac{2}{21}$. **9.** $4\frac{1}{8}$.
10. $1\frac{11}{45}$. **11.** $\frac{1}{6}$. **12.** $\frac{5}{48}$. **13.** $16\frac{1}{12}$. **14.** 13.
15. $11\frac{2}{3}$. **16.** $10\frac{7}{8}$. **17.** $2\frac{1}{4}$. **18.** 11. **19.** $\frac{1}{3}$.
20. 3. **21.** 0. **22.** 1. **23.** $3\frac{789}{1000}$. **24.** $5\frac{137}{1000}$.

V. k. Page 95. **26.** $6\frac{19}{22}$. **27.** $5\frac{41}{63}$. **28.** $4\frac{117}{140}$. **29.** $\frac{37}{40}$.
30. $3\frac{82}{189}$. **31.** $\frac{23}{420}$. **32.** $14\frac{44}{483}$. **33.** $\frac{37}{40}$. **35.** $\frac{7}{8}$.
36. $1\frac{8}{11}$. **37.** $\frac{1}{24}$. **38.** $1\frac{11}{18}$. **39.** First, by $\frac{1}{24}$.
40. 3*s.* **41.** $\frac{1}{5}$. **42.** $7\frac{1}{21}$; $\frac{1}{21}$. **43.** (i) $4\frac{5}{6}$; (ii) $\frac{8}{15}$.

V. l. Page 97. **1.** $\frac{9}{16}$. **2.** $\frac{3}{16}$. **3.** $18\frac{11}{12}$.
4. $5\frac{7}{12}$. **5.** $4\frac{11}{15}$. **6.** $9\frac{14}{15}$. **7.** $\frac{2}{15}$. **8.** $\frac{1}{6}$.
9. $1\frac{5}{12}$. **10.** $\frac{5}{28}$. **11.** 1. **12.** $8\frac{2}{3}$. **13.** $\frac{4}{15}$.
14. 1. **15.** 6. **16.** $8\frac{1}{6}$. **17.** 2.
18. $\frac{5}{9}$. **19.** $1\frac{1}{15}$. **20.** 10*s.* **21.** £3. 5*s.* 3*d.*
22. $\frac{7}{24}$; £50, £72, £48, £70. **23.** *A* pays *B* 5*s.*, *B* pays *A* 4*d.*
24. 9 ft. 6 in. **25.** £14400. **26.** 58333.

V. m. Page 101. **16.** $50\frac{1}{2}$. **17.** $74\frac{1}{2}$. **18.** 159.
19. $27\frac{1}{2}$. **20.** $1\frac{1}{8}$. **21.** $1\frac{1}{8}$. **22.** $\frac{11}{18}$. **23.** $\frac{7}{10}$.
24. (i) 12, 4, $12\frac{4}{9}$; (ii) 7, 8, $7\frac{8}{17}$; (iii) 15, 5, $15\frac{5}{13}$. **25.** 7*s.* 6*d.*
26. $\frac{10}{11}$ yds. **27.** $\frac{2}{5}$ ton. **28.** £6. 13*s.* 6*d.*
29. 570 metres. **30.** 3*s.* 4*d.* **31.** $14\frac{1}{6}$ yds. = 14 yds. 6 in.
32. 10; $13\frac{3}{4}$; $62\frac{1}{2}$. **33.** 22; 77; $160\frac{3}{5}$. **34.** $45\frac{1}{2}$; $73\frac{1}{2}$; 175.
35. 5; 250; $\frac{1}{2}$ of an acre.

V. n. Page 103. **9.** $198\frac{1}{4}$. **10.** $88\frac{1}{2}$. **11.** $662\frac{1}{2}$.
12. 336. **13.** (i) $\frac{mx}{n}$; (ii) mx; (iii) $\frac{a^2}{b}$. **14.** £4351. 13*s.* 4*d.*
15. £2118. 15*s.* **16.** 280. **17.** £15000. **18.** 5.
19. £3652, £2739, £1826; £1826. **20.** £7. 10*s.*

V. p. Page 107. **17.** $10\frac{1}{2}$. **18.** 9.
19. $1\frac{7}{8}$. **20** $26\frac{2}{5}$. **21.** $\frac{5}{32}$. **22.** 1.
23. $\frac{4}{7}$. **24.** 1. **25.** 2. **26.** £22. 10*s.*
27. $115\frac{35}{36}$ miles. **28.** 10 tons. **29.** $\frac{y}{4b}$. **30.** $\frac{m}{3n}$.
31. (i) $10\frac{5}{6}$; (ii) 17; (iii) $63\frac{8}{9}$. **32.** (i) $27\frac{1}{2}$; (ii) $41\frac{1}{4}$; (iii) 154.
33. £3. 14*s.* 3*d.* **34.** 13*s.* 9*d.* **35.** £77. **36.** 11*s.* 6*d.*
37. A, 25; M, 60. **38.** £8333. 6*s.* 8*d.*
39. 22 ft. and $18\frac{1}{3}$ ft.; $20\frac{4}{13}$ ft. and $16\frac{12}{13}$ ft.
40. $\frac{13}{30}$. **41.** $\frac{11}{50}$. **42.** 14*s.*

V. q. Page 110. **13.** $13\frac{1}{2}$. **14.** $1\frac{1}{5}$. **15.** $\frac{2}{3}$. **16.** $\frac{7}{8}$.
17. $1\frac{7}{8}$. **18.** $4\frac{1}{8}$. **19.** $12\frac{1}{2}$. **20.** $2\frac{11}{26}$. **21.** mb. **22.** $\frac{xn}{ym}$.
23. $\frac{m}{ny}$. **24.** am. **25.** $\frac{12}{13}$. **26.** $3\frac{7}{16}$. **27.** 10. **28.** 2.
29.
[For (i), (ii), (iii) use 1 Km. $=\frac{5}{8}$ mi. For (iv), (v), (vi) use 1 m. $=39\frac{3}{8}$ in.]
(i) $1\frac{3}{5}$ Km.; (ii) 1600 m.; (iii) 160 Km.; (iv) $\frac{32}{35}$ m.;
(v) $2\frac{34}{63}$ cm.; (vi) $44\frac{4}{5}$ m.
30. (i) $\frac{5}{11}$ Kg.; (ii) $50\frac{10}{11}$ Kg.; (iii) $\frac{4}{7}$ litre; (iv) $4\frac{4}{7}$ litres;
(v) 121 lbs.; (vi) $218\frac{3}{4}$ gals.; (vii) $15\frac{2}{5}$ grs.

V. r. Page 113. **1.** $4\frac{1}{5}$. **2.** $2\frac{11}{12}$. **3.** $26\frac{1}{4}$. **4.** $4\frac{7}{10}$.
5. $9\frac{1}{12}$. **6.** $3\frac{19}{30}$. **7.** $4\frac{17}{22}$. **8.** $2\frac{17}{30}$. **9.** $1\frac{11}{15}$.
10. $4\frac{1}{2}$. **11.** $2\frac{19}{50}$. **12.** $3\frac{3}{5}$. **13.** $7\frac{1}{2}$. **14.** $2\frac{7}{10}$.
15. $5\frac{1}{10}$. **16.** $\frac{23}{66}$. **17.** $\frac{263}{440}$. **18.** $1\frac{109}{120}$. **19.** $\frac{759}{800}$.
20. $\frac{1}{30}$. **21.** $1\frac{40}{63}$. **22.** 3. **23.** $21\frac{1}{8}$. **24.** $1\frac{2}{49}$.
25. $\frac{15}{57}$. **26.** $1\frac{17}{35}$. **27.** $2\frac{1}{2}$. **28.** 12. **29.** $2\frac{19}{66}$.
30. $\frac{4}{21}$. **31.** $5\frac{31}{40}$. **32.** $4\frac{11}{18}$. **33.** $17\frac{13}{14}$. **34.** $\frac{1}{9}$.
35. $1\frac{1}{12}$. **36.** 1. **37.** $3\frac{5}{12}$. **38.** $23\frac{1}{3}$. **39.** $3\frac{1}{3}$.

V. s. Page 115. **12.** $\frac{1}{5}$. **13.** 1. **14.** $\frac{1}{3}$.
15. 1. **16.** 1. **17.** $\frac{8}{27}$. **18.** 1. **19.** $\frac{1}{17}$. **20.** $1\frac{13}{35}$.
21. $1\frac{1}{2}$. **22.** $\frac{1}{5}$. **23.** $2\frac{1}{4}$. **24.** $\frac{1}{2}$. **25.** $1\frac{19}{41}$. **26.** $\frac{1}{240}$.
27. 5. **28.** 1. **29.** $\frac{69}{814}$. **30.** $1\frac{2}{3}$. **31.** $1\frac{1}{7}$. **32.** $1\frac{69}{71}$.

V. t. Page 116. **4.** (i) $1\frac{1}{2}$; (ii) $1\frac{1}{3}$; (iii) $\frac{1}{4}$; (iv) $\frac{1}{4}$.
5. (i) $\frac{1}{4}$; (ii) $\frac{1}{12}$; (iii) $\frac{2}{15}$; (iv) $\frac{1}{2}$.
6. (i) $\frac{1}{3}$; (ii) $\frac{1}{4}$; (iii) $\frac{2}{5}$; (iv) $\frac{4}{5}$.
7. (i) 6; (ii) $1\frac{1}{5}$; (iii) $1\frac{2}{3}$; (iv) $1\frac{5}{7}$.
8. (i) $\frac{4}{7}$; (ii) $1\frac{1}{3}$; (iii) $1\frac{1}{2}$; (iv) $\frac{7}{40}$.
9. (i) $2\frac{1}{2}$; (ii) $7\frac{1}{2}$; (iii) 2; (iv) $\frac{2}{3}$.
10. (i) $\frac{3}{4}$; (ii) $2\frac{1}{4}$; (iii) $\frac{2}{3}$; (iv) $1\frac{4}{5}$.
11. 6. **12.** 12. **13.** 8. **14.** 20. **15.** 3. **16.** 3.
17. 6. **18.** 5. **19.** 7. **20.** $\frac{1}{2}$. **21.** 15. **22.** 16.
23. 15. **24.** $2\frac{4}{5}$. **25.** $8\frac{1}{6}$. **26.** 35, 15. **27.** 11, 17.
28. *A*, 12*s*.; *B*, 8*s*. **29.** 6. **30.** (i) 8; (ii) $\frac{7}{12}$; (iii) 119.

VI. a. Page 120.
1. 6*s*. 8*d*.; 3*s*. 4*d*.; 2*s*. 6*d*.; 1*s*. 8*d*.; 1*s*. 3*d*.; 7*s*. 6*d*.; 12*s*. 6*d*.; 17*s*. 6*d*.
2. $7\frac{1}{2}$*d*.; 3*s*. 9*d*.; 1*s*. 8*d*.; 13*s*. 4*d*.; 16*s*. 8*d*.
3. 4 in.; 10 in.; 2 cwt.; 12 cwt. **4.** 1 pt.; 2 pts.; 1 pt.
5. 1*s*. 3*d*. **6.** 10*s*. **7.** £1. 9*s*. 2*d*. **8.** 10*s*. 6*d*.
9. £3. **10.** £1. 15*s*. 7*d*. **11.** 10*s*. 8*d*.
12. 18 tons 13 cwt. 2 qrs. 24 lbs. **13.** £1. 9*s*. 2*d*.
14. 9 yds. 1 ft. 8 in. **15.** £1. 7*s*. 6*d*.
16. 1 ton 19 cwt. 3 qrs. **17.** £65. 9*s*. 1*d*.
18. £8. 16*s*. $3\frac{3}{4}$*d*. **19.** 7 tons 9 cwt. 17 lbs. 8 oz.
20. £6. 0*s*. $3\frac{1}{4}$*d*. **21.** £36. 2*s*. $0\frac{1}{2}$*d*. **22.** 1 ac. 1 r. $11\frac{1}{5}$ p.
23. £137. 1*s*. 5*d*. **24.** 1 cwt. 2 qr. $7\frac{5}{9}$ lbs.
25. 3*s*. 3*d*. **26.** £1. **27.** 6 in.
28. 230 yds. **29.** £1. 5*s*. 5*d*. **30.** 962 yds.
31. 3 qrs. **32.** £4. 10*s*. **33.** 10*s*.

VI. b. Page 122. **6.** $\frac{11}{60}$. **7.** $\frac{1}{3}$. **8.** $\frac{11}{120}$. **9.** $\frac{3}{16}$.
10. $\frac{1}{7}$. **11.** $\frac{4}{5}$. **12.** $\frac{9}{160}$. **13.** $\frac{8}{15}$. **14.** $\frac{3}{100}$.
15. $\frac{27}{160}$. **16.** $\frac{2}{3}$. **17.** $\frac{5}{9}$. **18.** $\frac{5}{9}$. **19.** $\frac{6}{13}$.
20. $\frac{1}{10}$. **21.** $\frac{1}{10}$. **22.** $\frac{3}{4}$. **23.** $1\frac{13}{16}$. **24.** $\frac{7}{8}$.
25. $\frac{19}{23}$. **26.** $\frac{19}{36}$. **27.** $\frac{3}{8}$. **28.** $\frac{2}{9}$. **29.** $\frac{2}{3}$.
30. $\frac{25}{48}$. **31.** $\frac{51}{80}$. **33.** 2. **34.** 40 : 27. **35.** $\frac{8}{135}$.

VI. c. Page 125. **5.** (i) 25 %; (ii) $16\frac{2}{3}$ %. **6.** £40.
7. 329. **8.** £67. 10*s*. **9.** 5 %. **10.** 32 %. **11.** 6 %.
12. 4580. **13.** £3550. **14.** £1080. **15.** £1500. **16.** 14070.
17. £4500. **18.** £7696. **19.** 200. **20.** 12 %.

VI. e. Page 128. **1.** £100. **2.** £120. **3.** £42.

4. £40. **5.** £30. **6.** £24.

7. £9765. **8.** £5054. 13*s*. 4*d*. **9.** £3078. 2*s*. 6*d*.

10. £1982. 10*s*. **11.** £840. **12.** £1173.

13. £802. 13*s*. 4*d*. **14.** £88. **15.** £255.

16. £108. **17.** £526. 10*s*. **18.** £780. 18*s*. 9*d*.

19. £2001. **20.** £1522. 18*s*. 4*d*. **21.** £847. 17*s*. 6*d*.

22. £2455. 3*s*. 9*d*. **23.** £500. 0*s*. 6*d*. **24.** £1360. 16*s*. 8*d*.

25. £401. 11*s*. 8*d*. **26.** £2483. 8*s*. $1\frac{1}{2}$*d*. **27.** £2748. 16*s*. 3*d*.

28. £1048. 16*s*. **29.** £33161. 14*s*. **30.** £1824. 7*s*. 6*d*.

Miscellaneous Examples II. Page 129. **2.** 15.

3. £91. **4.** (i) $\frac{3}{8}$; (ii) $\frac{7}{13}$; (iii) $\frac{3}{7}$. **5.** 1 m. 65 cm.

6. 18 lbs. **7.** The first by $\frac{2}{15}$. **8.** $4\frac{7}{12}$, $3\frac{37}{165}$.

9. $\frac{5}{8}$ mile. **10.** 40. **11.** 902,100. **13.** (i) $\frac{14}{81}$; (ii) $\frac{7}{16}$.

14. 9 weeks. **15.** $51\frac{1}{3}$. **16.** £312. 10*s*. **17.** £25044. 3*s*. 9*d*.

18. $\frac{10}{38}$, $\frac{10}{31}$, $\frac{10}{55}$; $\frac{2}{11}$, $\frac{5}{19}$, $\frac{10}{31}$. **19.** $42\frac{3}{5}$ miles. **20.** 25, 24.

21. 1 m. 60 cm. **22.** 2275. **23.** 4. **25.** $\frac{9}{32}$.

26. First, third; 28, 41580. **27.** 56732, 11346. **28.** 6 fr.

29. 30 mi. an hour. **30.** $10\frac{33}{35}$; $18\frac{5}{21}$, $81\frac{16}{21}$.

31. $\frac{24}{91}$. **32.** 2520, 2688, 2856. **34.** 1 sq. yd.

35. $2^7 . 3^2 . 7 . 31 = 62 . 63 . 64$. **36.** £2. 18*s*.

37. (i) $10\frac{1}{42}$; (ii) $1\frac{1}{220}$. **38.** (i) $12\frac{97}{120}$; (ii) $\frac{19}{21}$; (iii) $\frac{5}{8}$.

39. 11200 Kg.; 9 cwt. 3 qr. 8 lbs. **40.** 20 ft. **41.** £2.

42. 216. **43.** $1\frac{1}{2}$*d*. **44.** (i) $1\frac{31}{90}$; (ii) $1\frac{1}{9}$. **45.** £425.

46. $1\frac{3}{4}$ pints. **47.** $\frac{1}{200}$. **48.** (i) $4\frac{19}{20}$; (ii) $3\frac{29}{70}$. **49.** $\frac{1}{15}$.

50. £49. 2*s*. **51.** 36. **52.** $1\frac{1}{4}$*d*. **53.** 25.

54. (i) $\frac{11}{24}$; (ii) $\frac{7}{8}$; (iii) $\frac{65}{96}$. **55.** 5 hrs. **56.** (i) 2; (ii) $8\frac{1}{2}$.

57. (i) 6 yds. 9 in.; (ii) £82. 12*s*. 5*d*. **58.** £25.

59. 160. **60.** 2*s*. 6*d*. **61.** $\frac{7}{80}$, $\frac{13}{40}$. **62.** $87\frac{1}{2}$; 225.

63. 1*s*., 3*s*., 4*s*. **64.** (i) 1; (ii) 2. **65.** 3*s*., 4*s*.

66. (i) $25\frac{3}{5}$ m.; (ii) $5\frac{1}{2}$ cwt.; (iii) $41\frac{1}{7}$ litres; (iv) 2240 Kg.; (v) 1*s*. 10*d*.; (vi) 2*d*. **67.** (i) $\frac{7}{19}$; (ii) $\frac{45}{56}$.

68. (i) $9\frac{8}{9}$; (ii) $\frac{1}{2}$. **69.** 5; $\frac{1}{7}$. **70.** 5 miles. **71.** 17920.

72. £6. 10*s*. 8*d*. **73.** $\frac{1}{4}$. **74.** $1\frac{1}{2}$*d*. **75.** 11.

76. 503 gm. **77.** £180. 3*s*. 3*d*. **78.** 4 hrs.

VII. a. Page 137.

11. 16. **12.** 21. **13.** 150. **14.** 8.
15. 13*s*. 4*d*. **16.** 56. **17.** 26. **18.** £3. 6*s*. 8*d*.
19. £7. 17*s*. 6*d*. **20.** 6 tons. **21.** £71. 13*s*. 4*d*. **22.** 9 hrs. 20 min.
23. (i) $1\frac{1}{2}$ hrs.; (ii) $\frac{px}{y}$ hrs. **24.** (i) 36 miles; (ii) $\frac{mx}{p}$ miles.
25. $8\frac{1}{2}$ miles, $\frac{x}{2}$ miles. **26.** (i) 36; (ii) $\frac{3a}{88}$.
27. (i) 132; (ii) $\frac{88m}{3}$. **28.** (i) $1\frac{1}{4}$ min.; (ii) $\frac{ax}{y}$ min.
29. 36 days. **30.** $7\frac{1}{3}$ hrs. **31.** 3 tons 17 cwt.
32. £80. 8*s*. **33.** £163. 7*s*. **34.** £23. 8*s*. 9*d*.
35. $28\frac{1}{2}$ min. **36.** £60. 4*s*. 6*d*. **37.** 2 yrs., 9 months.
38. £55. 14*s*. 8*d*. **39.** 47 ac. 1 r. 35 p. **40.** 75 days.

VII. b. Page 140.

1. £248. **2.** £93. 10*s*.
3. 63 in. **4.** 34 miles. **5.** £110. **6.** £1917.
7. £45. 10*s*. **8.** The whole. **9.** $\frac{5}{6}$. **10.** £150.
11. £50. **12.** 1*s*. 8*d*. **13.** $1\frac{1}{4}$*d*., $2\frac{3}{4}$*d*. **14.** $8\frac{3}{4}$*d*.
15. 3*s*. 8*d*. **16.** 2*s*. 8*d*. **17.** $8\frac{3}{4}$*d*. **18.** 2*d*.
19. Approximately equal. **20.** $81\frac{3}{5}$ Km. **21.** £5. 12*s*. 6*d*.
22. £124. **23.** 12. **24.** 2*s*. **25.** £63. 6*s*. 8*d*.
26. 468 ac. **27.** $\frac{1}{4}$. **28.** 128 miles. **29.** £4326. **30.** £603. 5*s*.

VIII. d. Page 150.

20. 234·5. **21.** 3·6401.
22. 10^2. **23.** 10^3. **35.** £4·1. **36.** £5·3.
37. £8·9. **38.** £17·7. **50.** 5·047. **51.** 9·302.
52. 0·416. **53.** 0·00626. **58.** 7 Dm. 2 m. 3 cm.; 0·07203.
59. 762. **60.** 2,518,000. **61.** 23900. **62.** 2 Km. 12 m.
63. (i) 10; (ii) 1000; (iii) 100,000. **64.** (i) 100; (ii) 1000; (iii) 10.
65. (i) $3{\cdot}816 \times 10^2$; (ii) $4{\cdot}068 \times 10^3$; (iii) $8{\cdot}67 \div 10$; (iv) $4{\cdot}25 \div 10^3$.

VIII. e. Page 153.

1. 48210. **2.** 60·2.
3. 4600. **4.** 0·8701. **5.** 146,800. **6.** 405·03.
7. 4·16. **8.** 5·0026. **9.** 8·14. **10.** 14·08.
11. 6·0606. **12.** 0·0707. **13.** 100. **14.** 10000.

VIII. f. Page 155. **1.** 3421. **2.** 80.
3. 4·09. **4.** 9000. **5.** 250. **6.** 6970. **7.** 46.
8. 16·2. **9.** 75. **10.** 1. **11.** 1000. **12.** 150.
13. 4750. **14.** 72·5. **15.** 25. **16.** 2·5. **17.** 88.
18. 720. **19.** 18·4. **20.** 0·92. **21.** 60 Kg.

VIII. g. Page 157. **1.** 9·4. **2.** 16. **3.** 17·5.
4. 16·88. **5.** 7. **6.** 20·5. **7.** 40·02.
8. 10. **9.** 309·702. **10.** 30. **11.** 2·13 m.
12. 7400 m. **13.** 4·3 Km. **14.** 29 fr. 40 c. ; 60 c.
15. 2·3. **16.** 1·41. **17.** 1·29. **18.** 0·81.
19. 2·97. **20.** 5·193. **21.** 0·25. **22.** 0·219.
23. 6·3. **24.** 0·994. **25.** 0·01 in. **26.** 3·2 m.
27. 500 m. **28.** 1 m. ; $\frac{1}{6001}$. **29.** 0·1.
30. 0·06 in. ; yes. **31.** 3·44. **32.** 1. **33.** –0·47.
34. –0·54. **35.** 0. **36.** –0·01. **37.** 6·2923.
38. –10. **39.** 0·001. **40.** 1·09. **41.** 1·307.
42. (i) 2·809 ; (ii) –2·16 ; (iii) –17·48. **43.** 25·81, 26.

VIII. h. Page 159. **14.** (i) 5·35 ; (ii) 53·5 ; (iii) 0·535.
15. (i) 3·28 ; (ii) 328 ; (iii) 0·0328.
16. (i) 1·29 ; (ii) 12900 ; (iii) 0·0129.
17. 6th decimal place. **18.** 0·000006.
19. (i) 16578·8 ; (ii) 1·65788 ; (iii) 165,788,000.
20. (i) 300 ; (ii) 0·003 ; (iii) 0·0003.
21. (i) 3654 ; (ii) 0·3654 ; (iii) 0·003654.
22. 6·9. **23.** 50·4°. **24.** 5468.
25. 1480 fr. **26.** 123 m. 75 cm.

VIII. k. Page 162. **1.** 5·04. **2.** 16·96.
3. 9·1. **4.** 1·19. **5.** 0·312. **6.** 99·2.
7. (i) 173·08 ; (ii) 25·962 ; (iii) 6·0578. **8.** 10·5525.
9. 1·17. **10.** 0·377. **11.** 0·02508. **12.** 1·932.
13. 1204·8. **14.** 3·348. **15.** 33. **16.** 843·5608.
17. 91·44 cm. ; 0·9144 m. **18.** 48 fr. **19.** 210.
20. 2·5029. **21.** 1002. **22.** 1·932. **23.** 0·10353.
24. 0·816. **25.** 745·821. **26.** 40 ; 0·4, 4.
27. 28·8 sq. in. **28.** 81·9 sq. m. **29.** 1·16 sq. m.
30. The first : $\frac{1}{100}$ sq. in. **31.** 4·275 sq. Km. **32.** 317·4 lbs.
33. (i) 0·049 ; (ii) 4·9 ; (iii) 0·000049.
34. (i) 2 ; (ii) 2·56 ; (iii) 76·2.

35. (i) 87·9 in.; (ii) 1414 ft.; (iii) 1885 ft. **36.** 378 gm.
37. 3780 gm. **38.** 61·6 gm. **39.** 5750 Kg. **40.** 480,000 Kg.
41. 3·5. **42.** 9·5. **43.** 1. **44.** 0·13.

VIII. l. Page 165.

20. 6; 12*s*. **21.** 1·87.
22. 0·263. **23.** 0·038. **24.** 0·0063. **25.** 0·076.
26. 0·4275. **27.** 2·73. **28.** 0·53. **29.** 0·318.
30. 7·5. **31.** 3·8. **32.** 6·5. **33.** Each = 0·4125.
34. 0·675. **35.** 0·474. **36.** 123. **37.** 0·59.
38. 265. **39.** 2·003. **40.** 5. **41.** 0·06.
42. 0·78. **43.** 0·0057. **44.** 30·25. **45.** 5·49.
46. 7·375. **47.** 0·125. **48.** 0·175. **49.** 36·25.
50. 21·550 Km. **51.** 104 dols. 61 cents. **52.** 90·1 in.; 0·01 in.
53. 12·09 m. **54.** 7·01″. **55.** 39. **56.** 0·7833. **57.** 0·9556.
58. 2·4286. **59.** 0·1111. **60.** 0·5714. **61.** 1·6364.
62. 3·7. **63.** 0·82. **64.** 0·09. **65.** 0·067.
66. 0·909. **67.** 0·4087. **68.** 0·6389. **69.** 0·0116.

VIII. m. Page 168.

1. 1·3. **2.** 1·1.
3. 2·3. **4.** 1·7. **5.** 1·6. **6.** 5·6.
7. 2·3. **8.** 17. **9.** 0·45. **10.** 0·28.
11. 111. **12.** 0·67. **13.** 0·14. **14.** 0·46.
15. 250. **16.** 0·0365. **17.** 7·02. **18.** 0·0046875.
19. 0·34; 34, 0·0034. **20.** 1·7; 0·017, 170.
21. 0·0365; 0·365, 3650. **22.** (i) 0·029; (ii) 7·1; (iii) 39; (iv) 6.
23. 214; *rem.* 90 c. **24.** 6000. **25.** 2·54. **26.** 3·28 ft.
27. 7·4″. **28.** 17·25 m. **29.** 10·8. **30.** 128.
31. 0·2. **32.** 0·0083. **33.** 80·0056. **34.** 0·2085.
35. 0·0033. **36.** 0·0078. **37.** 1·3240. **38.** 0·0365.
39. (i) 6·329; (ii) 0·516; (iii) 2·665.
40. (i) 15·8 in.; (ii) 40·34 in. **41.** 45; 0·1 in. *rem.*
42. (i) 0·0007; (ii) 0·0001; (iii) 0·0271; (iv) 0·0022.

VIII. n. Page 173.

1. $\frac{13}{20}$. **2.** $\frac{17}{20}$. **3.** $\frac{9}{25}$.
4. $\frac{11}{25}$. **5.** $5\frac{19}{50}$. **6.** $\frac{1}{80}$. **7.** $7\frac{11}{40}$. **8.** $\frac{3}{16}$.
9. 0·23. **10.** 3·009. **11.** 8·27. **12.** 0·00041.
13. 0·013. **14.** 0·00052. **15.** 0·000073. **16.** 0·175.
17. 3·875. **18.** 9·3125. **19.** 1·88. **20.** 0·0015625.
21. (i), (iii), (iv), (vi), (viii), (x) terminate; the rest do not.
22. 0·104. **23.** 2·077. **24.** 3·114. **25.** 0·579.

26. 7·609. **27.** (i) 0·4 ; (ii) 0·35 ; (iii) 0·353 ; (iv) 0·3529.
28. $0{\cdot}714\frac{2}{7}$; 0·714. **29.** $0{\cdot}764\frac{12}{17}$; 0·765. **30.** $0{\cdot}478\frac{6}{23}$; 0·478.
31. $0{\cdot}016\frac{5}{16}$; 0·016. **32.** $0{\cdot}023\frac{4}{7}$; 0·024.
33. (i) $0{\cdot}\dot{3}$, $0{\cdot}\dot{6}$; (ii) $0{\cdot}333\frac{1}{3}$, $0{\cdot}666\frac{2}{3}$; (iii) 0·333, 0·667.
34. (i) $0{\cdot}\dot{1}$, $0{\cdot}\dot{4}$, $0{\cdot}\dot{7}$; (ii) $0{\cdot}111\frac{1}{9}$, $0{\cdot}444\frac{4}{9}$, $0{\cdot}777\frac{7}{9}$; (iii) 0·111, 0·444, 0·778.
35. (i) $0{\cdot}\dot{0}\dot{9}$, $0{\cdot}\dot{2}\dot{7}$, $0{\cdot}\dot{7}\dot{2}$; (ii) $0{\cdot}090\frac{10}{11}$, $0{\cdot}272\frac{8}{11}$, $0{\cdot}727\frac{3}{11}$; (iii) 0·091, 0·273, 0·727.
36. (i) $0{\cdot}\dot{1}4285\dot{7}$, $0{\cdot}\dot{4}2857\dot{1}$, $0{\cdot}\dot{7}1428\dot{5}$; (ii) $0{\cdot}142\frac{6}{7}$, $0{\cdot}428\frac{4}{7}$, $0{\cdot}714\frac{2}{7}$; (iii) 0·143, 0·429, 0·714.
37. (i) $0{\cdot}1\dot{4}$; (ii) $0{\cdot}144\frac{4}{9}$; (iii) 0·144.
38. (i) $0{\cdot}29\dot{5}\dot{4}$; (ii) $0{\cdot}295\frac{5}{11}$; (iii) 0·295.
39. (i) $4{\cdot}\dot{1}5384\dot{6}$; (ii) $4{\cdot}153\frac{11}{13}$; (iii) 4·154.
40. (i) $2{\cdot}\dot{2}9\dot{7}$; (ii) $2{\cdot}297\frac{11}{37}$; (iii) 2·297.
41. (i) $5{\cdot}\dot{1}4\dot{8}$; (ii) $5{\cdot}148\frac{4}{27}$; (iii) 5·148.
42. 2·1375. **43.** 0·32625. **44.** 3·19875. **45.** $5{\cdot}32\dot{4}\dot{5}$.
46. $\frac{7}{16}$. **47.** $\frac{3}{16}$. **48.** $\frac{1}{32}$. **49.** $\frac{23}{320}$.
50. (i) 0·12 ; (ii) 0·625 ; (iii) 0·014 ; (iv) 0·055.

IX. a. Page 175. **1.** 0·5. **2.** 0·25. **3.** 0·75.
4. 0·375. **5.** 0·875. **6.** 0·6875. **7.** 0·5625.
8. $0{\cdot}\dot{3}$. **9.** 0·25. **10.** 0·75. **11.** 0·125.
12. 0·225. **13.** 0·625. **14.** 0·375. **15.** 0·575.
16. 0·0625. **17.** 5·325. **18.** 7·425. **19.** 8·725.
20. £4·575. **21.** £3·775. **22.** £1·3125. **23.** £0·4375.
24. £8·8875. **25.** £5·40625. **26.** £11·694. **27.** £0·831.
28. £8·767. **29.** £2·5344. **30.** £5·3729.
31. £0·1677. **32.** 5·6875 tons. **33.** 4·475 ac.
34. 0·003125. **35.** 0·430625. **36.** 0·7925.

IX. b. Page 177. **1.** 6*s.* **2.** 14*s.*
3. 18*s.* **4.** 1*s.* **5.** 11*s.* **6.** 13*s.*
7. 16*s.* **8.** 19*s.* **9.** 9*d.* **10.** $1\frac{1}{2}$*d.*
11. $7\frac{1}{2}$*d.* **12.** $10\frac{1}{2}$*d.* **13.** £3. 4*s.* 6*d.*
14. £5. 8*s.* 6*d.* **15.** £9. 17*s.* 6*d.* **16.** £2. 1*s.* 9*d.*
17. £11. 11*s.* 3*d.* **18.** 8*s.* $1\frac{1}{2}$*d.* **19.** £4. 12*s.* 7*d.*
20. £5. 14*s.* 5*d.* **21.** £14. 16*s.* 6*d.* **22.** £9. 17*s.* $9\frac{1}{2}$*d.*
23. 12*s.* $8\frac{1}{4}$*d.* **24.** £11. 0*s.* $11\frac{1}{4}$*d.* **25.** 11*s.* 3*d.*
26. £6. 6*s.* $4\frac{1}{2}$*d.* **27.** £15. 11*s.* $10\frac{1}{2}$*d.* **28.** £5. 6*s.* 3*d.*
29. £65. 14*s.* 9*d.* **30.** £8. 11*s.* $7\frac{1}{4}$*d.* **31.** 0·9422.
32. 0·9333. **33.** 0·3971. **34.** 0·0911.

35. 1 ton 17 cwt. 3 qrs. **36.** 6 ac. 1 r. 36 p.
37. 21 hr. 28 min. 48 sec. **38.** 2 m. 55 yds.
39. 8 cwt. 2 qrs. **40.** 6 tons 11 cwt. 1 qr. **41.** £31. 6*s*. $0\frac{3}{4}$*d*.
42. 0·6071. **43.** 0·325. **44.** 0·0375.
45. £4. 6*s*. 6*d*. **46.** £3. 1*s*. $5\frac{1}{4}$*d*. **47.** 14*s*. $2\frac{1}{2}$*d*.
48. 2*s*. **49.** £7. 15*s*. $6\frac{3}{4}$*d*. **50.** 15*s*. 1*d*.
51. 3. **52.** (i) 3; (ii) 5. **54.** 3.
55. 4238 mm. **56.** 7100 m. **57.** 39 lbs.
58. 385 yds. **59.** 755 sq. po. **60.** 304 yds.

IX. c. Page 180.

1. £0·25. **2.** £0·4.
3. £0·85. **4.** £0·6. **5.** £0·575. **6.** £0·475.
7. £0·825. **8.** £0·075. **9.** £0·005$\frac{5}{24}$. **10.** £0·010$\frac{5}{12}$.
11. £0·015$\frac{5}{8}$. **12.** £0·021$\frac{7}{8}$. **13.** £0·031$\frac{1}{4}$. **14.** £0·034$\frac{3}{8}$.
15. £0·041$\frac{2}{3}$. **16.** £0·047$\frac{11}{12}$. **17.** (i) £2·354$\frac{1}{6}$; (ii) £2·354.
18. (i) £3·566$\frac{2}{3}$; (ii) £3·567. **19.** (i) £7·720$\frac{5}{6}$; (ii) £7·721.
20. (i) £4·429$\frac{1}{6}$; (ii) £4·429. **21.** (i) £5·927$\frac{1}{12}$; (ii) £5·927.
22. (i) £18·040$\frac{13}{24}$; (ii) £18·041. **23.** £2·5625. **24.** £5·1125.
25. £0·70625. **26.** £16·08125. **27.** £0·19375.
28. £22·21875. **29.** £0·459375. **30.** £11·221875.
31. £0·865625. **32.** £0·20417. **33.** £3·70833.
34. £0·45208. **35.** £1·16406. **36.** £0·37188.
37. £0·56042. **38.** £8·51771. **39.** £0·94271.
40. £3·71146. **41.** £0·82604. **42.** £2·73333. **43.** £0·64688.

IX. d. Page 182.

1. 9*s*. 6*d*. **2.** 16*s*. 6*d*.
3. 6*s*. $1\frac{3}{4}$*d*. **4.** 11*s*. $2\frac{1}{4}$*d*. **5.** 13*s*. $2\frac{3}{4}$*d*. **6.** 9*s*. $2\frac{1}{2}$*d*.
7. 1*s*. $9\frac{1}{4}$*d*. **8.** 3*s*. 10*d*. **9.** 9*s*. $11\frac{1}{4}$*d*. **10.** £3. 1*s*. $11\frac{1}{2}$*d*.
11. £11. 2*s* $2\frac{3}{4}$*d*. **12.** £7. 10*s*. $2\frac{1}{4}$*d*. **13.** £4. 6*s*.
14. £4. 0*s*. $7\frac{1}{4}$*d*. **15.** £4. 0*s*. $0\frac{3}{4}$*d*. **16.** £9. 8*s*. 7*d*.
17. 7*d*. **18.** £4. 16*s*. 6*d*. **19.** £4. 12*s*. 8*d*.
20. £7. 8*s*. 4*d*. **21.** 18*s*. 5*d*. **22.** £2. 7*s*. 3*d*.
23. 7*s*. 5*d*. **24.** £2. 2*s*. **26.** £0·000042.
28. £13. 2*s*. $4\frac{3}{4}$*d*. **29.** £32. 8*s*. 3*d*. **30.** £25. 5*s*. 7*d*.

X. a. Page 187.

1. 79000; 79. **2.** 481000; 481.
3. 370000; 370. **4.** 2·764; 2764. **5.** 0·013; 13.
6. 3·401; 3401. **7.** 7·417; 7417. **8.** 7·472; 7472.
9. 0·910; 910. **10.** 0·002; 2. **11.** 0·001; 1.
12. 10·000; 10000. **13.** 6·27. **14.** 81·5.
15. 705. **16.** 0·682. **17** 0·0919.

18. 0·00403. **19.** 4·78. **20.** 8·62.
21. 0·0182. **22.** 37300. **23.** 70100.
24. 462000. **25.** 263. **26.** 1120.
27. 249. **28.** 16. **29.** 760.
31. £3·465 and £3·475, or £(3·47 ± ·005).
32. 41·625 tons and 41·635 tons, or (41·63 ± ·005) tons.
33. 3·595 miles and 3·605 miles, or (3·60 ± ·005) miles.
34. 0·065 m. and 0·075 m., or (0·07 ± ·005) m.
35. (i) £(40·76 ± ·005); (ii) £(0·4076 ± ·00005);
(iii) £(4076 ± ·5); (iv) £(407600 ± 50).
36. (i) (9·347 ± ·0005) Km.; (ii) (93·47 ± ·005) Hm.
(iii) (9347 ± ·5) metres; (iv) (934700 ± 50) cm.
37. (i) (74·20 ± ·05) fr., (7420 ± 5) fr.,
(7·420 ± ·005) fr., (74200 ± 50) fr.;
(ii) (74·20 ± ·005) fr., (7420 ± ·5) fr.,
(7·420 ± ·0005) fr., (74200 ± 5) fr.
38. Absolute errors (i) £0·018, (ii) £0·18, (iii) £0·00018. Relative and percentage errors the same in each case, namely 0·00048 and 0·048.
39. Absolute errors (i) 0·48 mi., (ii) 0·0048 mi., (iii) 0·0000048 mi. Relative error in each case 0·00053; percentage error 0·053.
40. 471, 0·00065, 0·065. **41.** 110 ac., 0·00011, 0·011.
42. £222000, 0·00029, 0·029. **43.** 2·77 ft., 0·00046, 0·046.
44. 32·35 grs., 0·0021, 0·21.

X. b. Page 189. **1.** 62·94. **2.** 5·90.
3. 6·78. **4.** 0·009. **5.** 4·375. **6.** 4·000.
7. 333. **8.** 41·6. **9.** 1000. **10.** 0·948.
11. 23,100,000. **12.** 27,700,000. **13.** £150,900,000.
14. 0·0064, 0·000512, 0·00004096. (i) 0·087; (ii) 0·087.
15. 0·25. **16.** £1047. **17.** 392·5 Km.
18. £3·7292, £0·7917, £12·7273. £17·2482; £17. 4*s*. 11½*d*.
19. £10. **20.** £10.

X. c. Page 192. **1.** 18·38. **2.** 3·32.
3. 49·54. **4.** 0·13. **5.** 15·94. **6.** 38·02.
7. 7·18. **8.** 8·50. **9.** 2·50. **10.** 376.
11. 30. **12.** 8. **13.** 243. **14.** 3·349.
15. 2·000. **16.** 10·173. **17.** 3·000. **18.** 50·9.
19. 0·0493. **20.** 30·0. **21.** 184. **22.** 3·312.
23. 1·751. **24.** 4·360. **25.** 1·846. **26.** 4·411.

27. 1.
28. 478,000,000.
29. 3,047,000.
30. 29,000,000.
31. 408,000,000.
32. 250,000,000.
33. 296 in.
34. 13·49 m.
35. 364 Km.
36. 15·65 Kg.
37. £103,100,000.
38. £12,600,000.

X. d. Page 196.

1. 8·18.
2. 17·7.
3. 0·331.
4. 0·0299.
5. 9·62.
6. 53·68.
7. 0·47.
8. 82·56.
9. 3·406.
10. 1·980.
11. 0·024.
12. 0·052.
13. 1·290.
14. 3·317.
15. 0·318.
16. 1·732.
17. 2·601.
18. 0·626.
19. 0·0982.
20. 24·9.
21. 0·0142.
22. 902.
23. 1132.
24. 1122.
25. 44,000.
26. 41900.
27. 8·120.
28. 92 sec.
29. 6082·7 ft.
30. £4. 10*s.* 2*d.*
31. 25 fr. 23 c.
32. 562, 150, 140.
33. (i) 1·152 miles; (ii) 0·868 knots.
34. (i) 0·025400 metres, 25·400 mm.; (ii) 20·12 metres; (iii) 1·609 Km.
36. $6·92, £1. 8*s.* 6¼*d.*
37. 277·3 cu. in.
38. £397. 1*s.* 3*d.*
39. £1539.
40. £18. 1*s.* 9*d.*, £30. 1*s.* 10*d.*, £2. 11*s.*

X. e. Page 199.

1. 0·87.
2. 3·68.
3. 0·63.
4. 4·41.
5. 9·35.
6. 6·73.
7. 1·07.
8. 0·51.
9. 219.
10. 703.
11. 344.
12. 192.
13. 2·011.
14. 13,170,000.
15. £2. 1*s.* 9*d.*
16. $4·8632.
17. $772·40.
18. 4·536 litres.
19. 2·828.
20. 2·000.
21. 0·00005, 0·00000.
22. (i) 0·000013; (ii) 0·0007.
23. 695.
24. 585.
25. 10·76.
26. 119·60 sq. yds.
27. 2·47 ac.
28. 16·387 cu. cm.
29. 1·4667 ft. per sec.; 1·4667.
30. 7·233 ft. lbs.

X. f. Page 205.

1. (13·08 ± 0·015) in.; 13·1″.
2. Within ± 20 miles.
3. 21·83 m.; 0·015 m.
4. 19·014, 19·008.
5. 10·01 correct to 2nd dec. place.
6. 2·56″ and 2·58″.
7. 48·5 metres, nearly.
8. 2·607 ± ·001.
9. 6·109 ± ·001.
10. 0·0045 and 0·0075.
11. 2·5785 and 2·5815.
12. ± 0·5 cm.

13. (i) To nearest 10 lbs., *i.e.* Error within ±5 lbs.
(ii) To nearest 100 lbs., *i.e.* Error within ±50 lbs.
(iii) Error within ±36·05 lbs.

14. (i) 29·0625 and 30·1625; (ii) 28·52 and 30·72.

15. (i) 21·7875 sq. cm. and 22·7375 sq. cm.
(ii) 29·4525 sq. cm. and 30·5525 sq. cm.
(iii) 560 sq. ft. and 660 sq. ft.
(iv) 215,000 sq. yds. and 234,000 sq. yds.

16. 21, units' figure doubtful (21·06 ± ·48, nearly).

17. 44, to nearest unit (44·21... ± ·06...).

18. 205, units' figure doubtful (204·8 ± ·5, nearly).

19. 2·1, to nearest tenth (2·066 ± ·004, nearly).

20. 2·16, the last digit doubtful (2·16 ± ·01, nearly).

21. 1354·0 ft. correct to the nearest tenth.

22. About 4620 lbs., units' figure doubtful.

23. About 213 lbs., units' figure doubtful.

24. 800·9 lbs. and 790·3 lbs. nearly.

25. Between £23,940 and £32,340.

26. 5·32..., remaining figures doubtful.

27. (i) 1 Hm.; (ii) 1 cm.; (iii) 1 Km.; (iv) 1 mm.; 3 figures.

28. £3·711458. (i) 2 figs.; (ii) 5; (iii) 1; (iv) 6.
(i) 7*s.* 5*d.*; (ii) £371. 2*s.* 11*d.*; (iii) $8\frac{3}{4}$*d.*; (iv) £3711. 9*s.* 2*d.*

29. 0·03612 lbs.; 252·88 grains; yes, to nearest hundredth.

30. 454 grms. nearly (453·6 ± ·2).

31. (i) 25·4 mm. (25·39 ± ·01). (ii) 2,590,000 sq. m. to nearest 10,000.

32. (i) 4·46 (± ·01). (ii) 14·91 (± ·01).
(iii) 2325 (± 3). (iv) 227,000 to nearest 1000.

33. (i) 2·32 metres, last figure doubtful.
(ii) About 7920 miles, last two figs. doubtful.

34. About 499 seconds (possible error ±2 seconds, nearly).

35. 10·4.

XI. a. Page 211. **1.** 0·933. **2.** 0·0375.
3. 5·84. **4.** 0·293. **5.** 0·125, 12·5%.
6. 0·56, 56%. **7.** 0·56, 56%. **8.** 0·76, 76%.
9. 22·1%. **10.** 2·85%. **11.** 3·46%. **12.** 3·40%.
13. 7·55%. **14.** 10·0%. **15.** 18*s.* **16.** £6. 16*s.*
17. £25. 15*s.* **18.** £12. 15*s.* 6*d.* **19.** £9. 3*s.* 6*d.*
20. £3. 4*s.* 2*d.* **21.** £22. 12*s.* **22.** £234. 12*s.* 7*d.*
23. £21. 5*s.* 2*d.* **24.** £80. 11*s.* 11*d.* **25.** 18*s.* 10*d.*
26. £19. 1*s.* 2*d.* **27.** £24. 8*s.* 9*d.* **28.** £3. 17*s.* 5*d.*

29. 3·71 %. **30.** 43·9 %. **31.** 49·5 %.
32. 81·6 %. **33.** 5·26 %. **34.** 22·4 %.
35. 17. **36.** 3·5 **37.** £728,668.
38. £14346. 11*s.* 5*d.*, £11955. 9*s.* 6*d.*, £9564. 7*s.* 7*d.*, £7970. 6*s.* 4*d.*, £3985. 3*s.* 2*d.*
39. 43 %, 35 %. **40.** 95·3 %, 88·3 %; £953, £883.

XI. b. Page 215. **1.** £2194. 6*s.* 8*d.* **2.** £901. 5*s.* 0*d.*
3. £1101. 3*s.* 2*d.* **4.** £4056. 6*s.* 7*d.* **5.** £1703. 18*s.* 1*d.*
6. £18. 3*s.* 7*d.* **7.** £193. 9*s.* 1*d.* **8.** £1874. 8*s.* 9*d.*
9. £667. 18*s.* 6*d.* **10.** £61. 9*s.* 7*d.* **11.** £3. 1*s.* 2*d.*
12. £734. 7*s.* 9*d.* **13.** £2737. 4*s.* 5*d.* **14.** £40. 4*s.* 2*d.*
15. £6. 11*s.* 4*d.* **16.** £1. **17.** £12. 19*s.* 9*d.*
18. £3072. 19*s.* 7*d.* **19.** £1209. 8*s.* 8*d.* **20.** £1144. 0*s.* 11*d.*
21. £10. 11*s.* 6*d.* **22.** £41. 4*s.* 2*d.* **23.** £36. 4*s.* 10*d.*
24. 139 fr. 87 c. **25.** 5779 fr. 49 c. **26.** 679 fr. 30 c.
27. £7427. 6*s.* $10\frac{1}{2}$*d.* **28.** £1439. 18*s.* 0*d.*
29. 9 hrs. 35′ 34″. **30.** 43 Kg. 175 gm.

XI. c. Page 218. **1.** £1. 10*s.* 9*d.* **2.** £34767.
3. 4·55 litres; 10·5 litres. **4.** 1094. **5.** £26. 16*s.* 3*d.*
6. 25·26 fr., 20·43 Mks., $4·85, 9·41 roubles, 15·00 rupees.
7. 29·739 gm. **8.** £2. 10*s.* 1*d.*
9. (i) 11·81, (ii) 22, (iii) 148·3. **10.** 1·4. **11.** £1. 11*s.*
12. £6. 3*s.* 3*d.* **13.** £9. 0*s.* 6*d.* **14.** 27 ft. 4·5 in.
15. 56716714. **16.** 19 %, 16 %, 15 %. **17.** 1*s.* 4*d.*
18. 17·9 %, 16·8 %, 15·5 %; 65,713, 62,113, 43,289. **19.** 1·03 Kg.
20. 5. **21.** (i) 25·27, (ii) 0·89; 36·2, 17·9. **22.** 326 ft.
23. 5700 tons. **24.** 193 cwt. **25.** 12.23 p.m. **26.** 276 lbs.
27. 37. **28.** 1·02 sq. metres, 204 sq. cm., 2 %.
29. 3·5 in. **30.** 29·4.

XII. a. Page 223. **1.** 4 days. **2.** 20 min.
3. 3 hrs. **4.** 1 day. **5.** 9.36 a.m. **6.** $56\frac{1}{4}$ min.
7. $1\frac{1}{5}$ hrs. **8.** $\frac{n-m}{mn}$, $\frac{mn}{n-m}$ min. **9.** $22\frac{1}{2}$ days.
10. $5\frac{5}{11}$ days; $\frac{1}{60}$ of the work. **11.** $\frac{4a}{7}$ days.
12. *A* 50 days, *B* 50 days. **13.** 30 days.
14. *A* 75 days, *B* 50 days. **15.** *A* 2*s.* 6*d.*, *B* 1*s.* 6*d.*, *C* 6*d.*
16. (i) 12 hrs.; (ii) 12 hrs.

XII. b. Page 226. **1.** 44. **2.** $73\frac{1}{3}$. **3.** 11.
4. $2\frac{3}{4}$. **5.** 15. **6.** 40. **7.** 3.
8. $5\frac{5}{8}$. **9.** 3. **10.** 1 min. **11.** 33 yds.
12. 3 sec. **13.** 30. **14.** 5 sec. **15.** 224 yds.
16. 4 hrs.; 14 miles from A. **17.** $2\frac{1}{2}$ hrs., $11\frac{1}{4}$ miles.
18. 6 hrs. **19.** 1.45 p.m.; $13\frac{1}{8}$ miles, $16\frac{7}{8}$ miles.
20. 12 o'clock, 12 miles. **21.** $4\frac{1}{2}$ secs. **22.** $27\frac{3}{11}$ sec.
23. 6 min. **24.** $11\frac{1}{2}$ sec.

XII. c. Page 228. **1.** 5 ft. 2 in.
2. 4853 tons, 422 tons. **3.** £432. **4.** 2*s*. 6*d*.
5. £94. 13*s*. 4*d*., £103. 6*s*. 8*d*. **6.** 80.
7. 6233 yds., $21\frac{1}{4}$ mi. per hour, very nearly. **8.** 16·7 yrs.
9. $\frac{9a+11b}{20}$; 18·1. **10.** 638·9 miles, 23·1 knots, 24·2 knots.
11. 1·52 in., 2·23 in.
12. (i) 0, 176, 352, 528, 704 yds.; (ii) 176, 352, 528, 704, 880 yds.
Average result = 2200 yds.
$s = 2200$ yds.

XII. d. Page 231. **1.** (i) $16\frac{4}{11}'$, (ii) $49\frac{1}{11}'$ past 3.
2. (i) $49\frac{1}{11}'$, (ii) $16\frac{4}{11}'$ past 9. **3.** $16\frac{4}{11}'$ past 6, $49\frac{1}{11}'$ past 6.
4. $10\frac{10}{11}'$ past 5, $47\frac{3}{11}'$ past 5. **5.** $25\frac{1}{11}'$, $51\frac{3}{11}'$ past 7.
6. $9\frac{9}{11}'$, 12′ past 2. **7.** 24′, or $30\frac{6}{11}'$ past 5.
8. 225 yds. **9.** 3 to 8. **10.** 5 to 13.
11. 8 of the former with 10 of the latter. **12.** 2 yds.
13. 10 yds. **14.** $8\frac{1}{3}$ yds. **15.** 24.
16. 20. **17.** 42. **18.** 176 yds.
19. In 3 min., when one has walked 330 yds. and the other 396 yds.
20. $6\frac{1}{4}$ yds. **21.** $5\frac{1}{2}$ min. **22.** *A*, 10 yds. **23.** 6 mi. per hr.
24. 440 yds., $4\frac{8}{9}$ yds. per sec., $4\frac{23}{30}$ yds. per sec.
25. 22 mi. per hr., 2 mi. per hr. **26.** 5 mi. per hr. **27.** 36 miles.

XII. e. Page 234. **1.** *A* 45 yrs., *B* 48 yrs.
2. *A* 16 yrs., *B* 25 yrs., *C* 10 yrs. **3.** Silk 6*s*., Linen 1*s*.
4. 140 at 1*s*. 6*d*., 60 at 2*s*. 6*d*. **5.** 180, 205. **6.** 40.
7. 36. **8.** 360. **9.** 55. **10.** 10 p.m.; half-way.
11. 7 miles. **12.** 15 miles from *A*'s starting point; $3\frac{1}{3}$ hrs.
13. $2\frac{1}{3}$ miles. **14.** $1\frac{3}{4}$ miles. **15.** 58 miles.

16. 60 miles, 5 p.m., 10 p.m. **17.** 2 p.m. or 3.40 p.m.
18. 2 hrs.; $+18$ must be changed to -18.
19. (i) $12'$ after A starts; 1 mile from Bath.
(ii) $42'$ after A starts; $3\frac{1}{2}$ miles from Bath. **20.** 5 miles.
21. 5 hrs. from the start. **22.** 9 parts of milk to 1 of water.
23. 6 gallons. **24.** 45 min. **25.** 2 miles an hour.
26. 12 noon; 125 miles from Bristol. **27.** 5 miles.

Misc. Ex. III. Page 237. **1.** 6*s.* 8*d.*, 6*s.* 10*d.*, 6*s.* 5*d.*
2. £11. 13*s.* 4*d.*, £16. 13*s.* 4*d.* **3.** 29,430 tons. **4.** 1*s.* 3*d.*
5. £11. 4*s.* 8*d.* **6.** 252·96. **7.** £3. 1*s.* 6*d.* **8.** 3*s.* $1\frac{3}{4}d.$
9. 17 ft.; 2 hrs. 58 min. **10.** 30 gallons, 6*s.*
11. 274·3000; 13·5. **13.** 6 yds. 2 ft. **15.** £19. 15*s.* 10*d.*
16. £26. 3*s.* 4*d.* **17.** 46·424, 6·80. **18.** $\frac{5}{12}$.
19. 21 ft. 9 in. **20.** £30. 10*s.* $3\frac{1}{4}d.$ **21.** 44 fr. 8 c.
22. £$244\frac{4}{9}$ = £244. 8*s.* 11*d.* **23.** 2.58 p.m. **24.** £3. 8*s.* 8*d.*
25. Till he is 55 yrs. old. **26.** 2 min. 15 secs. **27.** £1. 4*s.* 11*d.*
28. No: loses 8*s.* 6*d.* **29.** £48. 7*s.* 6*d.*
30. £5327. 2*s.* gained. **31.** 2·1635 Kg. and 2·1725 Kg.
32. 79900. **33.** 90 miles. **34.** C and D: by 264 yds.
35. 18 min.; $1\frac{1}{5}$ miles. 4 miles; and 42 min. from the start.

PART II.

XIII. a. Page 244. **1.** 19. **2.** 23. **3.** 29.
4. 31. **5.** 37. **6.** 43. **7.** 48. **8.** 59.
9. 58. **10.** 73. **11.** 193. **12.** 232. **13.** 140.
14. 309. **15.** 741. **16.** 708. **17.** 905. **18.** 1679.
19. 4321. **20.** 3702. **21.** 1003. **22.** 20515. **23.** 92829.
24. 34021. **26.** (i) 5345616; (ii) 105616. **27.** 4297.

XIII. b. Page 245. **1.** 3·4. **2.** 4·5. **3.** 0·79.
4. 0·17. **5.** 2·71. **6.** 6·42. **7.** 7·01.
8. 31·6. **9.** 9·99. **10.** 34·91. **11.** 2·403.
12. 0·0374. **13.** 3·083. **14.** 23·605. **15.** 0·61263.
16. 17·453. **17.** 52·004. **18.** 260·04. **20.** 0·018496.

XIII. c. Page 248. **1.** 1·732. **2.** 2·236.
3. 2·449. **4.** 3·162. **5.** 2·646. **6.** 1·581.
7. 0·686. **8.** 0·265. **9.** 0·714. **10.** 0·226.
11. 0·379. **12.** 15·349. **13.** 11·535. **14.** 4·178.
15. 17·029. **16.** 0·756. **17.** $1\frac{3}{4}$. **18.** $1\frac{1}{12}$.
19. $2\frac{5}{7}$. **20.** $\frac{19}{24}$. **21.** $6\frac{3}{29}$. **22.** $\frac{73}{309}$.
23. $\frac{109}{203}$. **24.** $33\frac{5}{9}$. **25.** 0·354. **26.** 1·183.
27. 0·845. **28.** 2·057. **29.** 1·029. **30.** 0·853.
31. 1·581. **32.** 3·953. **33.** 0·23769. **34.** 0·189736.
35. 0·60698. **36.** 1·751846.

XIII. d. Page 250. **1.** 1·1547. **2.** 6·7082.
3. 0·7071. **4.** 2·3094. **5.** 19·5959. **6.** 0·8165.
7. 3·7796. **8.** 7·5378. **9.** 0·2887. **10.** 0·2041.
11. 0·2679. **12.** 1·3660. **13.** 3·2361. **14.** 0·2679.
15. 5·0379. **16.** 22·1803. **17.** 24. **18.** 102.
19. 0·051. **20.** 2·71. **21.** 6·44.

XIII. e. Page 252. **1.** 289 yds. **2.** 105 ft
3. 219 yds. **4.** 1458 yds. **5.** 510 m., 721 m.
6. (i) 324 ; (ii) 325 ; (iii) 19·74. **7.** £38. 15*s*. 0*d*.
8. 30 min. **9.** 30·1 yds., 90·3 yds. **10.** £104. 10*s*. 0*d*.
11. 18 ft. **12.** 12 min. nearly. **13.** 6600 sq. yds. ; 81·2 yds.
14. (i) 314·16 sq. cm.; (ii) 17·7 cm. **15.** (i) 35·5 sq. in.; (ii) 43·3 sq. in.
16. 5·64 cm. **17.** 39·9 cm. **18.** 344 metres per sec.

XIV. a. Page 256. **1.** 98 sq. ft. **2.** 143 sq. ft.
3. 42 sq. ft. **4.** 20 sq. ft. **5.** 162 sq. ft. **6.** $116\frac{1}{4}$ sq. ft.
7. 99 sq. m. **8.** 47·56 sq. m. **9.** 101·6 sq. m. **10.** 54 yds.
11. 17·5 cm. **12.** 14 ft. **13.** 5 ft. 8 in.
14. 6 sq. yds. 2 sq. ft. 36 sq. in. **15.** 3 sq. yds. 5 sq. ft. 16 sq. in.
16. 1 sq. yd. 1 sq. ft. 81 sq. in. **17.** 3 sq. yds. 1 sq. ft. 64 sq. in.
18. £1. 2*s*. 2*d*. **19.** £4. 10*s*. **20.** 29 fr. 70 c.
21. £1. 9*s*. 4*d*. **22.** 6*d*. **23.** 5*s*. **24.** $16\frac{1}{2}$ ac.
25. 1·89 ac. **26.** 4 ac. 2 r. 9 p. **27.** 47 ac. 1 r. 9 p.
28. 21 ch. 25 lks. **29.** 23 ch. 45 lks. **30.** £156. 5*s*.
31. (i) 5·957 Ha., (ii) 15 ac. **32.** £21. 5*s*. ; £5.
33. 88 m. **34.** £149. 4*s*. 6*d*. **35.** 1029.
36. £3. 7*s*. 6*d*. **37.** 24 ft. 6 in. **38.** 1 ft. 3 in.
39. Length of tile must be parallel to width of floor ; 6*s*. 8*d*.
40. 2132, 41. **41.** 549 ; 540 whole, 9 divided.
42. 232 sq. yds. **43.** 19*s*. 6*d*. **44.** 3904.
45. 78 sq. yds. **46.** £49. 2*s*. 6*d*. **47.** £5. 6*s*. 1*d*.
48. 1728 tiles, 288 boards. **49.** £570. 12*s*.

XIV. b. Page 262. **1.** 40 yds. **2.** 28 yds. 1 ft.
3. 13 ft. 6 in. **4.** £20. **5.** 8 ; £9 ; $\frac{8}{9}$.
6. £4. 3*s*. 4*d*. **7.** 36 yds. ; £9. 18*s*. ; $13\frac{1}{2}$ sq. ft.
8. (i) 90 sq. yds. 6 sq. ft. ; (ii) 128 sq. yds. 3 sq. ft. **9.** £4.
10. £3. 10*s*. 6*d*. **11.** (i) 13 sq. yds. 1 sq. ft. ; (ii) 30 sq. ft. 132 sq. in.
12. 3*s*. **13.** 10. **14.** 12. **15.** £3.
16. £2. 3*s*. 3*d*. **17.** 10 ft. 6 in. **18.** 46 sq. ft.
19. 2*s*. 6*d*. **20.** 552 fr. **21.** 131 sq. ft.

XIV. c. Page 264. **1.** (i) 533 sq. ft. : (ii) 6*s*. $2\frac{1}{4}$*d*. ; (iii) 1*s*. 3*d*.
2. (i) 14·08 m. ; (ii) 46 ft. 2 in. **3.** 10·1 yds. **4.** £100. 11*s*. 8*d*.
5. £1. 12*s*. $3\frac{1}{2}$*d*. **6.** 1561·6 ac. **7.** 20 sq. cm.
8. 4 ways ; 39690, 4410, 810, 90. **9.** £1. 11*s*. 6*d*.
10. £2. 3*s*. $10\frac{1}{2}$*d*. **11.** 4 ac. 0 r. 2 p. **12.** 11 m. 73 cm.

13. 633 sq. m. **14.** £5. **15.** (i) 10·18 sq. m.; (ii) £2. 13*s*. 9*d*.
16. 628·3 sq. ft. **17.** 698 sq. yds. **18.** £7. 15*s*. 7*d*.
19. £4. 13*s*. 4*d*. **20.** £96. 6*s*. **21.** 15 sq. in.
22. 107·5 sq. ft., 65 sq. ft. **23.** 110 yds.
24. 2663 fr. 25 c. **25.** 682·6 ac. **26.** 171 yds.
27. 15 in., $12\frac{1}{2}$ in., 30 sq. ft. **28.** £62831. 4*s*.

XIV. d. Page 269.

1. (i) 15 cu. ft., (ii) 63 cu. ft., (iii) 60 cu. ft., (iv) 162 cu. ft.
2. 121·5 sq. ft., 91·125 cu. ft. **3.** 90 cu. yds.
4. 36 cu. ft. **5.** (i) 3 cu. m., (ii) 720 cu. m.
6. 4 m. **7.** 1·5 m. **8.** 150 lbs. **9.** 400 gallons.
10. 168 sq. ft. **11.** 2 ft. **12.** 1 ft. 6 in.
13. 10 sq. yds. **14.** 2836 tons. **15.** 93 tons.
16. 680625 gallons. **17.** 133·125. **18.** 3185 lbs.
19. 8640. **20.** 12 sq. ft. **21.** 12 ft.

XIV. e. Page 272.

1. 21·42 Kg.
2. (i) 18·75 gm.; (ii) 27·648 gm. **3.** 2·5 tons.
4. 1275 lbs. **5.** 3630 tons. **6.** 117·48 Kg., 258 lbs.
7. 10 cm. **8.** 218750 litres. **9.** 50 cm.
10. 203 lbs. **11.** 240 cu. in. **12.** 327 cu. in.
13. Capacity 819 cu. in.; Material 301 cu. in.
14. $11\frac{1}{4}$ hrs. **15.** 110,000 gallons. **16.** 25 cm.
17. $\frac{275}{504}$ tons; £2. 19*s*. 9*d*. **18.** 13 cu. ft.; 15438 lbs.
19. $4\frac{1}{2}$ in. **20.** 1105. **21.** 7·8.
22. 27000 blocks; 10, roughly; £11. 14*s*. 5*d*. **23.** 39 %.
24. 406 acres; 15 sq. mi. **25.** 255 days.
26. 2,374,600. **27.** 265 m.

XV. a. Page 278.

9. 36. **10.** 120.
11. (i) and (ii) 5; (iii) and (iv) 17; (v) and (vi) 37.
12. 10 mi. **13.** 13 mi. **14.** 10 mi.

XV. b. Page 280.

1. $x=5$, $y=8$. **3.** $(-5, 0)$, $(0, 10)$.
5. The three lines are parallel. **6.** A straight line through the origin.
7. A circle whose radius is 5 units, and whose centre is at the origin.

XV. c. Page 282.

1. $x=4$, $y=0$. **2.** $x=2$, $y=1$.
3. (i) 3; (ii) 1·3 nearly. **4.** 25; -5. **5.** 26 nearly; 1·28.

XV. d. Page 288. 6 27·8 and 45·3 millions.
8. 3·85 in.; 17·6 in. 9. 54·5° F.; 86·9° F. 10. 30·5; 5·9.
11. 8·1 in.; 24·375 oz. 12. £2. 12*s.*; £3. 8*s.* 13. 17*s.*; 36*s.* 6*d.*
15. 36, 32·5, 79. 16. 30·4 cm. 18 lbs.
17. P, 36700; Q, 42,000. In 1883; 45000 in each. 18. 45·96, 39·40.

XVI. a. Page 293. 1. $\frac{5}{7}$. 2. $\frac{7}{25}$. 3. $\frac{4}{15}$. 4. $\frac{3}{13}$.
5. $\frac{11}{3}$. 6. (i) 1 : 40; (ii) 7 : 80; (iii) 19 : 160; (iv) 17 : 64.
7. The second, the third; (i) 54·5 %, (ii) 63·6 %, (iii) 45·5 %.
8. (i) 35 : 32; (ii) 11 : 5; (iii) 7 : 4; (iv) 5 : 2.
9. 20 : 33. 10. 12 : 7. 11. 4 : 3. 12. 9 : 20.
13. 8 : 9. 14. 16. 15. $2\frac{1}{2}$. 16. 19·3.
17. (i) 8·8; (ii) 11·3; (iii) 7·0. 18. The first; 10 : 9.
19. $\frac{1}{1,267,200}$, 65 mi., $1\frac{3}{4}$ in.
20. $11\frac{1}{9}$ mi. to 1 in., $\frac{1}{704,000}$, 73 mi., 43 mi.

XVI. b. Page 297. 1. 45. 2. 7·65.
3. 24. 4. £2. 10*s.* 5. £3. 17*s.* 6. 1·05 Kg.
7. 221. 8. $\frac{21}{20}$. 9. 5·175. 10. 1.
11. 4·5. 12. $9\frac{1}{3}$. 13. 0·126. 14. $\frac{b^2}{a}$.
15. 21. 16. $\frac{2}{5}$. 17. 0·91. 18. a^2b^2.
19. £45. 20. 252 m. 21. 9·24 tons. 22. 945 mi.
23. £5. 8*s.* 24. 2750 m. 25. 132 fr. 26. 0·061.
27. 1018 Kg. 28. 2667 m. 29. 219 gals. 30. 449·8 lbs.

XVI. c. Page 300. 1. (i) £22; (ii) 9 tons; (iii) £9: (iv) 1·19 Km.; (v) £16. 5*s.*; (vi) 1 lb. 14 oz.
2. £132. 3. 333 mi. 4. 9 in. 5. 39 days.
6. 324 hrs. 7. $7\frac{1}{2}$ min. 8. £2. 12*s.* 6*d.* 9. 60 yds.
10. 25 knots. 11. 3 min. 12. (i) £58. 16*s.*; (ii) $8\frac{1}{4}$*d.* per hr.
13. (i) 44 mi.; (ii) 4 hrs. 12 min.; (iii) $21\frac{1}{3}$ mi. per hr. 14. £1.
15. 132 Kg. 16. £187. 10*s.* 17. 840 fr. 18. 31 cwt.
19. $54\frac{2}{3}$ mi.; $51\frac{1}{4}$ mi. per hr. 20. 131·22 oz. 21. 136·5 m.
22. 34·55 m. 23. 296. 24. 170. 25. 0·40 oz.
26. 20·2 mi. 27. 2 mi. to 1 in.; 3·8 in.; 5·1 mi.
28. 15·8 mi. to 1 in., $\frac{1}{1,001,088}$, 109 mi.
29. 710 lit., 710 Kg. 30. 8 min.

XVI. d. Page 303. 1. £263. 5*s.* 2. £96. 5*s.* 7*d.*
3. £121. 2*s.* 4. 18 cwt. 3 qr. 6 lbs. 5. 65 mi.

6. £3. 3*s.* **7.** 2 hrs. **8.** 40. **9.** 49.
10. 4. **11.** 33 mi. **12.** £1. 2*s.* 2*d.* **13.** 18 cwt. 80 lbs.
14. 2 tons 8 cwt. 0 qr. 24 lbs. **15.** £40. 5*s.* **16.** 15*s.*
17. 4*s.* 7*d.* **18.** £85. 8*s.* 4*d.* **19.** £105. 15*s.* 9*d.*
20. £937. 10*s.*; £37. 10*s.* **22.** £1511. 7*s.* 8*d.* **23.** £490. 10*s.* 10*d.*
24. £1424. **25.** £82. 10*s.* **26.** £2500.
27. £745. 1*s.* 3*d.* **28.** £524. 5*s.* **29.** £640.
30. £580. 13*s.* 4*d.*; £563. 7*s.* 10*d.* **31.** £528. 10*s.*
32. 3·5 in.; 15 mi. to 1 in.; $\frac{1}{950400}$. **33.** £176. 4*s.*
34. 3 hrs. **35.** £20. 8*s.* 6*d.* **36.** £1233.
37. 825 gm. **38.** £999. **39.** No gain or loss.
40. 1·4 Kg. **41.** 129646 lit. **42.** 0·78 cu. in.

XVI. e. Page 313. **7.** (i) 29*s.*; (ii) 14·4 dollars.
8. (i) 11·4 litres: (ii) 4·6 gals. **10.** (i) 54·1 grains; (ii) 0·2 gram.
11. 2300 yds. **12.** 50·25 cm.; 12·7 cm.
13. 7*s.* 8*d.*, 22*s.*, 53*s.*; 61 days. **14.** 27·3, 44, 53·2 mi. per hour.
15. £2, £2. 8*s.*, £3. 4*s.*, £3. 12*s.* **17.** 1*s.* 5*d.*, 2*s.*, 3*s.* 8*d.*; 5 hrs.
18. 90, 72.

XVI. f. Page 316. **1.** 320 sq. ft. **2.** $3\frac{1}{2}$ cwt.
3. 45 mi. **4.** £12. **5.** $\frac{3}{4}$ ton. **6.** $1\frac{1}{4}$ lbs.
7. 12 wks. **8.** £270. **9.** £11. 5*s.* **10.** 10.
11. £192. **12.** 24. **13.** 11 men. **14.** 60 men.
15. 26 days. **16.** $28\frac{1}{3}$ days. **17.** £100. **18.** 13 wks.
19. £39. 7*s.* 6*d.* **20.** 56. **21.** (i) 5·6; (ii) 11·2; (iii) 40.
22. 1000 men. **23.** 28 days. **24.** 3*s.* 6*d.* **25.** 33·6.
26. $5\frac{1}{4}$ hrs. **27.** 34 mi. an hr. **28.** English; £8000.

XVI. g. Page 320. **1.** £24, £56. **2.** 114 mi., 209 mi.
3. 90 tons, $112\frac{1}{2}$ tons, $247\frac{1}{2}$ tons. **4.** £119, £136, £153.
5. £5. 8*s.*, £6. 15*s.*, £8. 2*s.* **6.** 60·6 yds., 90·9 yds., 151·5 yds.
7. 25 Kg., 45 Kg., 50 Kg., 60 Kg. **8.** £550, £880, £990.
9. £30, £20, £15. **10.** 176 tons, 198 tons, 55 tons.
11. £3. 7*s.* 6*d.*, £5. 8*s.*, £8. 2*s.* **12.** £15, £20, £22. 10*s.*, £24.
13. 1·60 Km., 1·25 Km., 2·15 Km. **14.** 139 gm., 324 gm., 537 gm.
15. 26·0 m., 43·3 m., 50·7 m. **16.** 67·5 yds., 94·5 yds., 108 yds.
17. 30, 48, 72. **18.** $1\frac{1}{4}$ tons; $18\frac{3}{4}$ %, 50 %, $31\frac{1}{4}$ %.
19. 153 m., 102 m., 34 m. **20.** 88 ft., 120 ft., 130 ft.
21. £33. 2*s.*, £41. 7*s.* 6*d.*, £91. 0*s.* 6*d.* **22.** £40, £24, £16.
23. *A* 25 %, *B* $33\frac{1}{3}$ %, *C* $41\frac{2}{3}$ %; £625.

24. £1275, £1575, £2250; 25 %.
25. £31. 5*s.*, £41. 5*s.*, £52. 10*s.*; 2*s.* 6*d.*
26. 17*s.*, 13*s.*, 10*s.*
27. 33900 grs.
28. 864 gm.
29. 298 gm.
30. 3·53 Kg.
31. *A* £42. 7*s.*, *B* £18. 3*s.*, *C* £12. 2*s.*
32. *A* £1. 2*s.* 6*d.*, *B* £1. 16*s.*, *C* £1. 4*s.*
33. *A* £5. 16*s.* 8*d.*, *B* £2. 6*s.* 8*d.*, *C* £4. 6*s.* 8*d.*
34. *A* £42, *B* £56, *C* £48.
35. 29 : 212 : 19.
36. *A* £170. 13*s.* 4*d.*, *B* £153. 12*s.*, *C* £59. 14*s.* 8*d.*
37. £75, £85.
38. £202. 10*s.*, £225, £281. 5*s.*
39. *A* £930, *B* £46. 10*s.*, *C* £581. 5*s.*
40. *A* £336, *B* £420, *C* £210.
41. 6 oz., 9 oz.
42. 42 lbs., 16 lbs., 20 lbs.
43. £486. 13*s.* 4*d.*, £730, £2433. 6*s.* 8*d.*
44. 49, 56, 105.
45. 7 : 5 : 4; 1·4 lit., 1 lit., 0·8 lit.

XVI. h. Page 326.

1. 4·5 in.
2. 42 ft.
3. 250 sq. cm.
4. 22·0 cm.; 153·9 sq. cm.
5. 10,125 sq. yds.
6. 2587 sq. mi.
7. (i) 16 ft.; (ii) 258 ft., $s=16{\cdot}1\times t^2$.
8. 0·41 lb., 11·07 lbs.
9. 9 cm., 6 cm., 4·725 Kg.
10. 2·5 lbs., 56 lbs.
11. 78.
12. 302·4 cu. in.
13. The second.
14. 10*d.*
15. £2700.
16. 0·01538 gm.; 13·5 %.
17. $2\frac{1}{2}$ in.
18. $y=\frac{1}{4}x^2$. 7·56, 10·56.
19.

C (in inches)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2
S (in tons)	0·1	0·4	0·9	1·6	2·5	3·6	4·9	6·4

20. (i) 1·62 lbs. per sq. ft.; (ii) 28 mi. per hr.

Miscellaneous Examples IV. Page 330.

A. **1.** (i) $1\frac{1}{7}$; (ii) $\frac{1}{5}$.
2. £37. 5*s.* $5\frac{3}{4}$*d.*
3. The train.
4. 10·56 in.
5. 2757 tiles, 3344 sq. in.
6. $3^3 . 7 . 11 . 13 . 37$.

B. **7.** £18. 13*s.* 11*d.*
8. $1\frac{2}{5}$.
9. £7. 11*s.* 4*d.*
10. 391 fr.
11. 15*s.* 1*d.*
12. 114 days.

C. **13.** 39.
14. 0·28845.
15. 13 ac.; £253. 10*s.*
16. 0·727.
17. 16511 Kg.
18. 700 yds.

D. **19.** Lighter.
20. £39. 19*s.* $9\frac{3}{4}$*d.*
21. $29172\times 17^2=8430708$.
22. 15*d.* and 10*d.*
23. £16.
24. *B* £35, *C* £23.

E. **25.** £5500. **26.** 18 days, 12 days.
27. 0·251875 ; 0·251$\frac{7}{8}$. **28.** 50 %, 16 %, 13 %.
30. First 5*s.*, Second 3*s.* 6*d.*

F. **31.** (i) $\frac{3}{4}$; (ii) 1. **32.** 384.
33. Man 5*s.*, Boy 2*s.* 6*d.* **34.** £2. 11*s.* 10$\frac{1}{4}$*d.*
35. £7475. 10*s.* **36.** 113 grains.

G. **37.** 3^2, 7, 11, 17, and 2^3, 7, 13, 17. H.C.F. = 119.
38. 6·8 oz. **39.** 5 fr. 44 c. **40.** 17*s.* 2*d.*
41. 57 nearly. **42.** 363 men.

H. **43.** 10 Kg., 8 Kg. ; 11 lbs., 38 lbs.
44. *A* £173. 7*s.* 9*d.*, *B* £140. 10*s.* 6*d.*, *C* £96. 5*s.*
45. 9. **46.** 1123. **47.** £187. 5*s.* 7*d.* **48.** 30.

I. **49.** 18 sq. in. **50.** £1. 2*s.* 3*d.*, £1. 16*s.*, £1. 19*s.* 5*d.*
51. 21 days. **52.** 2*s.* **53.** 60 mi. per hr. **54.** £15. 5*s.* 4*d.*

J. **55.** 4·735, 12·3125, 5·1325, 7·82 ; 30. **56.** (i) $\frac{2}{9}$; (ii) $\frac{8}{39}$.
57. 132 sq. yds. 4 sq. ft. **58.** 22$\frac{1}{2}$ weeks.
59. 2·11 Kg., 4 Kg. per sq. cm. ; 32. **60.** £20.

K. **61.** £294. 7*s.* 3$\frac{1}{2}$*d.* **62.** 7. **63.** 210·00004.
64. (i) 3 mi. per hr. ; (ii) 4$\frac{1}{2}$ hrs. **65.** 51 litres.
66. 3 mins. 31$\frac{1}{5}$ secs.

L. **67.** 3$\frac{1}{2}$*d.* **68.** £134. 2*s.* 8*d.* **69.** £1600.
70. 25 in. to 1 mile, 62·5 sq. in. **72.** 4*s.* 10*d.*

M. **73.** £235. 5*s.* 6*d.* **74.** 10 days.
75. *A* £65. 8*s.* 9*d.*, *B* £196. 6*s.* 3*d.*, *C* £261. 15*s.*
76. 6·3 secs., 31·5 secs. **77.** 1 ft. 2 in. **78.** 52 ; 2*s.* 4*d.*, 5*s.*

N. **79.** 2·2 lbs. **80.** 31 oz. **81.** 1$\frac{1}{3}$ miles an hour.
82. 20 miles. **83.** 88 yds. ; 22 mi. per hr.
84. 2·83, 4·95, 12·16, 5·5.

O. **85.** 123 ; $93 = 105 - 15 + 3$. **86.** 4 ft. **87.** 138.
88. 3*s.* 2$\frac{3}{4}$*d.* **89.** 1 m. 85 cm. ; 1·815 m. and 1·885 m.
90. (i) 44 sq. ft. ; (ii) 164 gm.

P. **91.** 54. **92.** (i) £43. 10*s.* ; (ii) £4. 6*s.* 7$\frac{1}{2}$*d.* ; (iii) 24 mi. 1265 yds.
93. £2. 12*s.* 8*d.* **94.** 1 in. to 22 miles. **95.** 2 hrs.
96. 6 p.m., 48 mi. from London. At 4 and 8 p.m.
(i) *B* 4 mi. behind *A* ; *C* 6 mi. behind *B*. (ii) 4.20 p.m.

XVII. a. Page 340.

1. (i) 28·6 %; (ii) 15·4 %; (iii) 29·4 %; (iv) 52·4 %; (v) 53·3 %.
2. (i) $\frac{1}{30}$; (ii) $\frac{3}{40}$; (iii) $\frac{1}{8}$; (iv) $\frac{1}{3}$. **3.** 14 %.
4. $28\frac{3}{4}$ %. **5.** 25 %. **6.** $1\frac{1}{4}$ %. **7.** £1. 8*s*.
8. £3. 12*s*. **9.** £15. 15*s*. **10.** £41. 8*s*. **11.** £41. 2*s*. 6*d*.
12. 118·86 fr. **13.** 20 %. **14.** 16 %. **15.** 8 %.
16. $4\frac{1}{4}$ %. **17.** 14·0 %. **18.** 7·66 %. **19.** 60·2 %.
20. 3·31 %. **21.** 6·54 %. **22.** £22. 18*s*. 5*d*.
23. £79. 0*s*. 7*d*. **24.** £29. 10*s*. 10*d*. **25.** £2. 10*s*.
26. £6. 0*s*. 6*d*. **27.** £46. 5*s*. 1*d*. **28.** (i) £9. 1*s*. 8*d*.; (ii) £86. 9*s*. 10*d*.
29. £12. 10*s*. **30.** 0·69 %. **31.** 15·2 %. **32.** 268; 5·7 %.

XVII. b. Page 343.

1. £240. **2.** 13,272.
3. 57,000. **4.** 108,290. **5.** 8*s*. 3*d*. **6.** 14*s*. 7*d*.
7. 142,800. **8.** £560. **9.** £8000. **10.** £12,562. 10*s*.
11. 172·425 Km. **12.** 3·71 %. **14.** 4 %.
15. Olives, 35 lbs. per ac.; 8 %. Currants, 2083 lbs. per ac.; 3 %.
Figs, 1154 lbs. per ac.; 1 %.
16. 21 %, 27 %, 10 %, 42 %. **17.** 67 %, 66 %, 65 %; 66 %.
18. The former. **19.** 15 %, 15 %, 11 %.

XVII. c. Page 346.

1. 20 % gain. **2.** $11\frac{1}{9}$ % loss.
3. 10 % loss. **4.** $12\frac{1}{2}$ % gain. **5.** 20 % gain.
6. £6. **7.** 6*s*. 2*d*. **8.** 67 fr. 50 c.
9. £10. 9*s*. 3*d*. **10.** 25 %. **11.** $22\frac{1}{2}$ %.
12. £72. **13.** 9*s*. 6*d*. **14.** 4*s*.
15. (i) 8·4 % gain; (ii) 13·7 % loss; (iii) 12·5 % gain.
16. 12 % gain. **17.** 11*s*. 8*d*. **18.** 15 % gain.
19. 21 % gain. **20.** 20 % gain. **21.** 8*s*. 4*d*.
22. 12*s*. 6*d*. **23.** £16. **24.** £72. 10*s*.
25. £52. 10*s*. **26.** 1*s*. $0\frac{1}{2}$*d*. **27.** £18. 13*s*. 4*d*.
28. 3*s*. 4*d*.; £28. 16*s*. **29.** 2*s*.

XVII. d. Page 349.

1. £440. **2.** 21*s*.
3. £1. 7*s*. **4.** 10*s*. 10*d*. **5.** 20 % loss.
6. 7 % loss. **7.** 5 % lost. **8.** 9*s*. 2*d*.
9. 2*s*. 6*d*. **10.** 8 %. **11.** 29·6 % gain.
12. 12 %. **13.** £415. **14.** £37. 10*s*.
15. £33. 6*s*. 8*d*. **16.** £3. 2*s*. 6*d*. **17.** 14*s*. 8*d*.
18. 48*s*. **19.** £34. 13*s*. **20.** £4. 10*s*.
21. £2100. **22.** £4. 3*s*. 4*d*. **23.** £5.

24. 4*s*. 9*d*. **25.** £2. 10*s*.; 25 %. **26.** 10 %.
27. 5 %. **28.** 20 %.
29. (i) 8*s*. 3*d*.; (ii) £1. 17*s*. $1\frac{1}{2}$*d*.; (iii) £10; (iv) $48\frac{1}{2}$ %.

XVII. e. Page 352. **1.** 80,000. Births 8800, Deaths 3200.
2. 12 %. **3.** 25 %. **4.** £1036. 16*s*. **5.** 5.
6. £393. 15*s*. **7.** £134. 5*s*. **8.** 88 %. **9.** $57\frac{1}{2}$ %.
10. $33\frac{1}{3}$ %. **11.** 5 %. **12.** 25 %. **13.** £1. 13*s*. 4*d*.
14. 2 : 3. **15.** 11 : 80. **16.** 66 %.
17. £14·6 millions, 71·7 millions. **18.** £24. **19.** $\frac{1}{3}$ of a quart.
20. £10. **21.** £50. **22.** £1. 10*s*. 10*d*.
23. £1. 2*s*. 2*d*. **24.** In equal quantities. **25.** £18.
26. 19 %, 77 %. **27.** 28·6 %, 26·0 %. **28.** 8·2 %, £20.

XVIII. a. Page 357. **1.** £36. **2.** £90. **3.** £420.
4. £84. **5.** £40. 10*s*. **6.** £405. **7.** £15. 15*s*.
8. £38. 5*s*. **9.** £234. **10.** £118. 2*s*. **11.** £117. 10*s*.
12. £38. 5*s*. **13.** £75. 15*s*. 9*d*. **14.** £48. 18*s*.
15. £404. 10*s*. 3*d*. **16.** £61. 13*s*. 11*d*. **17.** £538. 6*s*. 8*d*.
18. £44. 5*s*. 5*d*. **19.** £313. 4*s*. **20.** £228. 9*s*. 4*d*.
21. £8. 3*s*. 3*d*. **22.** £166. 6*s*. 6*d*. **23.** £271. 10*s*. 10*d*.
24. £1953. 5*s*. 8*d*. **25.** £1284. 15*s*. 11*d*. **26.** £10. 6*s*. 8*d*.
27. $12,34. **28.** £449. 3*s*. 4*d*. **29.** fr. 6350,65.

XVIII. b. Page 360. **1.** £761. 5*s*. **2.** £1853. 7*s*.
3. £1748. 5*s*. **4.** £253. 11*s*. 11*d*. **5.** £1702. 16*s*.
6. £869. 8*s*. 5*d*. **7.** £431. 12*s*. 4*d*. **8.** £222. 18*s*. 5*d*.
9. £385. 9*s*. 2*d*. **10.** £791. 10*s*. 9*d*. **11.** £1. 8*s*. 7*d*.
12. £2. 0*s*. 6*d*. **13.** £65. 19*s*. 6*d*. **14.** £7. 13*s*. 5*d*.
15. £10. 7*s*. 5*d*. **16.** £415. 18*s*. 10*d*. **17.** £513. 7*s*. 1*d*.
18. £2473. 14*s*. 7*d*. **19.** £1406. 10*s*. 3*d*. **20.** £407. 9*s*.
21. £1268. 15*s*. **22.** £204. 13*s*. 2*d*., £301. 18*s*. 8*d*.
23. £632. 19*s*. 10*d*.

XVIII. c. Page 362. **1.** £300. **2.** £450. **3.** £2700.
4. £250. **5.** £126. 13*s*. 4*d*. **6.** £137. 10*s*. **7.** £443. 6*s*. 8*d*.
8. £75. **9.** £1090. **10.** £800. **11.** £750.
12. £700. **13.** £5400. **14.** £485. **15.** £200.
16. £236. 6*s*. 8*d*. **17.** £1351. **18.** £237. **19.** £1076.
20. £310. 16*s*. **21.** £1778. **22.** 10 yrs. **23.** 5 yrs.
24. 8 yrs. **25.** $4\frac{1}{2}$ yrs. **26.** $3\frac{3}{4}$ yrs. **27.** 4 yrs.
28. 8 yrs. **29.** 4 yrs. **30.** $2\frac{1}{2}$ yrs. **31.** 103 days.

32. 100 days. **33.** 112 days. **34.** 3 %. **35.** 6 %.
36. $3\frac{1}{2}$ %. **37.** 5 %. **38.** $4\frac{1}{2}$ %. **39.** $3\frac{1}{2}$ %.
40. $2\frac{1}{2}$ %. **41.** $4\frac{1}{2}$ %. **42.** 4 %. **43.** 8 %.
44. 3·55 %. **45.** 6·76 %. **46.** £2057.
47. (i) £236. 7*s.*; (ii) £55. 6*s.* **48.** 240 days.
49. Jan. 9th. **50.** £1500.

XVIII. d. Page 367.

1. £20. 4*s.*
2. £92. 6*s.* $6\frac{1}{2}$*d.* **3.** £28. 12*s.* 8*d.* **4.** £6. 7*s.* 6*d.*
5. £5. 10*s.* 5*d.* **6.** £51. 18*s.* 4*d.* **7.** £12. 3*s.* 3*d.*
8. £14. 12*s.* 2*d.* **9.** £14. 13*s.* 11*d.* **10.** £1. 9*s.* 4*d.*
11. £23. 2*s.* 8*d.* **12.** £13. 9*s.* 2*d.* **13.** £39. 9*s.* 6*d.*
14. £37. 5*s.* 8*d.* **15.** £27. 15*s.* **16.** £18. 10*s.*

XVIII. e. Page 369.

1. £3. 0*s.* 4*d.* **2.** £28. 11*s.* 8*d.*
3. £4. 2*s.* 8*d.* **4.** £3. 6*s.* 4*d.* **5.** £11. 10*s.* 11*d.*
6. £313. 13*s.* 1*d.* **7.** £1899. 7*s.* 8*d.* **8.** £3646. 5*s.* 11*d.*
9. £737. 5*s.* 3*d.* **10.** £1024. 13*s.* 9*d.* **11.** 4 %. **12.** $2\frac{1}{2}$ %.
13. 9 months. **14.** 4 months. **15.** 5 %. **16.** 10·26 %.

XVIII. f. Page 371.

1. £220. 10*s.* **2.** £2704.
3. £3978. 7*s.* 6*d.* **4.** £4051. 13*s.* 9*d.* **5.** £811. 16*s.* 6*d.*
6. £4011. 3*s.* 5*d.* **7.** £18. 7*s.* 3*d.* **8.** £53. 1*s.* 4*d.*
9. £90. 19*s.* 11*d.* **10.** £558. 18*s.* 9*d.* **11.** £373. 19*s.* 3*d.*
12. £516. 15*s.* **13.** £1371. 3*s.* 10*d.* **14.** $533,04.
15. £5384. 9*s.* 1*d.* **16.** £4890. 12*s.* 7*d.* **17.** £5266. 8*s.* 2*d.*
18. £7702. 17*s.* 5*d.* **19.** £5478. 4*s.* 6*d.* **20.** £1762. 9*s.* 8*d.*
21. £260. 2*s* 6*d.* **22.** £580. 3*s.* 3*d.* **23.** £325. 5*s.* 4*d.*
24. £239. 17*s.* 2*d.* **25.** £54. 14*s.* 3*d.* **26.** £340. 7*s.* 8*d.*
27. £574. 17*s.* 4*d.* **28.** £17,104. 9*s.* 8*d.* **29.** £106,408. 4*s.* 4*d*
30. £262. 14*s.* 9*d.* **31.** (i) 10·250 %; (ii) 5·095 %; (iii) 2·524 %.

XVIII. g. Page 375.

1. £410. 11*s.* 4*d.* **2.** £982. 9*s.* 1*d.*
3. £1104. 2*s.* 3*d.* **4.** £279. **5.** £312.
6. £3000. **7.** £4750. **8.** £3300.
9. £250. **10.** £266. 14*s.* **11.** £770. 19*s.* 6*d.*
12. £1830. 5*s.* 8*d.* **13.** £5600. **14.** £3600.
15. £280. 16*s.* **16.** £19,512. **17.** £2500.
19. £317. **20.** 33,600; 30,000. **21.** 374,600.
22. £15. 4*s.* **23.** £9. 17*s.* 2*d.* **24.** £18. 6*s.*
25. £4. 3*s.* 3*d.* **26.** £18. 7*s.* **27.** £240.
28. £3125. **29.** £3750. **30.** £5000.
31. £14,283. **32.** £22,103. **33.** £27. 11*s.* 11*d.* **34.** £1407.

XVIII. h. Page 380.

1. (i) £3·8; (ii) 16*s.*; (iii) 15; (iv) 32; (v) £100.
2. (i) £355; (ii) 33 years. **3.** £2. 2*s.*, 10*s.* 6*d.*; £1, £1. 8*s.*
4. Each Principal amounts to £450 in 25 years. £200; £300.

XIX. a. Page 383.

1. £2632. **2.** £3825.
3. £3811. **4.** £4230. 9*s.* **5.** £4125. 12*s.* 6*d.*
6. £485. 12*s.* 6*d.* **7.** £400. **8.** £2660.
9. £4000. **10.** £1600. **11.** £600.
12. £5600. **13.** £2736. **14.** 92.
15. 80. **16.** £714. **17.** £976.
18. 86. **19.** £2751. 16*s.* 10*d.* **20.** £7256. 8*s.* 2*d.*
21. £3911. 6*s.* 10*d.* **22.** £39. **23.** £21. 2*s.* 6*d.*
24. £760. **25.** £442. **26.** £137. 10*s.*
27. £37. 10*s.* **28.** £213. 18*s.* **29.** £50. 18*s.* 4*d.*

XIX. b. Page 385.

1. £102. **2.** £95. 12*s.* 6*d.*
3. £128. 12*s.* 6*d.* **4.** £44. **5.** £160.
6. £45. **7.** The 2nd by £10. 10*s.* **8.** £76.
9. £6. 5*s.* **10.** £21. 12*s.* 4*d.* **11.** £113. 16*s.* 9*d.*
12. £15. 3*s.* 11*d.* **13.** £2700. **14.** £6600.
15. £1200. **16.** £420. **17.** £1694.
18. £960. **19.** £1414. 10*s.* **20.** £6933.
21. £2414. **22.** 5%. **23.** 3%.
24. 3·79. **25.** 4·44. **26.** 3·35.
27. The 2nd by 0·64%. **28.** The 2nd by 0·07%.
29. The 1st by 0·12%. **30.** The 2nd; £1.
31. £1512 in each. **32.** £1000. **33.** 80.
34. 110. **35.** 75. **36.** 96. **37.** 110.
38. $135\frac{3}{4}$. **39.** 115. **40.** £480. **41.** £2007.

XIX. c. Page 391.

1. £7158. 3*s.* 6*d.* **2.** £2502. 10*s.*
3. £3500. **4.** £7350. **5.** £7256. 8*s.* 2*d.*
6. £3. 15*s.* **7.** £158. 6*s.* 8*d.* **8.** £126.
9. £120. **10.** £12,800. **11.** £7090.
12. £18,319. **13.** $94\frac{3}{4}$. **14.** $92\frac{3}{8}$.
15. £12. **16.** Gain of £5. 10*s.* **17.** Loss of £12.
18. £5. **19.** £60. **20.** Gain of £112. 11*s.* 6*d.*
21. £477. 7*s.* 6*d.* **22.** £1207. 5*s.* 2*d.* **23.** £12. 14*s.* 6*d.*
24. £951. 1*s.* 11*d.* **25.** £29. 12*s.* 8*d.* **26.** £419. 10*s.*
27. £1142. 10*s.* 6*d.*

XIX. d. Page 395. **1.** £1937. 10*s.* **2.** £1128. 15*s.*
3. £635. 6*s.* 6*d.* **4.** £875. **5.** £678. 2*s.* 6*d.*
6. 20. **7.** (i) £15. 15*s.*; (ii) £15. 5*s.* 6*d.*; (iii) £86. 5*s.*
8. £5. 15*s.* **9.** (i) £6; (ii) £5. 6*s.* 8*d.*; (iii) £4. 14*s.* 5*d.*
10. 50; £150; £20. 12*s.* 6*d.* **11.** (i) 56; (ii) £389. 5*s.*; (iii) £26. 5*s.*
12. £133. 6*s.* 8*d.*

XIX. e. Page 396. **1.** $95\frac{1}{2}$. **2.** £34. 9*s.* 4*d.*
3. The first by 12*s.* 6*d.* **4.** $96\frac{3}{4}$. **5.** $174\frac{3}{4}$.
6. £29. 3*s.* 4*d.*; £25. **7.** $103\frac{1}{2}$; £51. **8.** $3\frac{1}{8}$.
9. £135. 7*s.* 10*d.* **10.** £1000. **11.** £3240.
12. Gain of £10. 16*s.* **13.** The second by £1. 7*s.* 6*d.*
14. £5480. 15*s.* 11*d.* **15.** £16,250. **16.** £4725.
17. £1600. **18.** 2 %. **19.** 1*s.* **20.** Gain of £5.
21. 155 : 147. **22.** 10*d.* **23.** Increase of £36. 3*s.* 11*d.*
24. £1785. 15*s.* 9*d.* **25.** 3000. **26.** £489. 15*s.* 5*d.*
27. £20,643; 3·08 %. **28.** £248. 8*s.*; 9·6 %.

Miscellaneous Examples V. Page 400.
A. **1.** 1016 Kg. **2.** £12. 9*s.* 3*d.* **3.** $\frac{2}{3}$.
4. 20 ft. **5.** £95. 16*s.* 8*d.* **6.** £225.

B. **7.** 90 ac. **8.** (i) 10*d.*; (ii) $4\frac{1}{6}$ %. **9.** 168 lbs.
10. 0·39 sq. mm. **11.** 6*s.* 2*d.*; 20·3. **12.** £121. 4*s.* 4*d.*
C. **13.** 0·17, 17 %. **14.** 3·011760. **15.** $32\frac{1}{2}$ %.
16. $\frac{1}{2}$. **17.** $5\frac{5}{8}$ hrs., $4\frac{1}{2}$ hrs., $3\frac{3}{4}$ hrs. **18.** 65 %.
D. **19.** 1100. **20.** 150 yds. **21.** £6. 13*s.* 4*d.*
22. 539 fr. **23.** £30 loss. **24.** £678 at 7 %; £452 at 2 %.

E. **25.** $13\frac{25}{72}$; $12\frac{2}{5}$. **26.** $\frac{7}{13}$. **27.** £1831. 4*s.*
28. £15. **29.** 16*s.* **30.** 20,200.

F. **31.** 44 %. **32.** 3·13 acres, nearly. **33.** £375.
34. £392. 1*s.* 2*d.* **35.** 12; $27\frac{1}{2}$ %.
36. Man 3*s.* 6*d.*, woman 2*s.*, boy 1*s.* 6*d.*

G. **37.** £560. 6*s.* 8*d.*, 474·1 cu. ft. **38.** $28\frac{3}{4}$ lbs.
39. £137. 10*s.* less. **40.** £266. 13*s.* 4*d.* **41.** £2. 2*s.*
42. £810 at 81, £1500 at 135.

H. **43.** 1·8814. **44.** $\frac{1}{5}$. **45.** 70 gm.
46. 24 gals. of 1st with 60 of 2nd. **47.** 4 hrs. 55 min. 2 secs.
48. £262. 10*s.*

I. **49.** 5 %. **50.** £6,074,000 ; 3*s*. $0\frac{1}{2}$*d*. **51.** 36. **52.** £684.
53. £4. 15*s*. $4\frac{1}{2}$*d*. **54.** *A* 2*s*. 8*d*., *B* 3*s*. 4*d*., *C* 4*s*.

J. **55.** (i) £319. (ii) 15,600. (iii) £30. (iv) The former.
56. 12 days. **57.** 25 yrs. **58.** 1 ft. 9 in.
59. 54. **60.** 88 yds.

K. **61.** 125. **62.** 38 Kg. **63.** £28. 9*s*.; £27. 5*s*. $3\frac{1}{2}$*d*.
64. £600 at 3 % ; £120 at 5 %. **65.** 50 %. **66.** £1. 6*s*. 8*d*.

L. **67.** (i) 1, (ii) $\frac{1}{123}$.
68. (i) 3 mi. per hr., (ii) 4·4 ft. per sec. 5 mi. an hr.
69. £5. **70.** £1,000,000.
71. 500 : 441. 2·2 quarts ; 5·3 litres. **72.** £532,000, £8000.

M. **73.** £23. 14*s*. 1*d*. **74.** £22,720. **75.** 972 sq. yds., 7·56 sq. ft.
76. 5·41 in. **77.** 8*s*. 4*d*. **78.** 5 %, £1613. 8*s*. 6*d*.

N. **79.** 3·8 yrs. **80.** 4·51 fr. **81.** $1\frac{1}{2}$ in.
82. 3·02 %. **83.** 44 yds. **84.** $2\frac{1}{2}$ %, £4500.

O. **86.** £4531. **87.** $2\frac{1}{2}$ in. **88.** $11\frac{1}{4}$ mi.
89. 15 %. **90.** £7150. **91.** 18, 40, 51.
92. £2. 16*s*.

P. **93.** £12. 3*s*. 8*d*. **94.** 6·77 fr. **95.** $7\frac{13}{16}$ days.
96. 3*s*. 4*d*.; 60. **97.** £4. 0*s*. 9*d*. **98.** £620.
99. 54·2 cu. mm. **100.** $33\frac{1}{3}$ cwt., $18\frac{3}{4}$ cwt., $14\frac{2}{7}$ cwt., $12\frac{1}{2}$ cwt.

XX. a. Page 410. **1.** $\frac{3}{8}$. **2.** $2\frac{1}{41}$. **3.** $4\frac{1}{4}$. **4.** $2\frac{7}{11}$.
5. $2\frac{1}{6}$. **6.** 4. **7.** $6\frac{17}{24}$. **8.** $3\frac{1}{4}$. **9.** $4\frac{3}{11}$.
10. $\frac{2}{5}$. **11.** 18. **12.** 1. **13.** $1\frac{5}{14}$. **14.** $1\frac{5}{13}$.

XX. b. Page 414. **1.** $\frac{2}{1}, \frac{7}{3}, \frac{9}{4}, \frac{25}{11}, \frac{134}{59}$.
2. $\frac{3}{1}, \frac{13}{4}, \frac{29}{9}, \frac{42}{13}, \frac{155}{48}, \frac{817}{253}$.
3. $\frac{1}{2}, \frac{2}{5}, \frac{7}{17}, \frac{9}{22}, \frac{43}{105}, \frac{95}{232}, \frac{613}{1497}$.
4. $\frac{1}{3+}\,\frac{1}{1+}\,\frac{1}{2+}\,\frac{1}{6+}\,\frac{1}{4}; \frac{19}{70}$. **5.** $\frac{1}{6+}\,\frac{1}{2+}\,\frac{1}{3+}\,\frac{1}{1+}\,\frac{1}{10}; \frac{9}{58}$.
6. $1+\frac{1}{2+}\,\frac{1}{1+}\,\frac{1}{10+}\,\frac{1}{2+}\,\frac{1}{3}; \frac{43}{32}$. **7.** $3+\frac{1}{4+}\,\frac{1}{3+}\,\frac{1}{4+}\,\frac{1}{6}; \frac{181}{56}$.
8. $1+\frac{1}{7+}\,\frac{1}{5+}\,\frac{1}{6+}\,\frac{1}{1+}\,\frac{1}{3}; \frac{254}{223}$.

9. $1+\frac{1}{2+}\,\frac{1}{2+}\,\frac{1}{2+}\,\frac{1}{2+}\,\frac{1}{5+}\,\frac{1}{3}$; $\frac{17}{12}$.

10. $\frac{1}{2+}\,\frac{1}{1+}\,\frac{1}{1+}\,\frac{1}{5+}\,\frac{1}{8+}\,\frac{1}{1+}\,\frac{1}{3}$; $\frac{11}{28}$.

11. $\frac{1}{3+}\,\frac{1}{3+}\,\frac{1}{3+}\,\frac{1}{6+}\,\frac{1}{1+}\,\frac{1}{2+}\,\frac{1}{1+}\,\frac{1}{10}$; $\frac{63}{208}$.

16. $\frac{151}{115}$. **17.** $\frac{1384}{423}$. **18.** $\frac{3}{1}$, $\frac{19}{6}$, $\frac{171}{37}$, $\frac{721}{228}$, ...
20. $\frac{52525}{10301}$. **21.** 4854.

XX. c. Page 417. **1.** (i) 8649; (ii) 1726.
2. 12193·46 sq. m. **3.** 93 m. **4.** (i) 4073 yds.; (ii) 614 yds.
5. (i) 3337 m.; (ii) 8046 m. **7.** (i) 1·60 m.; (ii) 1·00 m.
8. 500 m. **9.** (i) 0·25000; (ii) 0·16667.
10. 0·648721. **11.** 0·346574. **12.** 3·1416.

XX. d. Page 424. **1.** (i) ±0·048; (ii) ±0·127.
2. (i) less than ±0·005; (ii) and (iii) less than ±0·0003.
4. Error = ±0·006, roughly.
5. 730 sq. metres, correct to nearest sq. metre.
6. 9480 sq. cm.; 46 sq. cm. **7.** 62·9 yds.; 0·08 yd.
8. 1·000429. **9.** 1·0054. **10.** 0·99284.
11. 1·0076. **12.** 1·00048. **13.** 1·017.
14. 1·06. **15.** 1·0015. **16.** 0·9998.
17. 4·9 %, nearly. **18.** 2·69 %; 1·34 %, nearly.
19. 0·0033, nearly. **20.** 3*p*, approx. **21.** 2·4 %.
22. 11206·25 cu. cm.; 2·4 %. **23.** 0·5 sq. cm., nearly.
24. 10,018 sq. ft. **25.** 8·04 cu. ft.

XXI. a. Page 429. **16.** 4. **17.** 8. **18.** 4.
19. 27. **20.** 4. **30.** 9. **31.** ·001.
32. ·001. **33.** ·001. **34.** ·001. **35.** 2143.
36. ·02143. **37.** 214300. **38.** ·0002143. **39.** ·002143.

XXI. b. Page 431. **1.** $5=\log_2 32$. **2.** $7=\log_3 2187$.
3. $4=\log_5 625$. **4.** $3=\log_7 343$. **5.** $2^7=128$.
6. $4^5=1024$. **7.** $3^6=729$. **8.** $a^c=b$.
9. $\log a-\log b-\log c$. **10.** $2\log a-\log b-3\log c$.
11. $\frac{1}{2}\log a+\frac{2}{3}\log b-5\log c$. **12.** $\frac{1}{4}\log b+\frac{1}{3}\log c-\frac{3}{2}\log a$.
13. $2\log 3+2\log 2$. **14.** $-3\log 3-2\log 2$.
15. $\frac{3}{2}\log 2+2\log 3$. **16.** $\frac{19}{6}\log 3+\frac{1}{2}\log 2$.

XXI. c. Page 435.

1. 3, 2, 0, $\bar{1}$, $\bar{1}$, $\bar{4}$, 1. **2.** 1·5745, $\bar{3}$·5745, 6·5745, $\bar{1}$·5745.
3. 2·9064, 4·9064, 0·00009064; 806100, 80·61, ·008061, ·8061.
4. $\bar{2}$·1043. **5.** $\overline{14}$·0476. **6.** $\overline{37}$·7984. **7.** $\bar{3}$·8291.
8. 0·9342. **9.** 2·8841. **10.** $\bar{7}$·6538. **11.** 2·3979.
12. $\bar{1}$·3141. **13.** $\bar{1}$·6354. **14.** $\bar{1}$·8253. **15.** $\bar{2}$·5979.

XXI. d. Page 438.

1. 4·994. **2.** 1·521.
3. 3008. **4.** ·7573. **5.** 467·3. **6.** 13·60.
7. ·05868. **8.** ·07612. **9.** 2·429. **10.** ·03055.
11. 1·923. **12.** 1·444. **13.** 19·97. **14.** 2·258.
15. 2·224. **16.** ·00008855. **17.** 1·784. **18.** 2·008.
19. $2{\cdot}391 \times 10^8$. **20.** $3{\cdot}908 \times 10^5$. **21.** $1{\cdot}772 \times 10^6$.
22. $8{\cdot}235 \times 10^5$. **23.** $3{\cdot}711 \times 10^5$. **24.** 4·354.
25. $4{\cdot}130 \times 10^6$. **26.** $6{\cdot}449 \times 10$. **27.** $2{\cdot}510 \times 10^{-1}$.
28. 5·802. **29.** $6{\cdot}900 \times 10^{-3}$. **30.** $6{\cdot}806 \times 10^{-4}$. **31.** 16.

XXI. e. Page 441.

1. £985. **2.** £352.
3. £1793. **4.** 20 years. **5.** 21 years. **6.** 3,794,000.
7. 9·29, 2560. **8.** 38·53. **9.** 3319 Kg. **10.** 1·23 sec.
11. 3·9 cm. **12.** 22·58 cm. **13.** 1427. **14.** 329.
15. 0·2905. **16.** 551 lbs. **17.** 11·3.
18. 10·2 cm., 12·1 cm., 15·2 cm. **19** 28 years. **20.** 0·3958 cm.
21. 716,900 m. **22.** 85·29 gallons.
23. 4·743, 0·06413. **24.** 280 sq. ft

Miscellaneous Examples VI. Page 454.

1. 2 p.m.; 2.52 p.m. **2.** 6 p.m.; (i) 3.30 p.m.; (ii) 7.30 p.m.
3. 47 mi. from *A*'s starting place at 12.42 p.m. 11.12 a.m. and 2.12 p.m.
4. (i) £80; (ii) £240. **5.** 16*s*. $9\frac{1}{4}$*d*. **6.** 16·7 gallons per head.
7. £18,880. **8.** 450 yds.
9. 9 mi. from Y at 12.48 p.m. 12.18 p.m. and 1.18 p.m.
10. 40 yds. *A* 16 yds. ahead, *C* 16 yds. behind. **11.** 5 secs.
12. 2 ft. **13.** £7. 3*s*. **14.** £3. 1*s*. 3*d*. **15.** £6. 18*s*. 9*d*.
16. 62. **17.** 7.36 p.m.; 3 p.m. and 5 p.m.; 19 mi. from Y.
18. 4 miles. **19.** 5 hrs. from the start.
20. 0·447 %. **21.** £430. 12*s*. $4\frac{1}{2}$*d*.
22. $765 = 3^2 \times 5 \times 17$; $7931520 = 2^7 \times 3^6 \times 5 \times 17$.

$$\left.\begin{matrix} 765 \\ 765 \times 2^7 \times 3^4 \end{matrix}\right\}. \qquad \left.\begin{matrix} 765 \times 2^7 \\ 765 \times 3^4 \end{matrix}\right\}.$$

23. 24 ft., 36 ft. **24.** £9294. 17*s.* 5*d.* **25.** $31\frac{1}{4}$ %.
26. £692. 19*s.* 9*d.* **27.** 1 lb. 2 oz., 1 lb. 14 oz., 2 lbs. 13 oz.
28. 7 hrs. 3.8 p.m. approx. **29.** 248 lbs., 296,500 lbs. nearly.
30. 35 mi. from London at 3.33 p.m. 3.9 p.m.; 3.51 p.m.; 36 mi.
31. £5. 17*s.* 1*d.* **32.** £4. 12*s.* **33.** £11.
34. 201,000,000 bush., 310,000,000 bush., 6 bush., 8 bush.
35. £690; 120 boys. **36.** £420. 20 for £480. **37.** 45 %.
38. 428·456 Kg. **39.** 4 %, 23 farthings.
40. 41·28 gals., 18·72 gals. **41.** 45 days. **42.** fr. 826·04.
43. £16. 16*s.* **44.** 28 hrs. 1 min. 14 secs. **45.** 45 miles.
47. 31,527. **48.** 20 : 31. **50.** £30.
51. Multiply by $\frac{5}{3}$. **52.** Between B and C, 3 mi. from B.
53. (i) *A* is $7\frac{1}{2}$ yds. behind; (ii) *B* is 6 yds. ahead; (iii) 96 yds.
54. 5 miles. **55.** 14·97 in. **56.** £2014. **57.** 30 days.
58. 6·08 %. **59.** 30 tons. **60.** 2·7. **61.** £141. 9*s.*
62. 50. **63.** 73,350. **64.** $\frac{3}{400}$ in.; $14\frac{17}{20}$ in.
65. 81,000 litres. **66.** 5 mi. per hour. **67.** 4 months later.
68. 5·5 %. **69.** £4000. **70.** 13*s.* 11*d.* **71.** 1·585 %.
72. The 2nd is 330 yds. from starting point. **73.** 5·04 %; 4·90 %.
… 3rd … 66 ………………………… . **74.** 10·97 c. per Km.
75. 30 mi., 40 mi.; 55 min. **77.** 120 days. **78.** £1066. 13*s.* 4*d.*
79. 6·015. **80.** Elder £7070, younger £5050. **81.** 7848 Kg.
82. £193. 7*s.* 1*d.* **83.** Between 5 and 10 miles. 60 mi. per hr.
84. 18·4 lbs. **85.** £13. **86.** (i) 0·09 %; (ii) 0·18 %.
87. 7·4 mm. **88.** £17. 10*s.*, £70, £157. 10*s.*
89. $15\frac{15}{28}'$ past noon; 1 hr. $2\frac{14}{23}'$. **90.** 112·64 yds. **91.** £605
92. 96·8 %, 2·66 %, 0·523 %. **93.** £3. 13*s.* 9*d.*
94. £303. 15*s.* **95.** 6300, 8150.
96. W. to N. 52 mi. per hr. N. to R. 39 mi. per hr. $59\frac{1}{2}$ mi.; 8.33 p.m.
97. £415. 5*s.* **98.** 37·5 Km. **99.** 5. **100.** 1·097.
101. 3·09. **102.** £29. 7*s.* 9*d.* **103.** $216\left(\frac{431}{432}\right)^{20}$. **104.** 20.
105. 23·37 in., 25·87 in., 25·70 in., 28·54 in., 33·34 in., 26·98 in.
106. 225 yds. **107.** £75. **108.** 16 cwt. **109.** $16\frac{1}{2}$ %.
110. $1\frac{1}{4}$ %. **111.** *A* £105, *B* £225, *C* £240.
112. 70 %, 30 %. **113.** £17. 10*s.* **115.** 0·028; 0·0078 Kg.
116. 81,097 sq. miles more. **118.** 3608 ft.
119. 24 min. **120.** 6 days. **122.** 185 oz.
123. 9*s.* $4\frac{1}{2}$*d.*, 5*s.* $7\frac{1}{2}$*d.*, 1*s.* $10\frac{1}{2}$*d.*, $2\frac{1}{2}$*d.*, $1\frac{1}{2}$*d.*, 1*d.* **124.** 617 or 167.
125. $\frac{3}{4}$ hr. **127.** 75 lbs. **128.** 53.
129. £1. 17*s.* 7*d.* **130.** 104 million gallons. **131.** 4·2 mi. per hr.

132. £5. 2*s*. 7*d*. **133.** 60, 70, 84, 105. **134.** £625.
135. 637 yards. **136.** 13·2 %. **137.** £116. 13*s*. 4*d*.
138. 0·703. **139.** £1040, £700.
140. $\frac{7}{34}$ water, $\frac{27}{34}$ other liquid. **141.** $37\frac{1}{2}$ miles.
142. 4·98 litres. **143.** 49·27. **144.** (i) 8·134 ; (ii) 1·229.
145. £614. 10*s*. **146.** $18\frac{2}{9}$ %. **147.** 308 lbs.
148. Between 2373 and 2387.
149. 12·5 grams bluestone, 7·81 grams lime. **150.** 19 lbs.
151. $2\frac{1}{2}$ miles. **152.** $37\frac{1}{2}$ %. **153.** £587·2. **154.** $166\frac{2}{3}$ min.
155. £171,429. **156.** 4.12 p.m. **157.** 61·1 of 1st, 38·9 of 2nd.
158. 59 in., 40 in. **159.** 5*s*. 11*d*. ; $7\frac{2}{3}d$. **160.** 10·4 in.
161. 75 in Latin, 80 in Greek. 74 and 50. **162.** 7·55 in. ; 6 in.
163. 1100 ft. per sec. **164.** 102·7 hrs.
165. 2nd station out from terminus. **166.** 1·02 grams.
167. $a=0{\cdot}314$, $b=90{\cdot}3$. $P=200$; $W=215$. **168.** 61 Kg., 8 Kg.
169. $C=0{\cdot}1838''$ by rule (*a*), $=0{\cdot}1836''$ by usual formula ;
$d=121{\cdot}8''$ by rule (*b*), $=121{\cdot}8''$ by usual formula.
170. 2·25 cu. ft. ; 31,700, to the nearest hundred.
171. 1,890,000 litres. **172.** £5. 3*s*. 1*d*. **173.** $1\frac{3}{4}$ % ; £124,490. 6*s*.
174. Yes. **175.** 17·5 lbs. **176.** 1·0025 %.
177. 8 : 17. **178.** 3·2 %. **179.** £62. 10*s*.
180. 12·6 in. **181.** 0·54. **182.** 211 m.
183. 12 %. **184.** 13 gallons wine, 19 gallons water.
185. 277 Kg. £4. 12*s*. 4*d*.

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